Three Jet Production in Neutral Current Deep Inelastic Scattering with ZEUS at HERA

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Deep Inelastic Kinematics

Neutral Current: \( e^+ p \rightarrow e^+ X (\gamma, Z) \)

\( q \equiv k-k': \) momentum transfer

\( Q^2 \equiv -q^2: \) exchanged boson virtuality

\( x \equiv Q^2 / 2pq: \) momentum fraction carried by struck quark (QPM)

\( y \equiv p \cdot q / p \cdot k: \) fraction of positron energy transferred (in proton rest frame)

\( Q^2 = x \cdot y \cdot s: \) kinematics relation (two degrees of freedom)

\( s = (p+k)^2: \) center of mass energy (fixed value)
“inelastic”:
  • proton breaks up -> quarks…

“deep”: photon $\lambda = 1/|q| \sim 2xM_p/Q^2$
  • large momentum transfer -> “small distance”

“deep inside proton”:
  • probe internal structure of proton
**DIS Cross Section**

\[
\frac{d^2\sigma(e^\pm p \to e^\pm X)}{dx \, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[ Y_+ F_2(x,Q^2) \mp Y_- x F_3(x,Q^2) - y^2 F_L(x,Q^2) \right]
\]

Where \( Y_\pm = 1 \pm (1-y)^2 \)

**Proton Structure Functions**

- \( F_2(x,Q^2) \): Interaction between transversely polarized photons & spin 1/2 quarks; related to the quark and anti-quark densities inside proton.

- \( F_L(x,Q^2) \): Interaction between longitudinally polarized photons & the quarks with transverse momentum; \( F_L = F_2 - 2x F_1 \)

- \( F_3(x,Q^2) \): Parity-violating structure function from \( Z^0 \) exchange; Contribution small for \( Q^2 \ll M_Z^2 = 8100 \text{ GeV}^2 \)
Naïve Quark Parton Model

- Proton is made of “point-like” partons
- No interaction between the partons
- Structure functions do not depend on $Q^2$ since “point-like” structure cannot be “probed” any further

Bjorken Scaling ($x$) dependence:

$$F_i(x, Q^2) \rightarrow F_i(x)$$

$F_L = 0$
Scaling Violation

\[ F_2(x,Q^2) = \sum q \frac{e_q^2(Q^2)}{x^q(x,Q^2) + xq(x,Q^2)} \]

\( F_2 \) has a \( Q^2 \) dependence:

- At small \( x \), strong rise of \( F_2 \) with \( Q^2 \)
- What happened at small \( x \)?
Quarks radiate and absorb gluons.

Gluons split into quark anti-quark pairs:
A “sea” of quark anti-quark pairs,
No longer “point-like” structure

The structure functions gain a $Q^2$ dependence: **Scaling Violation**

Quark and quark interactions mediated by gluons: generate transverse momentum: **Non zero $F_L$**

at small $x$, gluon-driven increase of “sea quarks”
Quarks and gluons cannot be observed as free particles: they are bound by the strong force to form colorless “hadrons”; The energy injected into a hadron creates new quark antiquark pairs and hence additional hadrons

$\alpha_s$ indicates the strength of the strong interaction between quarks and gluons: strong coupling “constant” $\alpha_s$ decreases at short distances and increases rapidly at large distances; In the very short distance (high energy), quarks and gluons are treated as free: Asymptotic Freedom
QCD only calculable at small distances: approximate solution by perturbative expansion of terms proportional to different orders of $\alpha_s$:

$$d\sigma = A_0 \alpha_s^0 + A_1 \alpha_s^1 + A_2 \alpha_s^2 + A_3 \alpha_s^3 + \ldots$$

Leading order (LO): first non-zero order of $\alpha_s$

Next-to-leading order (NLO): second non-zero order

However, divergencies appear in summing self-interactions, when the loop momenta tend to infinity:

