

Introduction to the Parton Model and Perturbative QCD

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- The Parton Model and Deep-inelastic Scattering
- From the Parton Model to QCD
- Factorization and Evolution
- As time allows . . .

This is an auspicious time for QCD, particle physics and whatever is beyond it.

Large Hadron Collider and Tevatron phenomenology relies on essentially all that follows.

The Context of QCD: “Fundamental Interactions”

- Electromagnetic
- + Weak Interactions \Rightarrow Electroweak
- + Strong Interactions (QCD) \Rightarrow Standard Model
- + ... = Gravity and the rest?
- **QCD: A theory “off to a good start”. Think of ...**
 - $\vec{F}_{12} = -GM_1M_2\hat{r}/R^2 \Rightarrow$ elliptical orbits
... 3-body problem ...
 - $L_{\text{QCD}} = \bar{q} \not{D}q - (1/4)F^2 \Rightarrow$ asymptotic freedom
... confinement ...

I. The Parton Model and Deep-inelastic Scattering

IA. Nucleons to Quarks

IB. DIS: Structure Functions and Scaling

IC. Getting at the Quark Distributions

**ID. Classic Parton Model Extensions:
Fragmentation and Drell-Yan**

Introduce concepts and results that predate QCD, led to QCD and were incorporated and explained by QCD.

IA. From Nucleons to Quarks

- The pattern: nucleons, pions and isospin:

$$\begin{pmatrix} p \\ n \end{pmatrix}$$

– p: $m=938.3$ MeV, $S = 1/2$, $I_3 = 1/2$

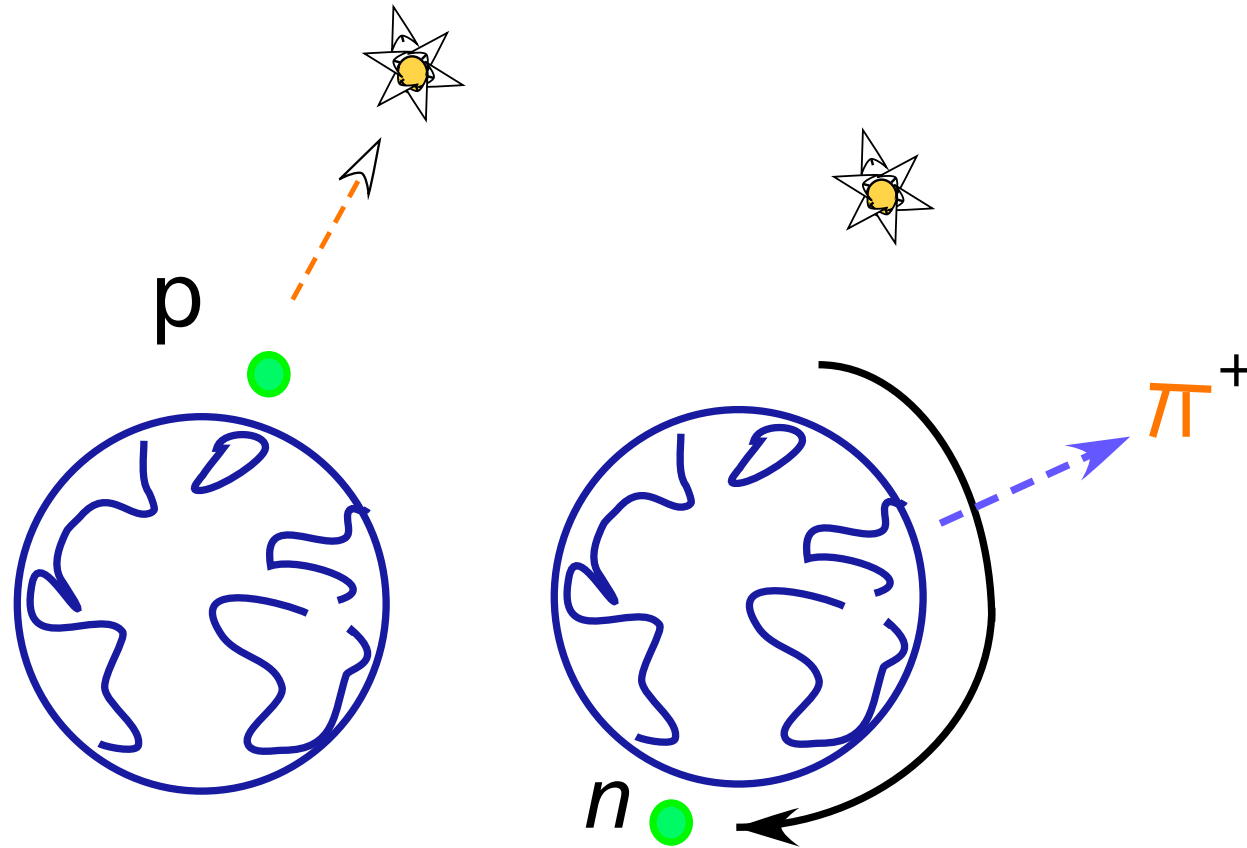
– n: $m=939.6$ MeV, $S = 1/2$, $I_3 = -1/2$

$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$$

– π^\pm : $m=139.6$ MeV, $S = 0$, $I_3 = \pm 1$

– π^0 : $m=135.0$ MeV, $S = 0$, $I_3 = 0$

- Isospin space . . .
- Globe with a “north star” set by electroweak interactions:



Analog: the rotation group (more specifically, $SU(2)$).

- **Explanation:** π , N common substructure: **quarks**

(Gell Mann, Zweig 1964)

- **spin** $S = 1/2$,
 $I = 1/2$ (u, d) & $I = 0$ (s)
with approximately equal masses (s heavier);

$$\left(\begin{array}{l} u \ (Q = 2e/3, I_3 = 1/2) \\ d \ (Q = -e/3, I_3 = -1/2) \\ s \ (Q = -e/3, I_3 = 0) \end{array} \right)$$

$$\pi^+ = (u\bar{d}) , \quad \pi^- = (\bar{u}d) , \quad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) ,$$

$$p = (uud) , \quad n = (udd) , \quad K^+ = (u\bar{s}) \dots$$

This is the quark model

- **Quark model nucleon has symmetric spin/isospin wave function (return to this later)**
- **Early success: $\mu_p/\mu_n = -3/2$ (good to %)**
- **And now, six: 3 'light' (u, d, s), 3 'heavy': (c, b, t)**
- **Of these all but t form bound states of quark model type.**

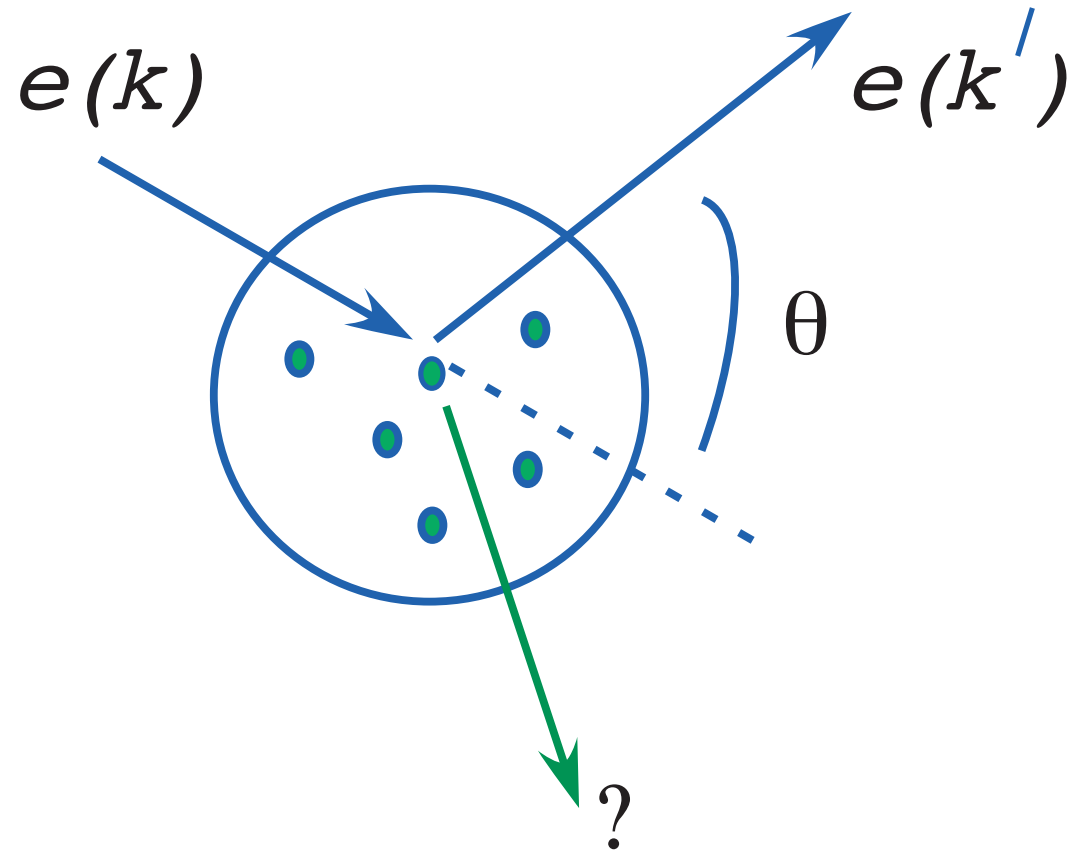
- **Quarks as Partons: “Seeing” Quarks.**

