Physics 535 - Problem Set 7

Scalar Bhabha scattering

Show that the amplitude for scalar $e^+e^- \Rightarrow e^+e^-$ is

$$M = e^2(\frac{u-s}{t} + \frac{u-t}{s})$$

and the differential cross section in the cm is

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s^3}\frac{1}{S^2}[\frac{s^2}{s-4m^2} + s(1-2S+2S^2) + 4m^2S(1-2S)]^2$$

where $S = \sin^2\frac{\theta^{CM}}{2}$. Hint: show that for elastic scattering of identical particles, $t = -(s-4m^2)S$ and $u = -(s-4m^2)(1-S)$.

For $e_2^+e_1^- \Rightarrow e_4^+e_3^-$ the single photon exchange scattering amplitude plus the annihilation-reformation amplitude is

$$M/e^2 = \frac{(-p_2-p_4)(p_1+p_3)}{(p_2-p_4)^2} + \frac{(-p_1-p_2)(p_3+p_4)}{(p_1+p_2)^2}$$

The Mandelstam variables are

$$s = (p_1+p_2)^2 = (p_3+p_4)^2$$

$$t = (p_3-p_1)^2 = (p_4-p_2)^2$$

$$u = (p_1-p_4)^2 = (p_3-p_2)^2$$

and with $p_1^2 = p_2^2 = p_3^2 = p_4^2 = m^2$ we have e.g. $u - s = -2p_1p_4 + 2p_1^2$ and

$$M/e^2 = \frac{u-s}{t} + \frac{u-t}{s}$$

In the cm we have where all particles have energy E and momentum p

$$t = 2m^2 - 2p_3p_1 = 2m^2 - 2(E^2 - p^2\cos\theta) = 2p^2(1-\cos\theta) = 4p^2\sin^2\frac{\theta}{2} = 4p^2S$$

and

$$s = 2m^2 + 2p_1p_2 = 2m^2 + 2(E^2+p^2) = 4m^2 + 2p^2$$

while

$$u = -(s-4m^2)(1-S)$$

The differential cross section is

$$\frac{d\sigma}{d\Omega} = \frac{M^2}{64\pi^2 s}$$

and substitution and simplification gives the required result.

Annihilation of fermions to bosons

Electron-positron annihilation into a virtual photon could produce a pair of heavy charged spin 0 point particles $e^+e^- \Rightarrow h^+h^-$. Combine a fermion annihilation current j_e and a charged scalar current j_h to derive the matrix element $M \propto e^2 j_f j_h/s$ then derive the cm differential cross section neglecting m_e

$$\frac{d\sigma}{d\cos\theta} = \pi\frac{\alpha^2}{4s}(1 - \frac{4m_h^2}{s})\sin^2\theta$$

The matrix element for $e_2^+ e_2^- \Rightarrow h_4^+ h_3^-$ is

$$iM = [(-ie)\bar{v}_2\gamma^\mu u_1]\frac{-ig_{\mu\nu}}{(p_1+p_2)^2}[(ie)(p_3-p_4)^n u] = e^2 j_f j_h/s$$

In the limit $m_e = 0$, the fermion transition current conserves helicity r=t and in the cm $j_f = 2E[0,1,it,0]$ while $j_h = [0, 2\mathbf{p}_3]$ so $j_f j_h = 4E(p_x + itp_y) = 2\sqrt{s}(p_x + itp_y)$ and

$$|M|^2 = (\frac{e^2}{s})^2 4s\mathbf{p}^2\sin^2\theta$$

where $\mathbf{p}^2 = s - 4m_h^2$. The differential cross section follows from

$$\frac{d\sigma}{d\Omega} = \frac{M^2}{64\pi^2 s}\ ;\ \frac{d\sigma}{d\cos\theta} = \frac{M^2}{32\pi s}$$