Introduce a cutoff $\mu_R$ on the loop momenta $\rightarrow$ Renormalization

$$\alpha_s \text{ dependence on } \mu_R: \alpha_s = 12\pi/[(33-2n_f)\ln(\mu_R^2/\Lambda^2)] \text{ (LO)}$$

$$\mu_R \rightarrow \infty, \alpha_s \rightarrow 0 \text{ (“asymptotic freedom”)}$$

$$\mu_R \rightarrow \Lambda, \alpha_s \rightarrow \infty \text{ (pQCD not valid, “color confinement”)}$$

$\mu_R$ is the scale at which $\alpha_s$ is evaluated (extract $\alpha_s(\mu_R)$!)
Parton Density Functions and Factorization Scale

At large distances, $\alpha_s$ is large: non-perturbative $\rightarrow$ Factorization

$$\sigma_{\text{DIS}} = \sum_a f_a(x, \mu_F^2) \otimes \sigma_a(x, \mu_F^2)$$

Scale $\mu_F$ is introduced to separate long-range (soft) and short-range (hard) processes:

- $K_T > \mu_F$, partons included in pQCD partonic cross-section
- $K_T < \mu_F$, partons absorbed in non-pQCD parton density functions

Several groups derive PDF from experimental data:

CTEQ, MRST, ZEUS-S
Jets

Quarks and gluons can’t be observed in the detector

Jets: colored partons evolve to a roughly collinear “spray” of colorless hadrons

Jet finding:
- Recombining energy deposits from one parton
- merging soft partonic radiation back to emitting parton

Jet algorithm:
- measurable
- calculable (infrared safe)
- accurate
Single Jet Dijet Trijet

Single Jet: $\alpha_s^0$

Dijet: $\alpha_s$

Trijet: $\alpha_s^2$

a) QCD Compton

b) Boson-Gluon Fusion
Motivation for Multijets Study

- Add a gluon radiation to dijet or split a gluon to $q\bar{q}$
  $\rightarrow$ direct test of QCD at $O(\alpha_s^2)$

- In the ratio $\sigma_{\text{trijet}}/\sigma_{\text{dijet}} = O(\alpha_s)$, cancellation of many correlated experimental and theoretical uncertainties.

- Measurement of $\alpha_s$ from $\sigma_{\text{trijet}}/\sigma_{\text{dijet}}$
  $\rightarrow$ first $\alpha_s$ measurement from $O(\alpha_s^2)$ jet rate at HERA

- Multijet NLO Calculations available

(Ref: Phys.Rev.Lett.87:082001,2001)
The proton and exchanged photon collide head-on, with the z-direction chosen to be the proton direction:

$$q + 2xp = 0$$

In single jet events, struck quark rebounds with equal and opposite momentum, the resulting jet has zero $E_T$ (transverse energy).

In multijets events, the outgoing jets are balanced in $E_T$:

$$E_T \rightarrow \text{multijets}$$
HERA Collider

World’s first electron-proton collider located at DESY, Hamburg

- 820/920 GeV proton
- 27.5 GeV electron or positron
- 300/318 GeV center of mass energy
- 220 bunches
- 96 ns crossing time
- 90mA protons
- 40mA positrons

2 collider detectors:
ZEUS, H1
2 fixed target detectors:
HERMES, HERA-B

Integrated luminosity:
HERA I (92-00): 121pb⁻¹, HERA II (03-07): 500pb⁻¹ (exp.)
Extended kinematic region over six-orders of x and $Q^2$

At $\sqrt{s} = 318$ GeV, HERA equivalent to 50 TeV fix-target experiment

H1 and ZEUS: DESY e-p
HERMES: DESY e-A
E665: Fermilab $\mu$-A
BCDMS: CERN $\mu$-A
CCFR: Fermilab $\nu$-A
SLAC: many experiments e-A
NMC: CERN $\mu$-A
ZEUS Detector