No isolated fractional charges seen (“confinement.”)

Can such a particle be detected? (SLAC 1969)

**Look closer: do high energy electrons
bounce off anything hard? (‘Rutherford-prime’)**

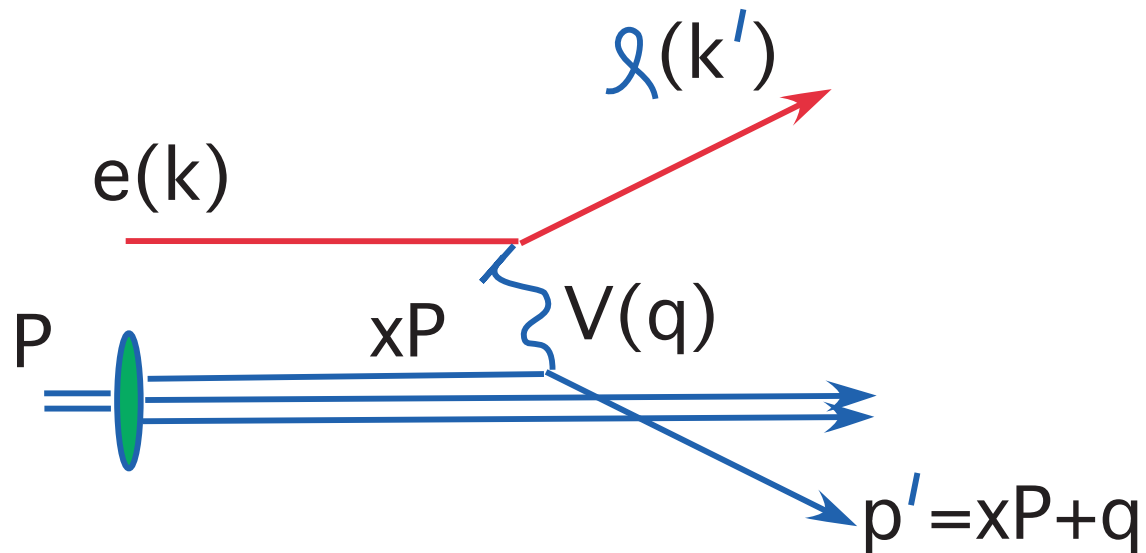
- So look for:



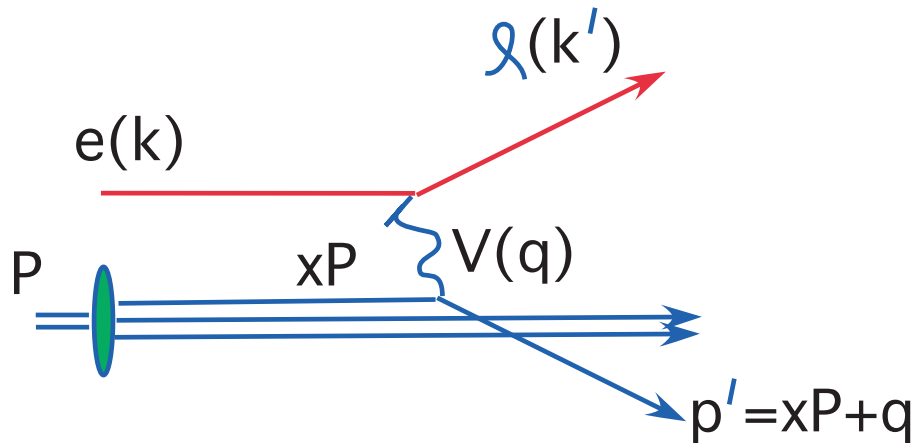
“Point-like’ constituents.

The angular distribution gives information on the constituents.

Kinematics ($e + N(P) \rightarrow \ell + X$)



- $V = \gamma, Z_0 \Rightarrow \ell = e, \mu$, “neutral current” (NC).
- $V = W^-(e^-, \nu_e), V = W^+(e^+, \bar{\nu}_e)$, or $e \rightarrow \mu$ “charged current” (CC).
- $p'^2 \gg m_{\text{proton}}^2$: **Deep-inelastic scattering (DIS)**



$Q^2 = -q^2 = -(k - k')^2$ momentum transfer.

$x \equiv \frac{Q^2}{2p \cdot q}$ momentum fraction (from $p'^2 = (xp + q)^2 = 0$).

$y = \frac{p \cdot q}{p \cdot k}$ fractional energy transfer.

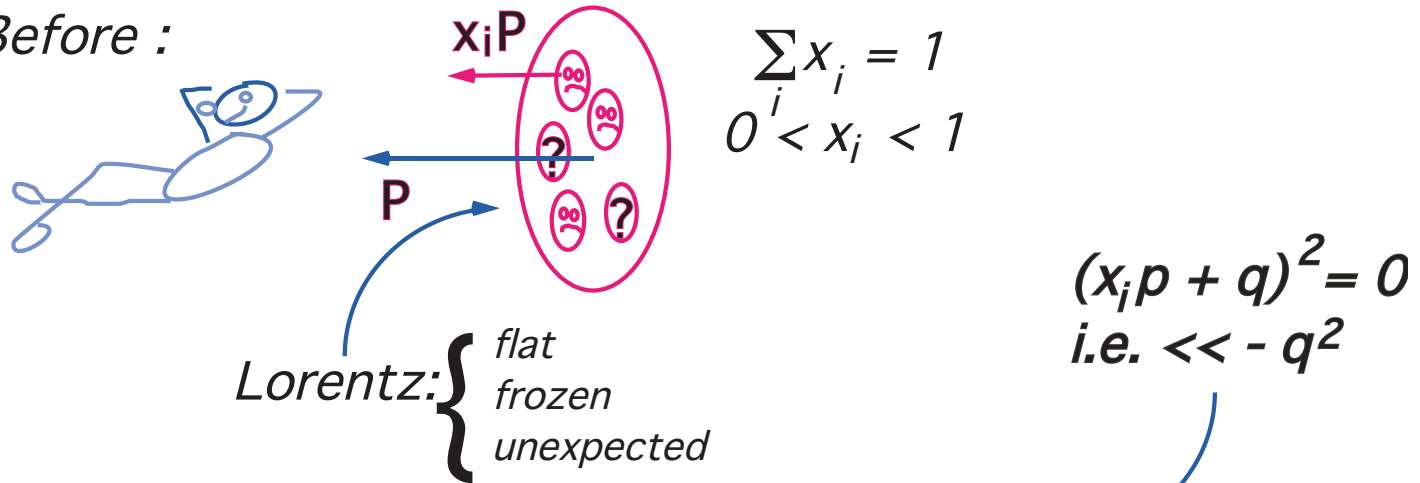
$W^2 = (p + q)^2 = \frac{Q^2}{x}(1 - x)$ squared final-state mass of hadrons.

