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THERMAL NOISE

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Abstract

The aim of this experiment is to observe the thermal noise in a resistor, to verify that the mean square noise voltage is proportional to the absolute temperature, and to obtain an an experimental value for the Boltzmann constant.

Theory

J.B. Johnson discovered that any resistor exhibits a small random alternating e.m.f. (now called Johnson noise) and that the noise is dependent on the temperature. Nyquist assumed that the noise was due to the thermal agitation of the electrons in the resistor and, using thermodynamics, developed the expression[1]

$$\bar{V}^2 = 4kT \int_{f_1}^{f_2} \text{Real } [Z(f)] df \tag{1}$$

where

- $\bar{V}^2$ is the mean square noise voltage.
- k is Boltzmann's constant
- T is the absolute temperature
- $Z(f)$ is the impedance of the device (a resistor) at the frequency f
- f_1 and f_2 are the frequency limits between which the noise is accepted by the measuring device.

If the device is a simple resistor, this becomes

$$\bar{V}^2 = 4kTR(f_2 - f_1). \tag{2}$$

In this experiment, we will not measure $\bar{V}^2$ directly but will use an amplifier. The amplifier output V_1 is related to the input V by

$$V_1 = A(f)V$$

where A is the frequency dependent amplification. Hence

$$\bar{V}^2 = 4kTR \int_{f_1}^{f_2} A^2(f) df. \tag{3}$$

Apparatus

Resistor

The resistor assembly is mounted in a copper tube (for good thermal conduction) on the end of a thin stainless steel tube (for lows thermal conduction). Two 500 kΩ metal film resistors in series are used and so R=1 MΩ. The nominal precision of the resistors is 1%.

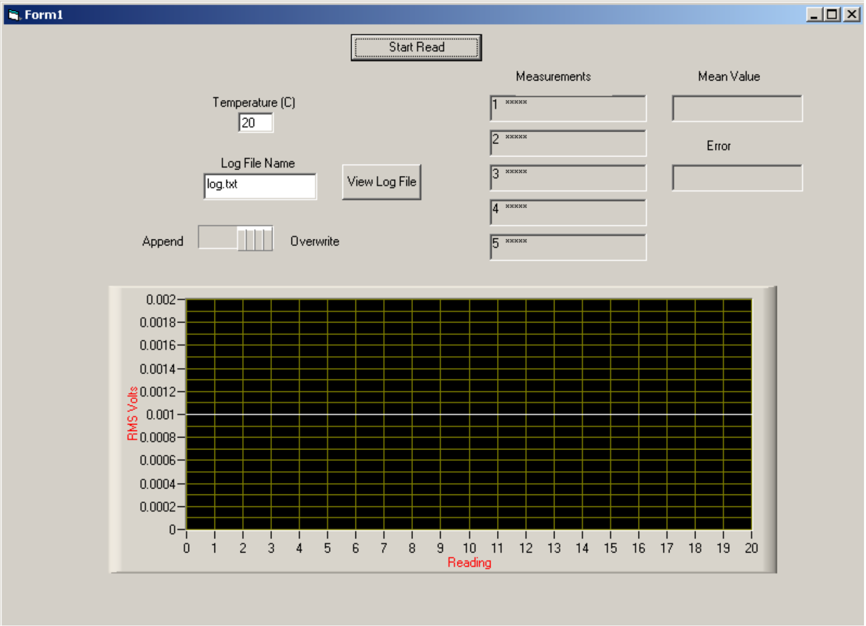
Amplifier (Stanford Research Systems Model SR560)

This solid state amplifier has an input impedance of 100 MΩ shunted by 25 pF and can be run either DC or AC coupled. There is a wide range of gain and bandwidth settings. The noise figure for the amplifier is $< 4\text{n}V/\sqrt{Hz}$ at 1 kHz. This is to be compared to the Johnson noise value of 129 $\text{nV}/\sqrt{Hz}$ at T = 300° K for a 1 MΩ resistor.

RMS Voltmeter (Schlumberger Model SI 7061)

The Schlumberger has a true RMS AC voltage function and is used on the most sensitive scale (100 mV). This voltmeter also has a General Purpose Interface Bus (GPIB) computer interface connected to a GPIB interface card in the PC.

A Visual Basic program THERMAL.EXE (icon on desktop) is used to read the meter and record the readings. The program makes five measurements when you 'Start Read', computes the mean and standard deviation and writes the result to a file. The temperature that you have entered in the temperature box also is written to the file. You can set the file name to a different name if desired. When the file logging is set to 'Append', subsequent readings are appended to the file. Below is the Windows screen that appears when THERMAL.EXE is opened.



Temperature (C) 20

Log File Name log.txt

1 *****
2 *****
3 *****
4 *****
5 *****

<u>Oven</u> (Leybold 200 watt)

The oven is operated from a variable autotransformer (General Radio “Variac”) and is used to heat the resistor up to 150° Higher temperatures will damage the thermocouple.