Muon systems

Uranium–Scintillator Calorimeter

Drift Chamber Central Tracker

27.5 GeV positrons

820 GeV protons

25 meters
**ZEUS Calorimeter**

Depleted Uranium and scintillator

**Energy Resolution**
- electromagnetic: $18\%/\sqrt{E}$
- hadronic: $35\%/\sqrt{E}$

**Cell Dimensions**
- electromagnetic: 5x20cm (FCAL, BCAL)
- hadronic: 20x20cm

**99.7% solid angle coverage**

$$-3.5 < \eta < 4.0$$

$$\eta = -\log \left[ \tan \left( \frac{\theta}{2} \right) \right]$$

27.5 GeV positrons

820 GeV protons
ZEUS Central Tracking Detector

CTD: Drift chamber inside 1.4T solenoid
Vertex Resolution: 4mm in z-direction, 1mm transvers
DIS Event

\[ Q^2 \sim 3600 \text{GeV}^2 \]
\[ X \sim 0.15 \]
\[ Y \sim 0.20 \]

(1+1) event

Scattered Positron
Proton remnant
Jet

\[ e^+ \]
Remnant
Jet
Low \[ Q^2 \]

\[ e^+ \]
Remnant
Jet
High \[ Q^2 \]
Jet Algorithm

Cone algorithm:
- conceptually simple, fast to run
- ambiguities related to overlapping jets and merging of jets
- infrared unsafe at NNLO
- seeding requirements

\[ R = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2} \]
Maximize \( E_T \) within a cone of radius \( R \)

\[ d_i = E_{T,i}^2 \]

\[ d_{ij} = \min \{ E_{T,i}^2, E_{T,j}^2 \} \cdot (\Delta \eta)^2 + (\Delta \phi)^2 \]
Combine \( i \) and \( j \) if \( d_{ij} \) is smallest of \( \{d_i, d_{ij}\} \)

\( k_T \) algorithm:
- no ambiguity in jets merging
- infrared safe at all orders
- seeding not necessary
- longitudinally invariant \( k_T \) algorithm in inclusive mode
**ZEUS Trigger**

**Challenge**

10 MHz bunch crossing rate  
Extract 10Hz Physics from 100kHz background

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**Level 1**

- Dedicated custom hardware
- **Pipelined and Deadtimeless:** decision made for every bunch crossing (96ns)  
  - ~5µs latency
- **Programable**  
  - Global and regional energy sums  
  - Isolated positron recognition  
  - Track quality information

**Level 2**

- "Commodity" Transputers  
- Calorimeter timing cuts  
- $E - p_z$ cuts (next slide)  
- Initial vertex information  
- Simple physics filters

**Level 3**

- Processor Farm (SGI)  
- Full event available  
- Offline tools available: jet finding, positron id, etc  
- Complete tracking algorithms

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Work on:

- Calorimeter
- First Level Trigger

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**Readout**

10^5Hz → 10^4Hz → 10^3Hz → 10^2Hz → 10^1Hz
\[ E - p_z = \sum_i E_i (1 - \cos \theta_i) \]

Sum runs over calorimeter cells

In a given frame, \( E - p_z \) is conserved

\[ E_{\text{beam}} \rightarrow p_{\text{beam}} \]

Before: \( E - p_z = 2E_{\text{beam}} = 55\text{GeV} \)

Unless energy escapes down rear beam pipe, \( E - p_z \) after collision will be near \( 2E_{\text{beam}} \) for interesting physics at the nominal interaction point

No timing cuts

\( \text{SLTEvents} \)

\( \text{E-}p_z \text{ (GeV)} \)

\( \text{10}^3 \)

\( \text{10}^{-3} \)

\( \text{0} \)

\( \text{10} \)

\( \text{20} \)

\( \text{30} \)

\( \text{40} \)

\( \text{50} \)

\( \text{60} \)

Contained NC DIS

\( E - p_z \approx 55 \)
"Distance" between FCAL and RCAL is ~10ns

On Time Event

Beam Gas Event

Calorimeter timing at Level 2

ZEUS Calorimeter timing resolution <1ns
LO Monte Carlo

LO Matrix Element + Parton Showers (MEPS) or Color Dipole Model (CDM)

- Acceptance corrections: detector independent cross sections
- Hadronization corrections, QED corrections: for NLO calculation
- LEPTO (6.5.1) and ARIADNE (4.0.8) program used