$$xy = \frac{Q^2}{S}$$

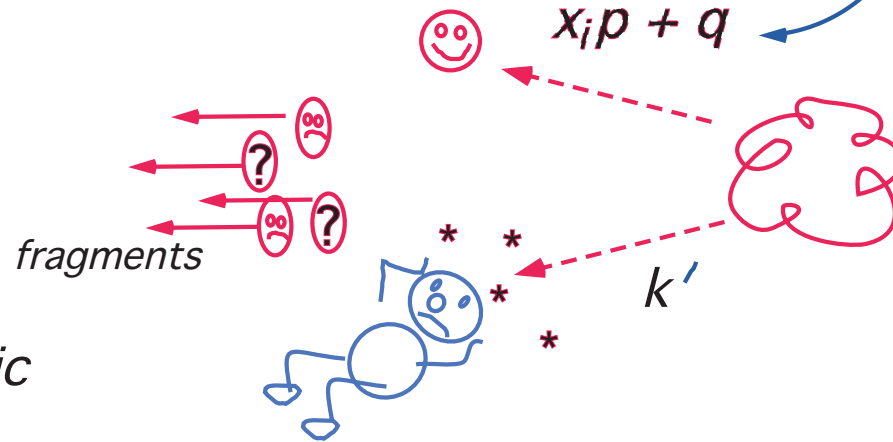
Parton Interpretation (Feynman 1969, 72)

Look in the electron's rest frame . . .

I) *Before* :



II) *After* :



"Deep-inelastic Scattering"

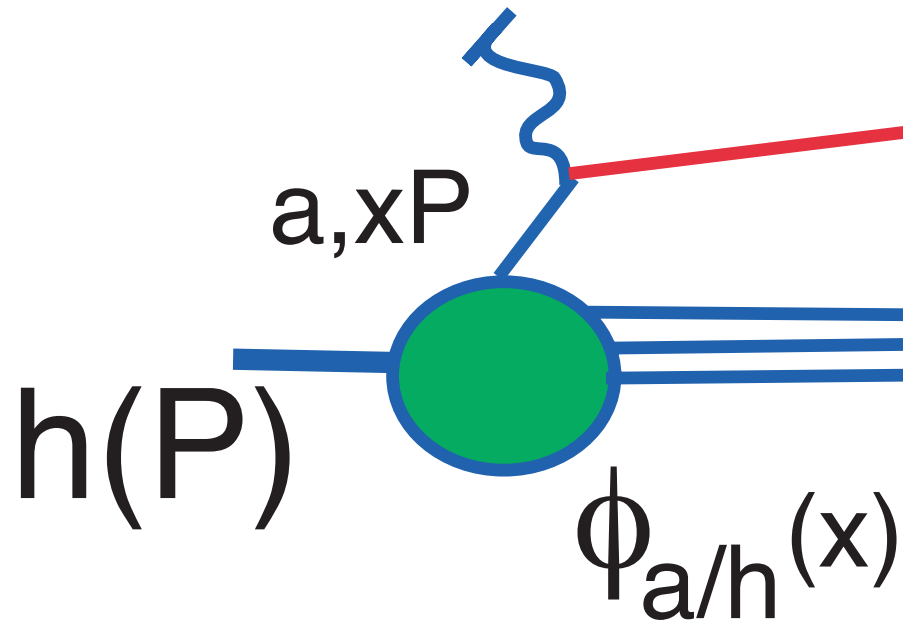
– Basic Parton Model Relation

$$\sigma_{eh}(p, q) = \sum_{\text{partons } a} \int_0^1 d\xi \hat{\sigma}_{ea}^{\text{el}}(\xi p, q) \phi_{a/h}(\xi),$$

- **where:** σ_{eh} is the cross section for $e(k) + h(p) \rightarrow e(k' = k - q) + X(p + q)$
- **and** $\hat{\sigma}_{ea}^{\text{el}}(xp, q)$ is the elastic cross section for $e(k) + a(\xi p) \rightarrow e(k' - q) + a(\xi p + q)$ which sets $(\xi p + q)^2 = 0 \rightarrow \xi = -q^2/2p \cdot q \equiv x$.
- **and** $\phi_{a/h}(x)$ is the **distribution of parton a in hadron h**, the “probability for a parton of type a to have momentum xp ”.

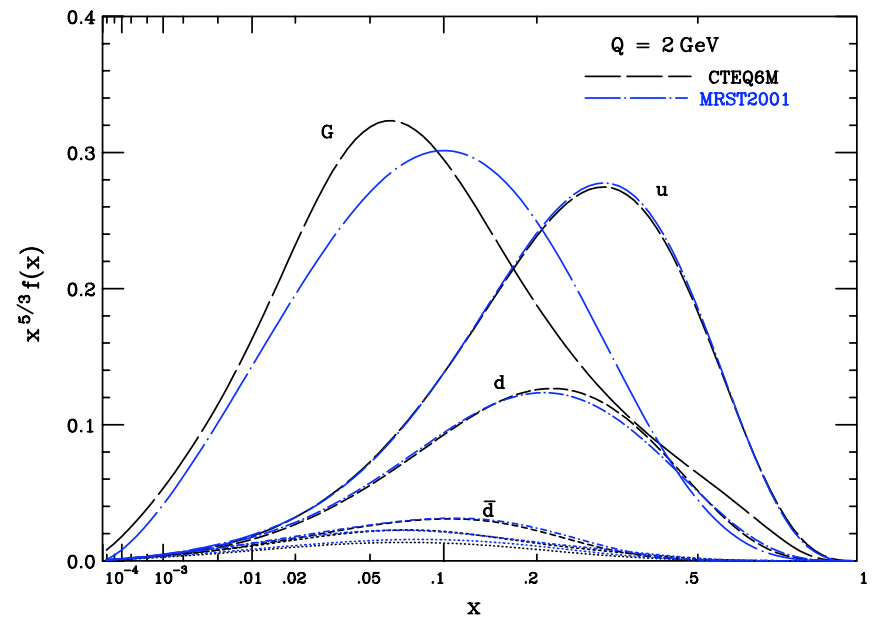
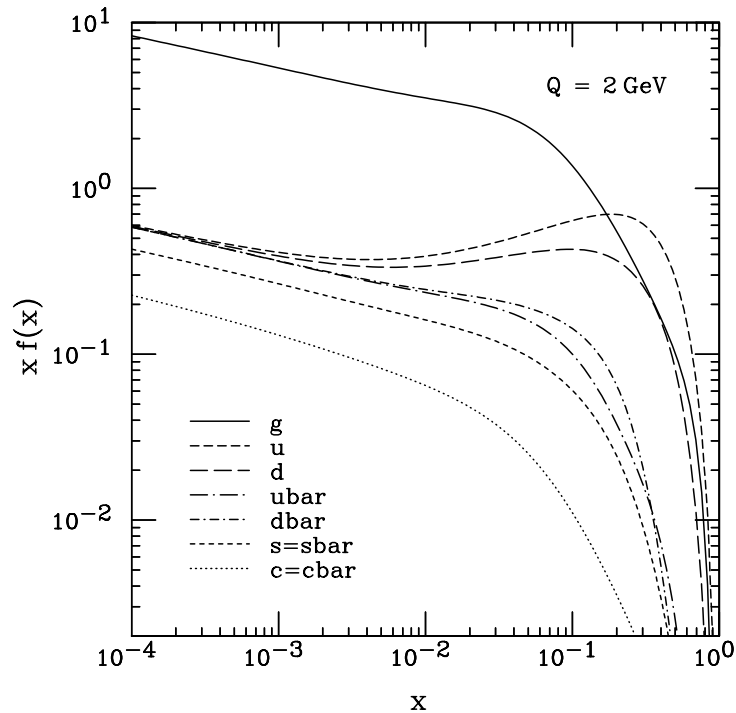
- in words: **Hadronic INELASTIC** cross section is the sum of convolutions of partonic **ELASTIC** cross sections with the hadron's parton distributions.
- The nontrivial assumption: **quantum mechanical incoherence of large- q scattering and the partonic distributions. Multiply probabilities without adding amplitudes.**
- Heuristic justification: the binding of the nucleon involves long-time processes that do not interfere with the short-distance scattering. **Later we'll see how this works in QCD.**

