From last time

1st law: *Law of inertia*
Every object continues in its state of rest, or uniform motion in a straight line, unless acted upon by a force.

2nd law: $F = ma$, or $a = F/m$
The acceleration of a body along a direction is
– proportional to the total force along that direction, and
– inversely the mass of the body

3rd law: *Action and reaction*
For every action there is an equal and opposite reaction.

**Gravitational force**

Gravitational force on apple by earth
Gravitational force on Earth by apple

These forces are equal and opposite,

$$m_{\text{Earth}} a_{\text{Earth}} = m_{\text{apple}} a_{\text{apple}}$$

But $m_{\text{Earth}} = 6 \times 10^{24}$ kg
$m_{\text{apple}} = 1$ kg

**Equal accelerations**

- If more massive bodies accelerate more slowly with the same force...

... why do all bodies fall the same, independent of mass?

$$F_{\text{gravity}} = mg$$

- Gravitational force on a body depends on its mass:
- Therefore acceleration is independent of mass:

$$a = \frac{F_{\text{Gravity}}}{m} = \frac{mg}{m} = g$$

**A fortunate coincidence**

- A force exactly proportional to mass, so that everything cancels nicely.
- But a bit unusual.
- Einstein threw out the gravitational force entirely, attributing the observed acceleration to a distortion of space-time.

**Velocity of the moon**

What is the direction of the velocity of the moon?

**Acceleration of the moon**

What is the direction of the acceleration of the moon?
Physics 107, Fall 2006

**Acceleration** = \[ \frac{\text{change in velocity}}{\text{change in time}} \]

- Velocity at time \( t_1 \)
- Velocity at time \( t_2 \)
- \( \text{Speed is same, but direction has changed} \)
- \( \text{Velocity has changed} \)

**How has the velocity changed?**

- Velocity at time \( t_1 \)
- Velocity at time \( t_2 \)
- Change in velocity

**Centripetal acceleration** = \( \frac{v^2}{r} \), directed toward center of orbit. \( r = \text{radius of orbit} \)

(In this equation, \( v \) is the speed of the object, which is the same at all times)

**Earth’s pull on the moon**

- The moon continually accelerates toward the earth,
- But because of its orbital velocity, it continually misses the Earth.
- The orbital speed of the moon is constant, but the direction continually changes.
- Therefore the velocity changes with time.

**True for any body in circular motion**

**Experiment**

- \( F = m_2 g \)
- \( F = m_1 \)
- \( m_1 \) accelerates inward in response to force \( m_2 g \)

**Newton’s falling moon**

- From Newton’s Principia, 1615

**Shoot the monkey**

- The dart gun is fired just as the monkey drops from the tree.
- After the dart leaves the gun, the only force is from gravity.
- The only deviation from straight-line motion is an acceleration directly downward.

- The monkey has exactly the same acceleration downward, so that the dart hits the monkey.

- Another example of superposition
Acceleration of moon

- The moon is accelerating at \( \frac{v^2}{r} \) \(m/s^2\) directly toward the earth!

- This acceleration is due to the Earth’s gravity.

- Is this acceleration different than \( g \), the gravitational acceleration of an object at the Earth’s surface?
  - Can calculate the acceleration directly from moon’s orbital speed, and the Earth-moon distance.

Distance and diam. of moon

- The diameter of the moon is the diameter of its shadow during a solar eclipse. From the diameter \( d \) and angular size \( d/r \approx 5 \text{ deg} \), infer distance \( r \approx 60^\circ \text{r}(\text{earth}) \).

The radius of the earth

- “Originally” from study of shadows at different latitudes by Eratosthenes!
- \( R(\text{earth}) = 6500 \text{ km} \)

Distance dependence of Gravity

- The gravitational force depends on distance.

Moon acceleration, cont

- Distance to moon = 60 earth radii \( \sim 3.84 \times 10^8 \text{ m} \)
- Speed of moon?
  - Circumference of circular orbit = \( 2\pi r \)
  - Speed = orbital distance \( / \) orbital time \( = 1023 \text{ m/s} \)
  - Centripetal acceleration = 0.00272 \( m/s^2 \)

- This is the acceleration of the moon due to the gravitational force of the Earth.