Parton Level

Hadron Level

Hadronization Models
- String Fragmentation (LUND)
- Cluster Model

Detector Simulation
NLO Calculation

- Four parton final states
- Most accurate pQCD calculation up to now
- Soft/collinear and virtual loop divergences cancel

- All finite order calculations have renormalization scale dependence (uncertainty), at NLO, scale uncertainty much reduced
- No hadronization and no high order QED effects
- Use LO MC to get these corrections
- NLOJET program: first NLO program for trijet
- DISENT program: double check NLOJET for dijet
Data Sample: Offline Selection

ZEUS 1998-2000 data

- 82.2 pb\(^{-1}\)
- \(\sqrt{s} = 318\) GeV
- \(E_p = 920\) GeV, \(E_e = 27.5\) GeV

Kinematic Range

- \(10\) GeV\(^2\) < \(Q^2\) < 5000 GeV\(^2\): good acceptance, best use of statistics
- 0.04 < \(Y\) < 0.6: good reconstruction of hadronic system, reduce background
- \(\cos \gamma_{\text{had}} < 0.7\): good reconstruction of jets in Breit frame

Jet Reconstruction

- Invariant KT algorithm in Breit frame (inclusive)
- \(E_{T,jet}^{\text{BRT}} > 5\) GeV for each jet: hard process
- \(-1 < \eta_{\text{jet}}^{\text{LAB}} < 2.5\): jets boosted back to lab to check CTD acceptance
- Invariant mass \(M_{2,3\text{jet}} > 25\) GeV: avoid unphysical theoretical calculation
Lepto provides a good description of data

Data

- Lepto

Area normalized to compare shapes
Dijet Kinematics

- Data
- Lepto

Area normalized to compare shapes

Lepto describes physics and detector, used for acceptance corrections
Trijet Cross Sections: \( E_T^{BRT} \) dependence

CTEQ6 PDF, \( \alpha_s = 0.1179 \)

Scale \( \mu_r^2 = \mu_f^2 = (E_T^2 + Q^2)/4 \)

Jets ordered in \( E_T^{BRT} \)

Measurement down to low \( E_T \)

Scale dependence

Good agreement for each jet

NLOJET describes data over large range of scales
Trijet Cross Sections: $\eta_{LAB}$ dependence

Jets ordered in $\eta_{LAB}$

Angular distribution

Good agreement for each jet

NLOJET describes data over whole $\eta$ range
**Multijet Cross Sections: Q^2 dependence**

**ZEUS**

- **Dijet NLO:** $O(\alpha_s^2)$
- **Trijet NLO:** $O(\alpha_s^3)$

Measurement down to low $Q^2$

Scale dependence

NLOJET describes both dijet & trijet data over 3 orders of magnitude in $Q^2$
Multijet Cross Sections Ratio: $Q^2$ dependence

$R_{3/2} = \frac{\sigma_{\text{trijet}}}{\sigma_{\text{dijet}}}$

Systematic uncertainties substantially reduced

Scale dependence reduced

Very sensitive test of QCD calculation

NLOJET describes data over large range of scales
Multijet Cross Sections in $Q^2$: MRST99 and CTEQ4M PDF

**ZEUS**

- ZEUS (prel.) 98-00 Dijets
- ZEUS (prel.) 98-00 Trijets
- Energy Scale Uncertainty

$\alpha_s = 0.1175$

\[ \frac{d\sigma}{dQ^2} (\text{pb/GeV}^2) \]

NLOJET describes both dijets and trijets over 3 orders of magnitude in $Q^2$ for both PDFs — PDF independence
Trijet to Dijet Cross Section Ratio
$R_{3/2} = \frac{\sigma_{\text{trijet}}}{\sigma_{\text{dijet}}}$

- As expected, predictions are sensitive to $\alpha_s$

**CTEQ4A1**: $\alpha_s = 0.110$

**CTEQ4A2**: $\alpha_s = 0.113$

**CTEQ4M**: $\alpha_s = 0.116$

**CTEQ4A4**: $\alpha_s = 0.119$

**CTEQ4A5**: $\alpha_s = 0.122$

**ZEUS**

*(prel.) 98-00 Energy Scale Uncertainty

$\mu_r^2 / (Q^2 + \mu_r^2) < 1$

$1 < 1/(Q^2 + \mu_r^2)$
Parameterisation of $R_{3/2}$ with the value of $\alpha_s(M_z)$