– The familiar picture



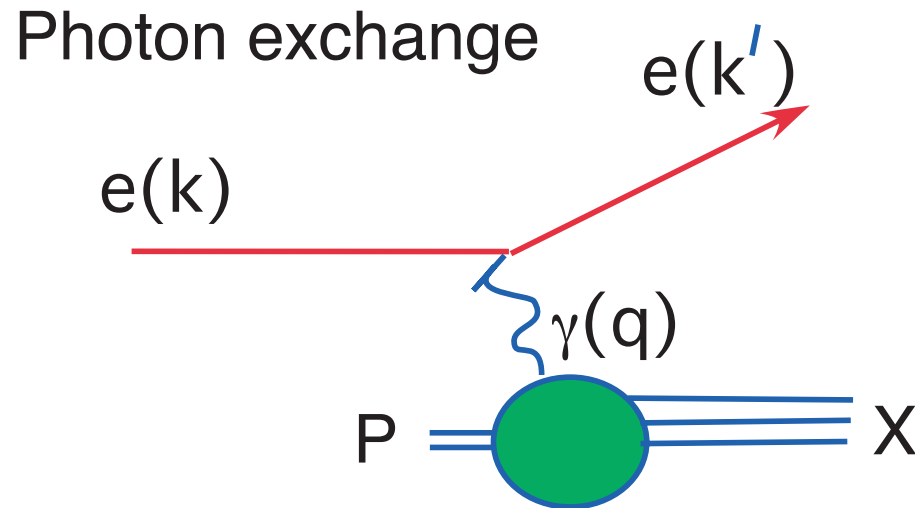
- “QM incoherence” \Leftrightarrow no interactions between of the outgoing scattered quark and the rest.

- Two modern parton distribution sets at moderate momentum transfer (note different weightings with x):



- We'll see where these come from.

IB. DIS: Structure Functions and Scaling



$$\begin{aligned}
 A_{e+N \rightarrow e+X}(\lambda, \lambda', \sigma; q) &= \bar{u}_{\lambda'}(k') (-ie\gamma_\mu) u_\lambda(k) \\
 &\quad \times \frac{-ig^{\mu\mu'}}{q^2} \\
 &\quad \times \langle X | eJ_{\mu'}^{\text{EM}}(0) | p, \sigma \rangle
 \end{aligned}$$

- **Historically** an assumption that the photon couples to hadrons by point-like current operator. **Now, built into the Standard Model.**

- **The cross section:**

$$d\sigma_{\text{DIS}} = \frac{1}{2^2} \frac{1}{2s} \frac{d^3k'}{(2\pi)^3 2\omega_{k'}} \sum_{\lambda, \lambda', \sigma} |A|^2 \times (2\pi)^4 \delta^4(p_X + k' - p - k)$$

In $|A|^2$, separate the known leptonic part from the “unknown” hadronic part:

- **The leptonic tensor:**

$$L^{\mu\nu} = \frac{e^2}{8\pi^2} \sum_{\lambda, \lambda'} (\bar{u}_{\lambda'}(k') \gamma^\mu u_\lambda(k))^* (\bar{u}_{\lambda'}(k') \gamma^\nu u_\lambda(k))$$

$$= \frac{e^2}{2\pi^2} (k^\mu k'^\nu + k'^\mu k^\nu - g^{\mu\nu} k \cdot k')$$

- Leaves us with the hadronic tensor:

$$W_{\mu\nu} = \frac{1}{8\pi} \sum_{\sigma, X} \langle X | J_{\mu} | p, \sigma \rangle^* \langle X | J_{\nu} | p, \sigma \rangle$$

- And the cross section:

$$2\omega_{k'} \frac{d\sigma}{d^3k'} = \frac{1}{s(q^2)^2} L^{\mu\nu} W_{\mu\nu}$$

- $W_{\mu\nu}$ has sixteen components,
but known properties of the strong interactions
constrain $W_{\mu\nu} \dots$

- An example: current conservation,

$$\partial^\mu J_\mu^{\text{EM}}(x) = 0$$

$$\Rightarrow \langle X | \partial^\mu J_\mu^{\text{EM}}(x) | p \rangle = 0$$

$$\Rightarrow (p_X - p)^\mu \langle X | J_\mu^{\text{EM}}(x) | p \rangle = 0$$

$$\Rightarrow q^\mu W_{\mu\nu} = 0$$

- With parity, time-reversal, etc . . .

$$W_{\mu\nu} = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1(x, Q^2) \\ + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) W_2(x, Q^2)$$

- Often given in terms of the dimensionless structure functions,

$$F_1 = W_1 \quad F_2 = p \cdot q W_2$$

- Note that if there is no other mass scale, the F 's cannot depend on Q except indirectly through x .

- **Structure functions in the Parton Model:
The Callan-Gross Relation**

From the “basic parton model formula”:

$$\frac{d\sigma_{eh}}{d^3k'} = \int d\xi \frac{d\sigma_{eq}^{\text{el}}(\xi)}{d^3k'} \phi_{q/h}(\xi) \quad (1)$$

Can treat a quark of “flavor” f just like any hadron and get

$$\omega_{k'} \frac{d\sigma_{ef}^{\text{el}}(\xi)}{d^3k'} = \frac{1}{2(\xi s)Q^4} L^{\mu\nu} W_{\mu\nu}^{ef}(k + \xi p \rightarrow k' + p')$$

Let the charge of f be e_f .

Exercise 1: Compute $W_{\mu\nu}^{ef}$ to find:

$$W_{\mu\nu}^{ef} = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \delta \left(1 - \frac{x}{\xi} \right) \frac{e_f^2}{2} \\ + \left(\xi p_\mu - q_\mu \frac{\xi p \cdot q}{q^2} \right) \left(\xi p_\nu - q_\nu \frac{\xi p \cdot q}{q^2} \right) \delta \left(1 - \frac{x}{\xi} \right) \frac{e_f^2}{\xi p \cdot q}$$

Ex. 2: by substituting in (1), find the Callan-Gross relation,

$$F_2(x) = \sum_{\text{quarks } f} e_f^2 x \phi_{f/p}(x) = 2x F_1(x)$$

And Ex. 3: that this relation is quite different for scalar quarks.

- **The Callan-Gross relation shows the compatibility of the quark and parton models.**
- **In addition: parton model structure functions are independent of Q^2 , a property called “scaling”.**
- **With massless partons, there is no other massive scale. Then the F 's must be Q -independent; see above.**
- **Approximate properties of the kinematic region explored by SLAC in late 1960's – early 1970's.**
- **Explore corrections to this picture in QCD “evolution”.**

IC. Getting at the Quark Distributions

- Relating the parton distributions to experiment
- Simplifying assumptions that illustrate the general approach.

$$\phi_{u/p} = \phi_{d/n} \quad \phi_{d/p} = \phi_{u/n} \quad \text{isospin}$$

$$\phi_{\bar{u}/p} = \phi_{\bar{u}/n} = \phi_{\bar{d}/p} = \phi_{\bar{d}/n} \quad \text{symmetric sea}$$

$$\phi_{c/p} = \phi_{b/N} = \phi_{t/N} = 0 \quad \text{no heavy quarks}$$

- Adequate to early experiments, but no longer.

- **With assumptions above, find for e , ν and $\bar{\nu}$ scattering (see appendix)**

$$F_2^{(eN)}(\mathbf{x}) = 2\mathbf{x}F_1^{(eN)}(\mathbf{x}) = \sum_{f=u,d,s} e_F^2 \mathbf{x} \phi_{f/N}(\mathbf{x})$$

$$F_2^{(W^+N)} = 2\mathbf{x} \left(\sum_{D=d,s,b} \phi_{D/N}(\mathbf{x}) + \sum_{U=u,c,t} \phi_{\bar{U}/N}(\mathbf{x}) \right)$$

$$F_2^{(W^-N)} = 2\mathbf{x} \left(\sum_{\bar{D}} \phi_{\bar{D}/N}(\mathbf{x}) + \sum_U \phi_{U/N}(\mathbf{x}) \right)$$

$$F_3^{(W^+N)} = 2 \left(\sum_{\bar{D}} \phi_{\bar{D}/N}(\mathbf{x}) - \sum_U \phi_{U/N}(\mathbf{x}) \right)$$

$$F_3^{(W^-N)} = 2 \left(- \sum_{\bar{D}} \phi_{\bar{D}/N}(\mathbf{x}) + \sum_U \phi_{U/N}(\mathbf{x}) \right)$$

- **Exercise: derive some of these for yourself.**

- The distributions are actually overdetermined with these assumptions, which checks the consistency of the picture.
- Further consistency checks: Sum Rules.

$$N_{u/p} = \int_0^1 dx \left[\phi_{u/p}(x) - \phi_{\bar{u}/p}(x) \right] = 2$$

etc. for $N_{d/p} = 1$.