<u>Thermocouple</u>

An iron-constantan thermocouple junction is mounted next to the resistor and is used to measure the temperature of the resistor. A Keithley Model 610C Electrometer is used to measure the thermocouple voltage. The Handbook of Chemistry and Physics contains thermocouple tables.

<u>Procedure</u>

The resistor output connection is the two BNC connectors on the resistor assembly box, corresponding to the two ends of the resistor. These outputs are connected to the A and B inputs of the preamplifier which is run in the A–B differential mode to eliminate common mode noise. The suggested bandwidth settings are 100 to 1000 Hz, and the suggested gain is 100. Use the preamp in the “low noise” setting.

1. Connect the resistor assembly to the preamp as described above. Look at the output of the preamp on the scope. The signal should be pure white noise with no extraneous 60 Hz pickup. The actual RMS voltage measurements are made using the Schlumberger 7061. Calculate the thermal noise expected at the output of the preamplifier at room temperature. For a simple calculation assume that the amplifier frequency response is simply $(f_{low} - f_{high})$ where A is the amplifier gain and $\mathrm{f_{low}}$ and $\mathrm{f_{high}}$ are the the 6 db low and high roll-off frequencies respectively. Is the actual measured voltage reasonable?

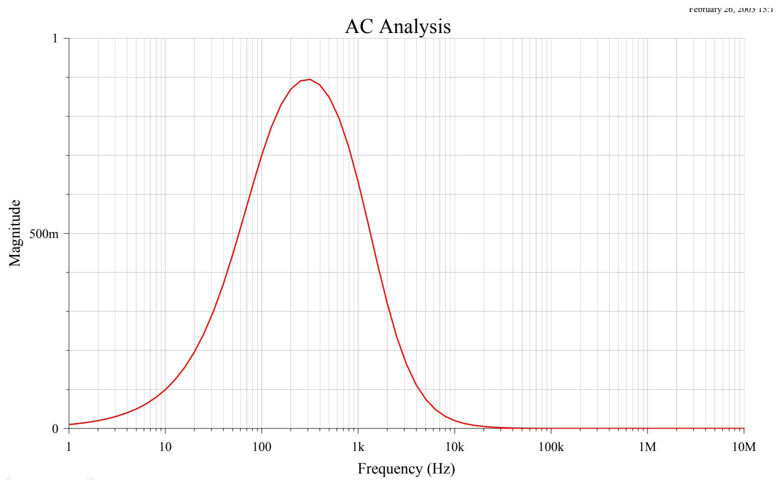
2. Take data above room temperature using the variac controlled oven. Do not exceed 150° C. The experimental fluctuations in the measured voltage will give you guidance as to how to space the data in temperature. Your data will be used to determine the slope and intercept of the V^2 vs. T curve. The intercept at 0° K will be a measure of the noise contributions from other than thermal noise. The slope is used to determine the Boltzmann constant.

3. On the basis of the high temperature data, decide if the result is improved by taking data well below room temperature. Liquid nitrogen can be used to cool the resistor assembly.

4. There are several checks you should make. The resistor value should be measured at the highest and lowest temperatures used during the experiment to check that the resistor maintains the same value. the value should be 1 MΩ to 1%. Also, at some point, the amplifier noise should be measured directly by shorting the A and B inputs and reading the output voltage. This noise voltage should be significantly smaller than the thermal noise voltages you are measuring from the resistor.

5. For analysis, first estimate the bandwidth integral analytically or numerically, using exact analytic expressions for 6 db/octave RC roll-off filters. This procedure will overestimate the bandwidth integral, since the system input capacitance effectively reduces the bandwidth integral by lowering the high frequency roll-off.

   It is also possible to simulate the complete circuit with a SPICE circuit simulation program. The frequency response obtained with such a circuit simulation is shown below.



Measure the bandwidth directly using a Stanford Instruments Model DS345 Function Generator. The output of the Function Generator should be connected through the small insertion box which contains a 1MΩ resistor to correctly simulate the output impedance of the thermal noise circuit. Doing this will correctly account for the low-pass filter representing the front-end input impedance and capacitance. After the bandwidth has been measured, use SigmaPlot to numerically calculate the bandwidth integral.

Once you have obtained the bandwidth integral, k can be determined by using the measured value of the slope of the V^2 vs. T curve together with eqn.(3). Your final value should include a full error analysis.

References

[1] R.E. Simpson, "Introductory Electronics for Scientists and Engineers", 2nd ed., (Allyn and Bacon, Inc., 1974), Sec. 8.3.

[2] P. Horowitz and W. Hill, "The Art of Electronics, Second Edition", (Cambridge University Press, 1989), pp. 430f, 453.