Procedures:

- Run NLOJET with several $\alpha_s$ values and fit with a linear function for each $Q^2$ bin
- Use this function to establish correlation of $R_{3/2}$ with $\alpha_s(M_z)$
- Extract $\alpha_s$ for each $Q^2$ bin and determine combined value with $\chi^2$-fit.
Extraction of $\alpha_S$ with CTEQ4 PDF

- Data
  ▲CTEQ4
  
- CTEQ4 provides more sets with different $\alpha_S(M_Z)$ than other PDF (i.e. MRST99)

\[
\alpha_S(M_Z) = 0.1179 \pm 0.0013 \text{(stat.)} ^{+0.0028}_{-0.0046} \text{(syst.)} ^{+0.0061}_{-0.0047} \text{(th.)}
\]
Systematic Uncertainties

Experimental (maximum change in any $Q^2$ bin)
- Jet pseudo-rapidity cut: 1%
- Use of different LO MC model: 2%
- Jet transverse energy and invariant mass cuts: 2%
- The absolute energy scale of the CAL: 2.5%
- Other sources which have negligible effects: 0.4%
  - Un-reweighted MC
  - $Z_{\text{Vertex}}$ cuts
  - $Y_{\text{JB}}$ cut
  - $E-P_Z$ cut
  - $\cos\gamma_{\text{had}}$ cut

Theoretical (maximum change in any $Q^2$ bin)
- Hadronisation correction factors: 2%
- Renormalization scale: 5%
- Uncertainties in the proton PDFs: 1.5%

\[ \Delta \alpha_s(M_Z) = +0.0028 -0.0046 \]

\[ \Delta \alpha_s(M_Z) = +0.0064 -0.0046 \]
Other $\alpha_s$ measurements

- Errors competitive
- See comparison with different PDF next page

- Inclusive jet cross sections in $\gamma p$

- Multi-jets in NC DIS
  ZEUS new analysis

- Subjet multiplicity in CC DIS
  ZEUS (Eur Phys Jour C 31 (2003) 149)

- Subjet multiplicity in NC DIS
  ZEUS (Phys Lett B 558 (2003) 41)

- Jet shapes in NC DIS
  ZEUS (DESY 04-072 - hep-ex/0405065)

- NLO QCD fit

- Inclusive jet cross sections in NC DIS
  ZEUS (Phys Lett B 547 (2002) 164)

- Dijet cross sections in NC DIS
  ZEUS (Phys Lett B 507 (2001) 70)

- World average
  (S. Bethke, hep-ex/0211012)
Extraction of $\alpha_S$ with MRST99 PDF

Excellent agreement between MRST99 and CTEQ4M ($\alpha_s(M_Z)=0.1179$), 0.1% difference

$\alpha_S(M_Z) = 0.1178 \pm 0.0010$ (stat.) $^{+0.0021}_{-0.0035}$ (syst.) $^{+0.0048}_{-0.0034}$ (th.)
Summary

- NLO predictions give a good description of the dijet and trijet cross sections and the cross-section ratio $R_{3/2}$ over the whole kinematic range.
- The value of strong coupling constant $\alpha_s$ at the conventional scale of $M_Z=91.9$ GeV is measured to be

$$\alpha_s(M_Z) = 0.1179 \pm 0.0013\,(\text{stat.}) \pm 0.0028\,(\text{syst.}) \pm 0.0064\,(\text{th.})$$

in good agreement with current world average

$$\alpha_s(M_Z) = 0.1182 \pm 0.0027$$
Outlook

- Higher order calculations, e.g. NNLO calculation needed
- LO MC models need to be improved
- HERA II program aims at collecting a high luminosity DIS data of high $E_T$ and $Q^2$
  - Greatly increase statistics, higher precision
  - Improved jet and kinematic reconstruction
  - Lower experimental uncertainties at high $E_T$ and $Q^2$