The most famous ones make predictions for structure functions . . .

- **The Adler Sum Rule:**

$$\begin{aligned}
 1 &= N_{u/p} - N_{d/p} \\
 &= \int_0^1 dx \left[\phi_{d/n}(x) - \phi_{\bar{u}/p}(x) - \left(\phi_{d/p}(x) - \phi_{\bar{u}/n}(x) \right) \right] \\
 &= \int_0^1 dx \left[\sum_D \phi_{D/n}(x) + \sum_U \phi_{\bar{U}/n}(x) \right] \\
 &\quad - \int_0^1 dx \left[\sum_D \phi_{D/p}(x) + \sum_U \phi_{\bar{U}/p}(x) \right] \\
 &= \int_0^1 dx \frac{1}{2x} \left[F_2^{(\nu n)} - F_2^{(\nu p)} \right]
 \end{aligned}$$

In the second equality, we've used isospin invariance, in the third, all the extra terms from heavy quarks $D = s, b, U = c, t$ cancel.

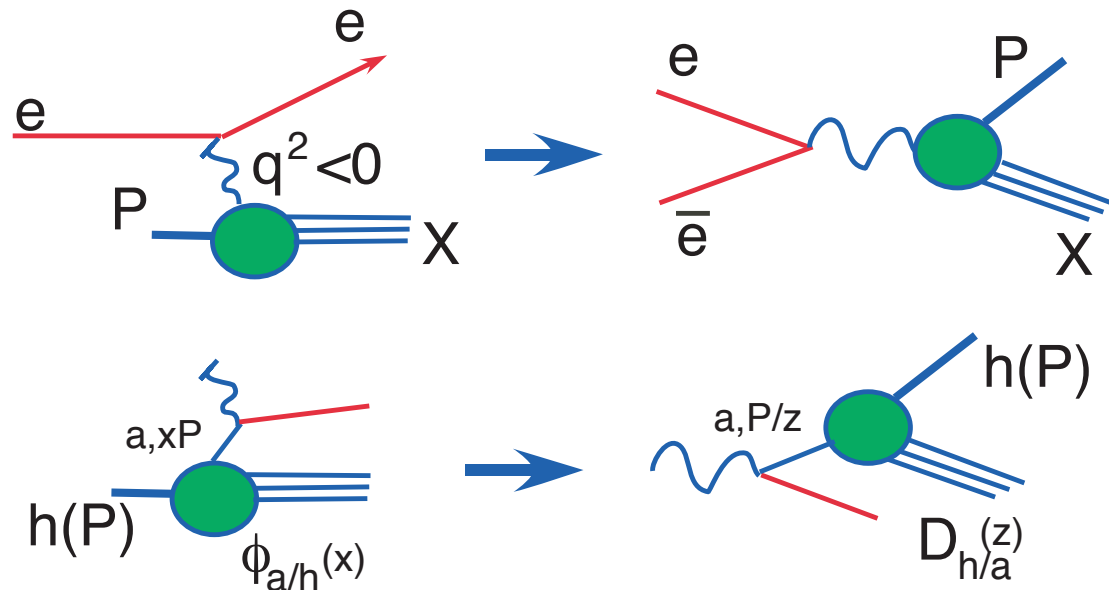
- **And similarly, the Gross-Llewellyn-Smith Sum Rule:**

$$3 = N_{u/p} + N_{d/p} = \int_0^1 dx \frac{1}{2x} \left[xF_3^{(\nu n)} + xF_3^{(\nu p)} \right]$$

ID. Classic Parton Model Extensions: Fragmentation and Drell Yan

- Fragmentation functions
- “Crossing” applied to DIS: “Single-particle inclusive” (1PI)
From scattering to pair annihilation.

Parton distributions become “fragmentation functions” .