Equation for force of gravity

\[ F_{\text{gravity}} = \frac{(\text{Mass of object 1}) \times (\text{Mass of object 2})}{\text{square of distance between them}} \]

\[ F = \frac{m_1 \times m_2}{d^2} \]

For masses in kilograms, and distance in meters,

\[ F = 6.7 \times 10^{-11} \frac{m_1 \times m_2}{d^2} \]

Newton: I thereby compared the force requisite to keep the Moon in her orb with the force of gravity at the surface of the Earth, and found them answer pretty nearly.
Example

- Find the acceleration of an apple at the surface of the earth

\[ \text{Acceleration of apple} = \frac{F_{\text{apple}}}{m_{\text{apple}}} = 6.7 \times 10^{-11} \text{mEarth} \] \( \text{d}^2 \)

This is also the force on the Earth by the apple!

\[ \text{Force on apple} = F_{\text{apple}} = 6.7 \times 10^{-11} \text{mEarth} \] \( m_{\text{apple}} \) \( d^2 \)

So moving farther from the Earth should reduce the force of gravity

- Typical airplane cruises at ~5 mi = 8000 m
  - \( d \) increases from 6,370,000 m to 6,378,000 m
  - only about a 0.25% change!

Gravitational force decreases with distance from Earth

\[ \text{Force on apple} = F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{Earth}} \times m_{\text{apple}}}{d^2} \]

So why is everyone floating around?

- International space station orbits at 350 km = 350,000 m
  - \( d = 6,370,000 \text{ m} + 350,000 \text{ m} = 6,720,000 \text{ m} \)
  - Again \( d \) has changed only a little, so that \( g \) is decreased by only about 10%.

The space station is falling...

...similar to Newton’s apple

- In its circular orbit, once around the Earth every 90 minutes, it is continuously accelerating toward the Earth at ~8.8 m/s².
- Everything inside it is also falling (accelerating toward Earth at that same rate).
- The astronauts are freely falling inside a freely-falling ‘elevator’. They have the perception of weightlessness, since their environment is falling just as they are.

So why is everyone floating around?

Edward M. (Mike) Fincke, Expedition science officer

Supreme Scream – 300 feet of pure adrenaline rush

A freefall ride

\[ d = \frac{1}{2} a t^2 \]

\[ t = \sqrt{\frac{2d}{a}} \]

\[ t = \sqrt{\frac{2 \times 300\text{ ft}}{32 \text{ ft/s}^2}} \]

4.3 sec of freefall
A little longer ride

Parabolic path of freely falling object

Accelerating gravity on moon

- On the moon, an apple feels gravitational force from the moon.
- Earth is too far away.

Force on apple on moon = \( F_{\text{apple}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}} \times m_{\text{apple}}}{r_{\text{moon}}^2} \)

Accel. of apple on moon = \( \frac{F_{\text{accel}}}{m_{\text{apple}}} = 6.7 \times 10^{-11} \frac{m_{\text{moon}}}{r_{\text{moon}}^2} \)

Compare to accel on Earth = \( 6.7 \times 10^{-11} \frac{m_{\text{Earth}}}{r_{\text{Earth}}^2} \)

Gravitational force at large distances:

Stars orbiting our black hole

- At the center of our galaxy is a collection of stars found to be in motion about an invisible object.

Orbits obey Newton’s gravity, orbiting around some central mass

- Scientists at the Max Planck Institute for Extraterrestrial Physik has used infrared imaging to study star motion in the central parsec of our galaxy.
- Movie at right summarizes 14 years of observations.
- Stars are in orbital motion about some massive central object

http://www.mpe.mpg.de/www_ir/GC/intro.html
What is the central mass?

- One star swings by the hole at a minimum distance $b$ of 17 light hours (120 A.U. or close to three times the distance to Pluto) at speed $v=5000$ km/s, period 15 years.
- From the orbit we can derive the mass.
- The mass is 2.6 million solar masses.
- It is mostly likely a black hole at the center of our Milky Way galaxy!