Physics 535 - Problem Set 11

Muon Collider

The Higgs couplings to fermions are proportional to mass so direct h production in e^+e^- colliders is suppressed. Suppose a $\mu^+\mu^-$ collider with $L = 10^{33}$ cm^{-1} − s^{-1}. The resonance cross section for $\mu\mu \Rightarrow h \Rightarrow X$ is

$$\sigma = \frac{4\pi\Gamma_{h\Rightarrow\mu\mu}\Gamma_{h\Rightarrow X}}{(s - m_h^2)^2 + m_h^2\Gamma_h^2}$$

For m_h= 100 GeV, estimate the inclusive event rate at the peak assuming the beam energy spread is small compared to the width of the resonance.

$$\sigma = \frac{4\pi}{m_h^2}\frac{\Gamma_{\mu\mu}}{\Gamma_h} = \frac{4\pi}{m_h^2}B_{h\Rightarrow\mu\mu}$$

A 100 GeV Higgs decays dominately to $b\bar{b}$ in three colors. The muonic branching fraction is

$$B_{\mu\mu} = \frac{m_\mu^2}{3(m_b^2 + m_c^2 + m_s^2 + m_u^2 + m_d^2) + m_\tau^2 + m_\mu^2 + m_e^2} \simeq \frac{1}{3}(\frac{m_\mu}{m_b})^2$$

so

$$\sigma = \frac{4\pi}{3}(\frac{m_\mu}{m_b})^2\frac{1}{m_h^2} \simeq 4(\frac{0.105}{5})^2(\frac{.2\text{ GeV} - \text{fm}}{100\text{ GeV}})^2 = .6 \times 10^{-34}\text{ cm}^2$$

and the event rate is 0.06 Hz. The rate of hadronic events from the nearby Z pole is

$$\sigma_{ee\Rightarrow Z\Rightarrow hadrons} = \frac{3\pi}{p^2}\frac{m_Z^2\Gamma_{Z\Rightarrow\mu\mu}\Gamma_{Z\Rightarrow had}}{(s - m_Z^2)^2 + m_Z^2\Gamma_Z^2}$$

is a factor of ten larger. The Higgs width is 5e-5 while the beam energy spread is around 1e-3 at LEP. Taking the LEP energy spread as representative, the Higgs peak would be smeared and peak excess reduced by at least a factor 10.

Z decay

Calculate the percentage change in the width of the Z boson if it can decay to an additional 4th generation light (massless) neutrino.

The partial width to a neutrino pair is ($S = \sin^2\theta_W$, $C = \cos^2\theta_w$)

$$\Gamma_\nu = \alpha m_Z/(24SC)$$

The rate to charged leptons is

$$\Gamma_{\bar{e}e+\bar{\mu}\mu+\bar{\tau}\tau} = 3(4S^2 + (2S-1)^2)\Gamma_\nu$$

The rate to up like quarks is

$$\Gamma_{\bar{u}u\bar{c}c} = 3[1 + (1 - \frac{8}{3}S)^2]\Gamma_\nu$$

The rate to down like quarks is

$$\Gamma_{\bar{d}d+\bar{s}s+\bar{b}b} = \frac{9}{2}[1 + (1 - \frac{4}{3}S)^2]\Gamma_\nu$$

The fractional change is for $N_\nu = 3$

$$\begin{aligned}\frac{\Delta\Gamma}{\Gamma} \quad &= \Gamma_\nu / [N_\nu \Gamma_\nu + \Gamma_{\bar{e}e+\bar{\mu}\mu+\bar{\tau}\tau} + \Gamma_{\bar{u}u\bar{c}c} + \Gamma_{\bar{d}d+\bar{s}s+\bar{b}b}] \\ &= 1/[21 - 40S + \frac{160}{3}S^2] \simeq 0.067\end{aligned}$$

Spectator Model for Charmed Hadron Decays

Assume the decay rate of a hadron containing a c plus light quarks is dominated by $c \Rightarrow sW^+$ with $W^+ \Rightarrow (e^+\nu_e), (\mu^+\nu_\mu), (u\bar{d})$ the latter in three colors. The decay rate is

$$\Gamma_{c\to x} = \frac{5}{192\pi^3} G_F^2 m_c^5 |V_{cs}|^2 f(m_s^2/m_c^2)$$

with $f(x) = 1 - 8x + x^3 - x^4 - 12x^2 \ln x$. Take $m_s = m_\phi/2$ and $m_c = m_{J/\psi}/2$ to predict the lifetime of any charmed hadron and compare to $\tau_{\Lambda_c^+ = udc} = 0.206 \pm 0.012$ ps, $\tau_{D^+ = c\bar{d}} = 1.057 \pm 0.015$ ps, $\tau_{D^0 = c\bar{u}} = 0.415 \pm 0.004$ ps.

The $c \Rightarrow sW \Rightarrow sf\bar{f}$ decay differs from $\mu \Rightarrow \nu W \Rightarrow \nu(e\nu)$ and the function f is a bit different but numerically the effect of a heavy final fermion is similar. We have $x = (m_s/m_c)^2 = (.510/1.548)^2 = 0.108$, and f=0.45,

$$\tau = \frac{\tau_\mu}{5f}(\frac{m_\mu}{m_c})^5 = 2.2 \times 10^6 \text{ps} \frac{1}{0.457 * 5}(\frac{.105}{1.548})^5 = 1.41 \text{ ps}$$

The lifetime is shorter presumably due to nonspectator diagrams. For example, in D^0 decay, the $c \Rightarrow sW^+$ can be followed by internal absorption $W^+\bar{u} \Rightarrow d$. $W \Rightarrow cb$

Show that

$$\Gamma_{W\Rightarrow cb} \quad = \frac{g^2}{16\pi}|V_{cb}|^2 m_W (1 - 2(x_c + x_b + x_c x_b) + x_c^2 + x_b^2)^{\frac{1}{2}}$$
$$\times (1 - \frac{x_b + x_c}{2} - \frac{(x_b - x_c)^2}{2})$$

The decay is similar to $W \Rightarrow e\nu$ but both fermions have mass. The decay rate follows from the expression for $W \Rightarrow e\nu$ with $E_\nu \Rightarrow (E_b + p)/2$, $E_e \Rightarrow E_c$:

$$d\Gamma = N_c \frac{g^2}{32\pi^2 m_W^2} \frac{1}{2}(E_b + p)p[(E_c + p) - 2p\cos^2\theta]d\Omega$$

with appropriate kinematics for p, E_c, E_b.