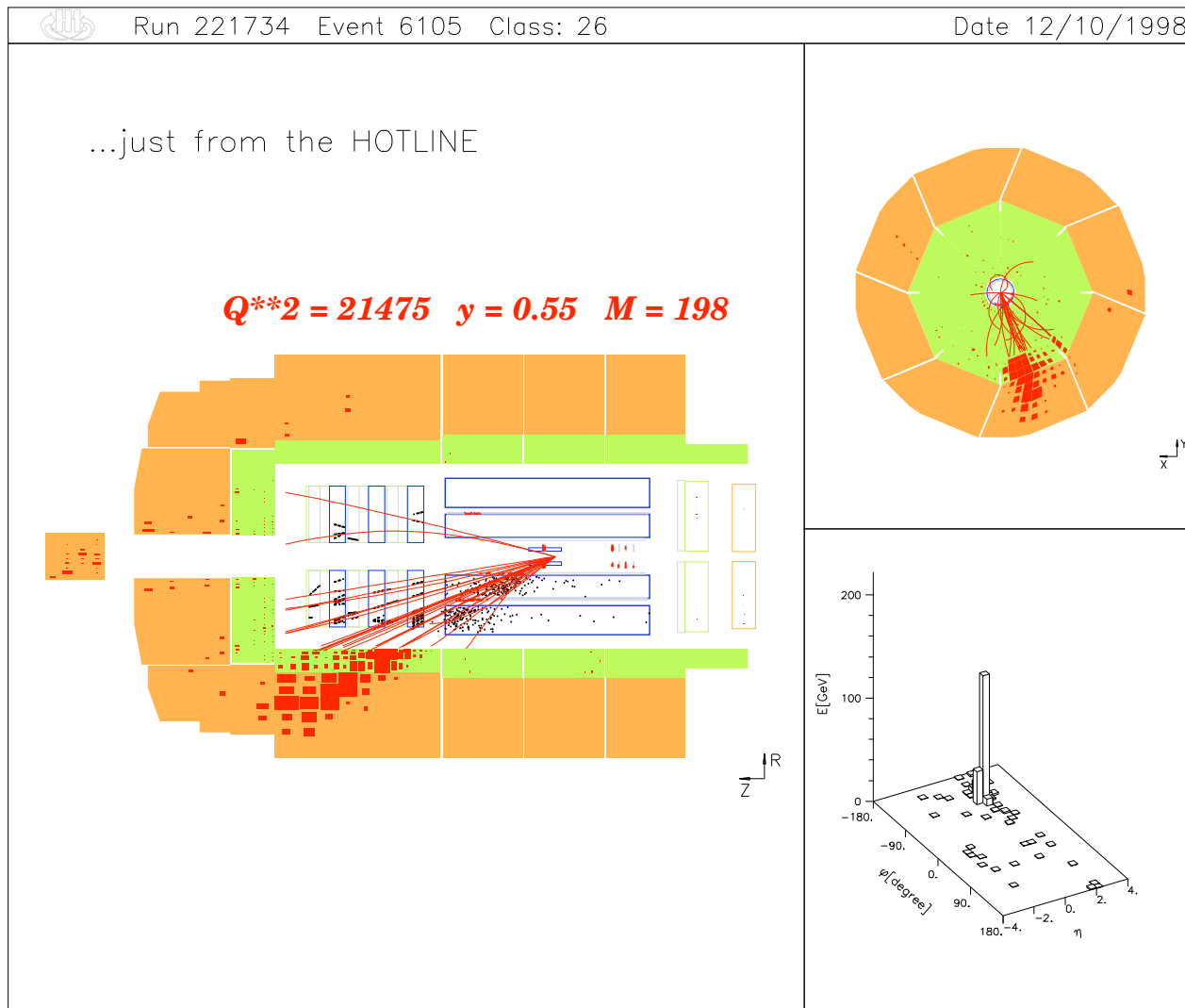


- Parton model relation for 1PI cross sections

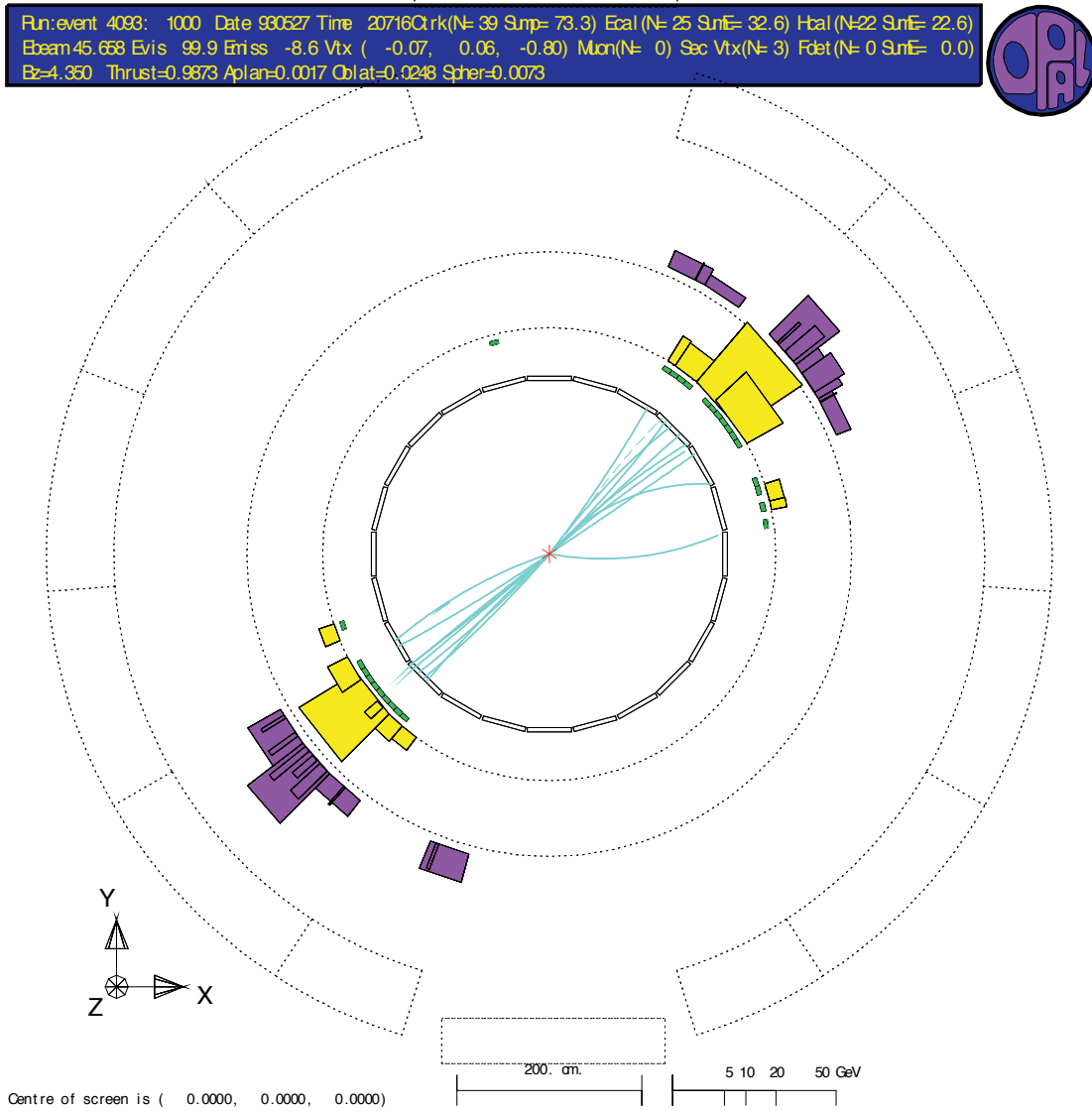
$$\sigma_h(P, q) = \sum_a \int_0^1 dz \hat{\sigma}_a(P/z, q) D_{h/a}(z)$$

- Heuristic justification: Formation of hadron C from parton a takes a time τ_0 in the rest frame of a , but much longer in the CM frame – this “fragmentation” thus decouples from $\hat{\sigma}_a$, and is independent of q (scaling).
- Fragmentation picture suggests that hadrons are aligned along parton direction \Rightarrow jets. And this is what happens.

• For DIS:

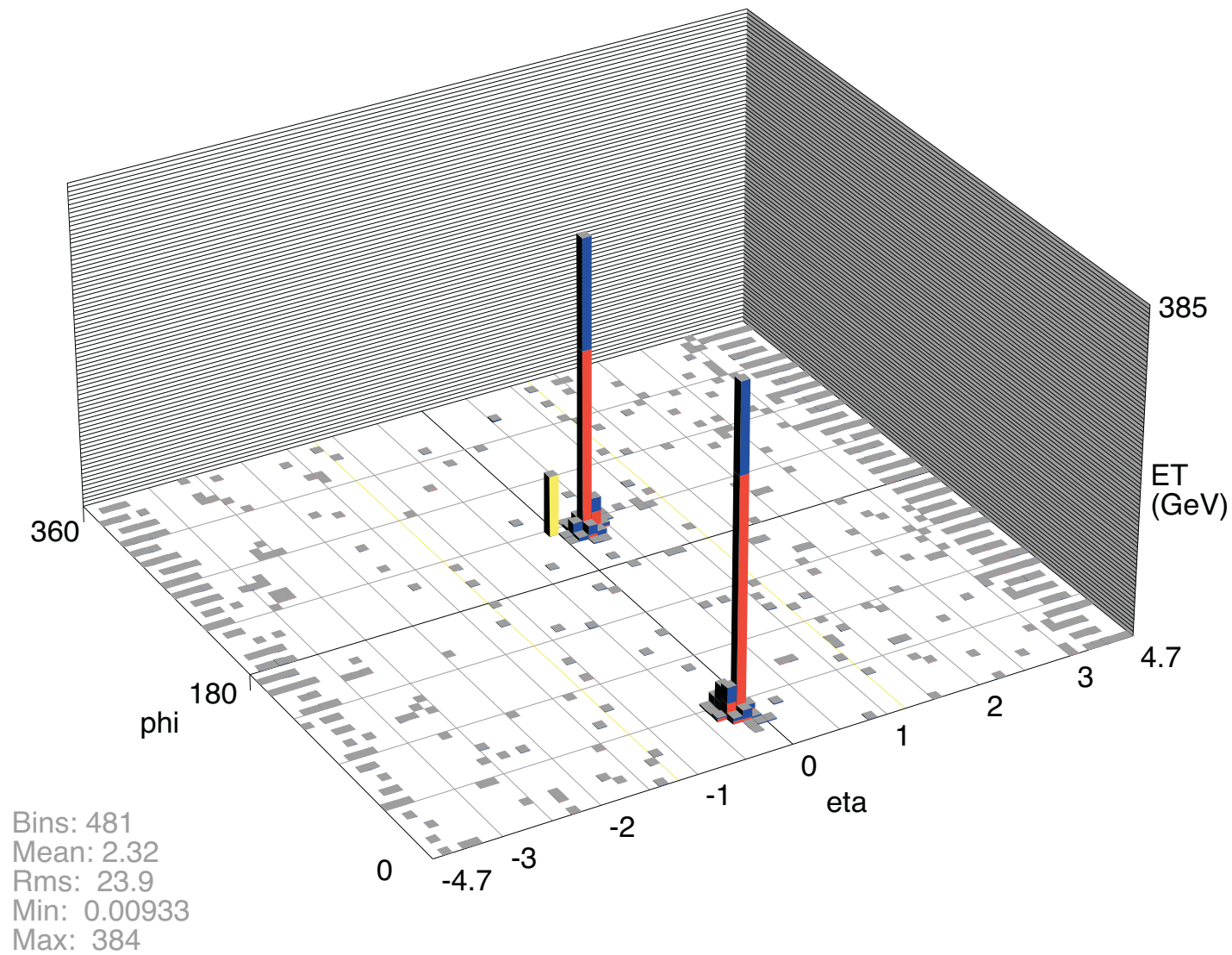


• For e^+e^- :



• And in nucleon-nucleon collisions:

Run 178796 Event 67972991 Fri Feb 27 08:34:03 2004



mE_t: 72.1
phi_t: 223 deg

- Finally: the Drell-Yan process
- In the parton model (1970).
Drell and Yan: look for the annihilation of quark pairs into virtual photons of mass Q ... any electroweak boson in NN scattering.

$$\frac{d\sigma_{NN \rightarrow \mu\bar{\mu} + X}(Q, p_1, p_2)}{dQ^2 d\dots} \sim$$

$$\int d\xi_1 d\xi_2 \sum_{a=q\bar{q}} \frac{d\sigma_{a\bar{a} \rightarrow \mu\bar{\mu}}^{\text{EW, Born}}(Q, \xi_1 p_1, \xi_2 p_2)}{dQ^2 d\dots}$$

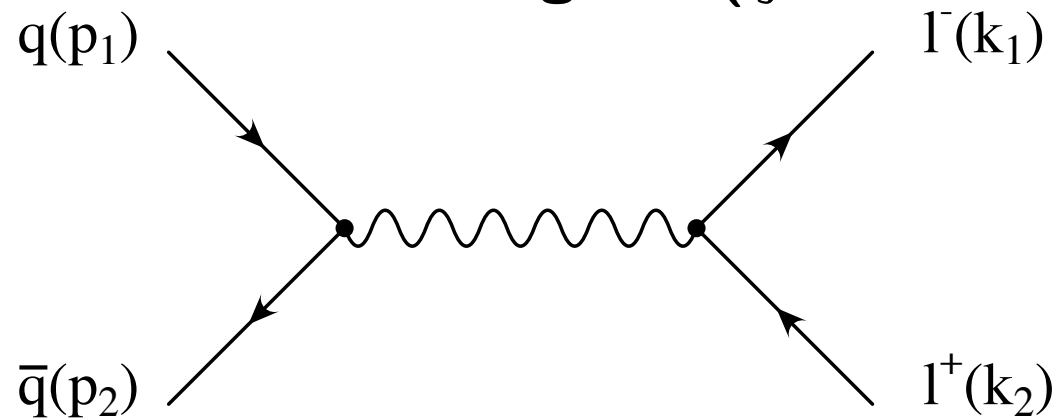
× (probability to find parton $a(\xi_1)$ in N)
 × (probability to find parton $\bar{a}(\xi_2)$ in N)

The probabilities are $\phi_{q/N}(\xi_i)$'s from DIS!

How it works (with colored quarks) ...

- **The Born cross section**

$\sigma^{\text{EW,Born}}$ is all from this diagram (ξ 's set to unity):



With this matrix element:

$$M = e_q \frac{e^2}{Q^2} \bar{u}(k_1) \gamma_\mu v(k_2) \bar{v}(p_2) \gamma^\mu u(p_1)$$

- **First square and sum/average M . Then evaluate phase space.**

- Total cross section at pair mass Q

$$\begin{aligned}\sigma_{q\bar{q}\rightarrow\mu\bar{\mu}}^{\text{EW, Born}}(x_1 p_1, x_2 p_2) &= \frac{1}{2\hat{s}} \int \frac{d\Omega}{32\pi^2} \frac{e_q^2 e^4}{3} (1 + \cos^2 \theta) \\ &= \frac{4\pi\alpha^2}{9Q^2} \sum_q e_q^2 \equiv \sigma_0(M)\end{aligned}$$

With Q the pair mass and 3 for color average.

- And measured rapidity:

Pair mass (Q) and rapidity

$$\eta \equiv (1/2) \ln \left(\frac{Q^+}{Q^-} \right) = (1/2) \ln \left(\frac{Q^0 + Q^3}{Q^0 - Q^3} \right)$$

- ξ 's are overdetermined \rightarrow delta functions in the Born cross section

$$\frac{d\sigma_{NN \rightarrow \mu\bar{\mu}+X}^{(PM)}(Q, p_1, p_2)}{dQ^2 d\eta} = \int_{\xi_1, \xi_2} \sum_{a=q\bar{q}} \sigma_{a\bar{a} \rightarrow \mu\bar{\mu}}^{\text{EW, Born}}(\xi_1 p_1, \xi_2 p_2) \times \delta(Q^2 - \xi_1 \xi_2 S) \delta\left(\eta - \frac{1}{2} \ln \left(\frac{\xi_1}{\xi_2} \right)\right) \times \phi_{a/N}(\xi_1) \phi_{\bar{a}/N}(\xi_2)$$

- and integrating over rapidity, back to $d\sigma/dQ^2$,

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha_{\text{EM}}^2}{9Q^4} \right) \int_0^1 d\xi_1 d\xi_2 \delta(\xi_1\xi_2 - \tau) \\ \times \sum_a \lambda_a^2 \phi_{a/N}(\xi_1) \phi_{\bar{a}/N}(\xi_2)$$

Found by Drell and Yan in 1970 (aside from 1/3 for color).

Analog of DIS scaling in x is DY scaling in $\tau = Q^2/S$.

- **Template for all hard hadron-hadron scattering**
- **Next, the quantum field theory of all this ... QCD**

▪

- **Appendix I: Quarks in the Standard Model**
Electroweak interactions of quarks: $SU(2)_L \times U(1)$. Their non-QCD interactions.
- **Quark and lepton fields: L(eftrightarrow) and R(ight)**
 - $\psi = \psi^{(L)} + \psi^{(R)} = \frac{1}{2}(1 - \gamma_5)\psi + \frac{1}{2}(1 + \gamma_5)\psi; \psi = q, \ell$
 - **Helicity: spin along \vec{p} (R=right handed) or opposite (L=left handed) in solutions to Dirac equation**
 - $\psi^{(L)}$: expanded only in L particle solutions to Dirac eqn.
R antiparticle solutions
 - $\psi^{(R)}$: only R particle solutions, L antiparticle
 - **An essential feature: L and R have different interactions in general!**

- L quarks come in “weak $SU(2)$ ” = “weak isospin” pairs:

$$q_i^{(L)} = \begin{pmatrix} u_i \\ d'_i = V_{ij} d_j \end{pmatrix} \quad u_i^{(R)}, d_i^{(R)}$$

(u, d')
 (c, s')
 (t, b')

$$\ell_i^{(L)} = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} \quad e_i^{(R)}, \nu_i^{(R)}$$

(ν_e, e)
 (ν_μ, μ)
 (ν_τ, τ)

(We've neglected neutrino masses.)

- V_{ij} is the “CKM” matrix.
- The electroweak interactions distinguish L and R.

- Weak vector bosons: electroweak gauge groups

- SU(2): three vector bosons B_i with coupling g

- U(1); one vector boson C with coupling g'

- The physical bosons:

$$W^\pm = B_1 \pm iB_2$$

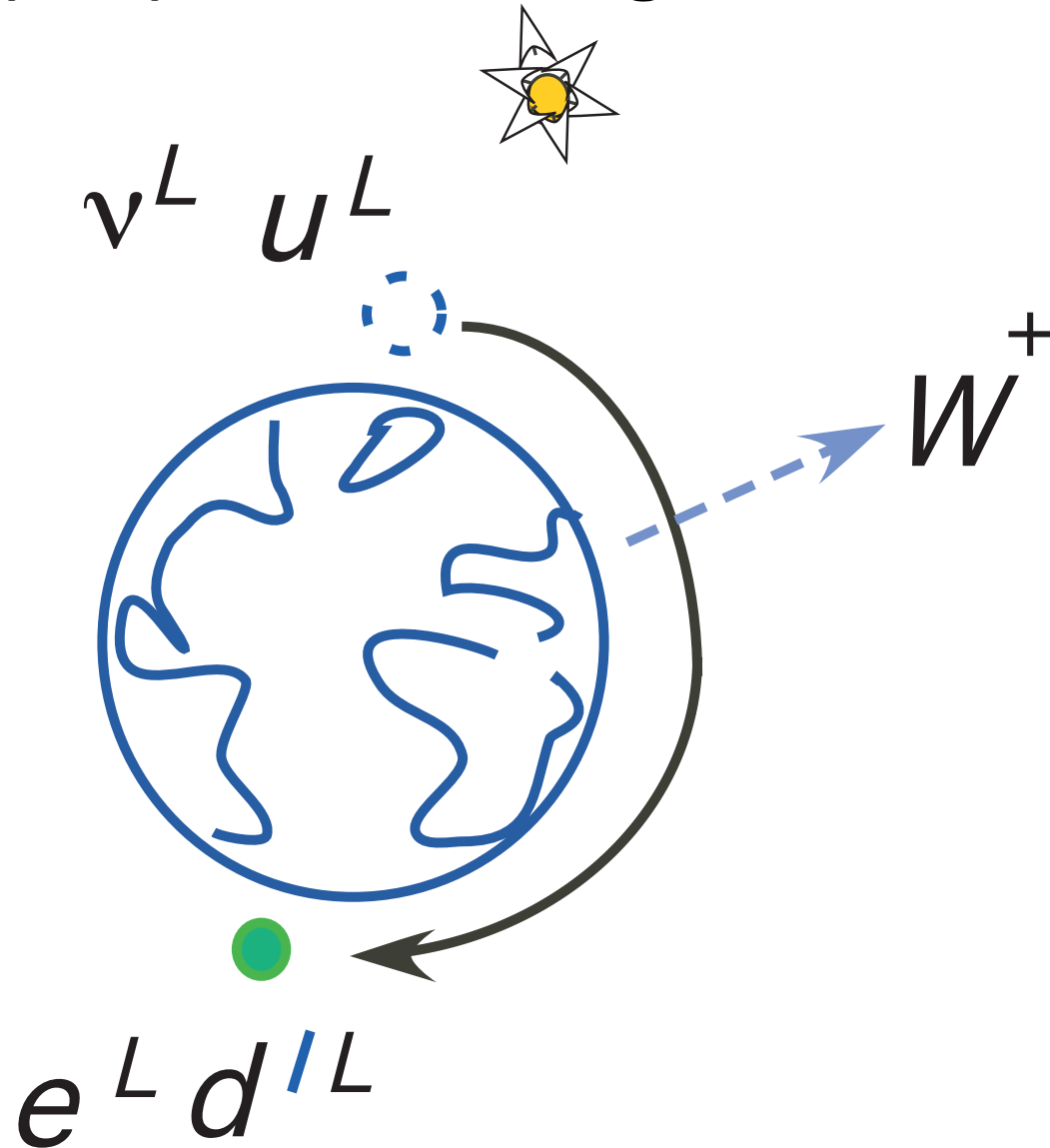
$$Z = -C \sin \theta_W + B_3 \cos \theta_W$$

$$\gamma \equiv A = C \cos \theta_W + B_3 \sin \theta_W$$

$$\sin \theta_W = g' / \sqrt{g^2 + g'^2} \quad M_W = M_Z / \cos \theta_W$$

$$e = gg' / \sqrt{g^2 + g'^2} \quad M_W \sim g / \sqrt{G_F}$$

- Weak isospin space: connecting u with d'



- Only left handed fields move around this globe.

- The interactions of quarks and leptons with the photon, W, Z

$$\mathcal{L}_{\text{EW}}^{(\text{fermion})} = \sum_{\text{all } \psi} \bar{\psi} (i\not{\partial} - e\lambda_{\psi} \not{A} - (gm_{\psi}2M_W)h) \psi$$

$$- (g/\sqrt{2}) \sum_{q_i, e_i} \bar{\psi}^{(L)} (\sigma^+ \not{W}^+ + \sigma^- \not{W}^-) \psi^{(L)}$$

$$- (g/2 \cos \theta_W) \sum_{\text{all } \psi} \bar{\psi} (v_f - a_f \gamma_5) \not{Z} \psi$$

- Interactions with W are through ψ_L 's only.
- Neutrino Z exchange depends on $\sin^2 \theta_W$ even at low energy.
- This observation made it clear by early 1970's that $M_W \sim g/\sqrt{G_F}$ is large \rightarrow a need for colliders.
- Coupling to the Higgs $h \propto$ mass (special status of t).

- **Symmetry violations in the standard model:**
 - W 's interact through $\psi^{(L)}$ only, $\psi = q, \ell$.
 - These are left-handed quarks & leptons; right-handed antiquarks, antileptons.
 - Parity (P) exchanges L/R; Charge conjugation (C) exchanges particles, antiparticles.
 - CP combination OK ($L \xrightarrow{P} R \xrightarrow{C} L$) if all else equal, but it's not (quite) ...

Complex phases in CKM V result in CP violation.

- **Appendix II: Structure Functions and Photon Polarizations**

In the **P** rest frame can take

$$q^\mu = (\nu; 0, 0, \sqrt{Q^2 + \nu^2}) , \quad \nu \equiv \frac{p \cdot q}{m_p}$$

In this frame, the possible photon polarizations ($\epsilon \cdot q = 0$):

$$\epsilon_R(q) = \frac{1}{\sqrt{2}} (0; 1, -i, 0)$$

$$\epsilon_L(q) = \frac{1}{\sqrt{2}} (0; 1, i, 0)$$

$$\epsilon_{\text{long}}(q) = \frac{1}{Q} (\sqrt{Q^2 + \nu^2}, 0, 0, \nu)$$

- **Alternative Expansion**

$$W^{\mu\nu} = \sum_{\lambda=L,R,long} \epsilon_{\lambda}^{\mu*}(q) \epsilon_{\lambda}^{\nu}(q) F_{\lambda}(x, Q^2)$$

- **For photon exchange (Exercise 4):**

$$F_{L,R}^{\gamma e} = F_1$$
$$F_{long} = \frac{F_2}{2x} - F_1$$

- **So F_{long} vanishes in the parton model by the C-G relation.**

- **Generalizations: neutrinos and polarization**
- **Neutrinos: flavor of the “struck” quark is changed when a W^\pm is exchanged. For W^+ , a d is transformed into a linear combination of u, c, t , determined by CKM matrix (and momentum conservation).**
- **Z exchange leaves flavor unchanged but still violates parity.**

- The Vh structure functions for $= W^+, W^-, Z$:

$$\begin{aligned}
W_{\mu\nu}^{(Vh)} = & \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) W_1^{(Vh)}(x, Q^2) \\
& + \left(p_\mu - q_\mu \frac{p \cdot q}{q^2} \right) \left(p_\nu - q_\nu \frac{p \cdot q}{q^2} \right) \frac{1}{m_h^2} W_2(x, Q^2) \\
& - i \epsilon_{\mu\nu\lambda\sigma} p^\lambda q^\sigma \frac{1}{m_h^2} W_3^{(Vh)}(x, Q^2)
\end{aligned}$$

- with dimensionless structure functions:

$$F_1 = W_1, \quad F_2 = \frac{p \cdot q}{m_h^2} W_2, \quad F_3 = \frac{p \cdot q}{m_h^2} W_3$$

- $F_i^{(\nu h)}$ gives $W^+ h$ scattering, $F_i^{(\bar{\nu} h)}$ gives $W^- h$

- And with spin (for the photon).

$$\begin{aligned}
W^{\mu\nu} &= \frac{1}{4\pi} \int d^4z e^{iq \cdot z} \langle h(P, S) | J^\mu(z) J^\nu(0) | h(P, S) \rangle \\
&= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) F_1(x, Q^2) \\
&\quad + \left(P^\mu - q^\mu \frac{P \cdot q}{q^2} \right) \left(P^\nu - q^\nu \frac{P \cdot q}{q^2} \right) F_2(x, Q^2) \\
&\quad + im_h \epsilon^{\mu\nu\rho\sigma} q_\rho \left[\frac{S_\sigma}{P \cdot q} g_1(x, Q^2) + \frac{S_\sigma(P \cdot q) - P_\sigma(S \cdot q)}{(P \cdot q)^2} g_2(x, Q^2) \right]
\end{aligned}$$

- Parton model structure functions:

$$F_2^{(eh)}(x) = \sum_f e_f^2 x \phi_{f/h}(x)$$

$$g_1^{(eh)}(x) = \frac{1}{2} \sum_f e_f^2 (\Delta\phi_{f/h}(x) + \Delta\bar{\phi}_{f/h}(x))$$

- Notation: $\Delta\phi_{f/h} = \phi_{f/h}^+ - \phi_{f/h}^-$ with $\phi_{f/h}^\pm(x)$ probability for struck quark f to have momentum fraction x and helicity with (+) or against (-) h 's helicity.