From last time

**Gravity and centripetal acceleration**

\[ a_c = \frac{v^2}{r} \text{ m/s}^2 \]

\[ F = \frac{G m_1 m_2}{r^2} \]

Used to explore interesting questions like what is at the center of the galaxy

**HW#2:** Due

**HW#3:** Chapter 5: Conceptual: # 22  Problems: # 2, 4
Chapter 6: Conceptual: # 18  Problems: # 2, 5

**Exam 1:** Next Wednesday, Review Monday, Scantron with XX questions, bring #2 pencil
Chapters 1 and 3-6
1 Page, front only, equation sheet allowed

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What is the central mass?

- One star swings by the hole at a minimum distance \( b \) of 17 light hours (120 A.U. or close to three times the distance to Pluto) at speed \( v = 5000 \text{ km/s} \), period 15 years.

\[ a_c = \frac{v^2}{r} = 6.7 \times 10^{-11} \frac{m}{r^2} \]

\[ F = \frac{G m_1 m_2}{r^2} \]

\[ m \approx \frac{v^2 r}{6.7 \times 10^{-11}} = 6.7 \times 10^{10} \times 25 \times 10^{18} = 6.7 \times 10^{36} \]

- Mass Sun: \( 2 \times 10^{30} \), 3.4 million solar mass black hole (approximate estimate) at the center of our Milky Way galaxy!

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Discussion so far…

- So far we have talked about
  - Velocity and Acceleration
  - Momentum and conservation of momentum
  - Momentum transfer changing the velocity of an object
  - That momentum transfer resulting from a force when the objects are in contact
  - Newton’s relation: Acceleration = Force / mass

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Something missing

- With these tools, can think about the world in many ways.
  - Collisions resulting in a momentum transfer
  - Gravitational forces resulting in acceleration of falling bodies and orbits of planets.
- But this leaves something out
- Think about firing a rifle:
  - Before pulling the trigger, both rifle and bullet are stationary: total momentum is zero.
  - After firing, the bullet and rifle move in opposite directions. Total momentum is still zero.
  - But clearly the situation before and after is different.

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Energy

- Both objects moving in final state.
- That movement represents energy.
- In addition to momentum, the energy is physical property of the system.
- We will see that it is also conserved.
- In the rifle - bullet example
  - Before firing, the energy is stored in the gunpowder.
  - After firing, most of the energy appears as the motion of the bullet and rifle
  - Some of the energy appears as heat.

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Before Energy Consider Work

- Work is done whenever a body is continually pushed or pulled through a distance.
- Twice as much work is done when the body is moved twice as far.
- Pushing twice as hard over the same distance does twice as much work.
- Work = Force x Distance
**Work, cont.**

- Force has units of Newtons (N)
  - Distance has units of meters (m)
  - So work has units of N·m, defined as Joules (J).

- **Example:**
  - The Earth does work on an apple when the apple falls.
  - *The force applied is the force of gravity*

- **Example:**
  - I do work on a box when I push it along the floor.
  - *The force applied is from my muscles*

**Multi-part question**

I lift a body weighing 1 N upward at a constant vertical velocity of 0.1 m/s. The Net force on the body is

A. 1 N upward
B. 1 N down
C. 0 N

Since the acceleration is zero, the net force must be zero.

**Question, cont.**

The force I exert on the body is

A. 1 N up
B. 1 N down
C. 0

Since net force is zero, and the gravitational force is 1 N down, I must be exerting 1 N up.

**Question, cont.**

After lifting the 1 N body a total distance of 1 m, the work I have done on the body is

A. 1 J
B. 0.1 J
C. 0 J

Work = Force x Distance
= 1 N x 1 m = 1 N-m = 1 Joule

**Question, cont.**

After lifting the 1 N body a total distance of 1 m, the Net work done on the body is

A. 1 J
B. 0.1 J
C. 0 J

Work = Force_{net} x Distance
= (1 N - 1 N) x 1 m = 0 Joule

**Energy**

A object’s energy is the amount of work it can do.

Energy comes in many forms

- **Kinetic Energy**
- **Gravitational Energy**
- **Electrical Energy**
- **Thermal Energy**
- **Solar Energy**
- **Chemical Energy**
- **Nuclear Energy**

It can be converted into other forms without loss (i.e. it is conserved)
Energy of motion
In outer space, I apply a force of 1 N to a 1 kg rock for a distance of 1 m.

I have done \( \text{Force} \times \text{Distance} = (1 \text{ N}) \times (1 \text{ m}) = 1 \text{ J} \) of work.

After applying the force for 1 m, the rock is moving at some final velocity \( v_{\text{final}} \) as a result of the acceleration \( \frac{\text{Force}}{\text{mass}} \).

So the energy I expended in doing work has caused the body to change its velocity from zero to \( v_{\text{final}} \).

Kinetic energy (energy of motion)

- Work = Force \times Distance
- A constant applied force leads to an acceleration.
- After the distance is moved, the body is traveling at some final velocity \( v_{\text{final}} \).
- So the result of the work done is to change the body’s velocity from zero to \( v_{\text{final}} \)

Work-energy relation

- The acceleration of the body is related to the net force by \( F=ma \)

\[
\begin{align*}
\text{Work} &= \int F_{\text{net}} \, dx = (ma) \times x = m \times (ad)
\end{align*}
\]

For a body initially at rest, constant accelerates says

\[
\begin{align*}
\frac{d}{dt} &= \frac{1}{2} a t^2, \text{ so } t = \sqrt{\frac{2d}{a}} \quad \text{or} \quad v_{\text{final}} = at = \sqrt{\frac{2ad}{a}} = \sqrt{2ad}
\end{align*}
\]

\[
\text{Work} = F_{\text{net}} \times x = \frac{1}{2} m v_{\text{final}}^2
\]

\[
\frac{1}{2} m v_{\text{final}}^2 \text{ is called Kinetic Energy, or energy of motion}
\]

A more general form

- If the object initially moving at some velocity \( v_{\text{initial}} \) it has kinetic energy \( \frac{1}{2} m v_{\text{initial}}^2 \)

- As the result of a net work \( W_{\text{net}} \), the velocity increases to \( v_{\text{final}} \)

\[
\frac{1}{2} m v_{\text{final}}^2 = \frac{1}{2} m v_{\text{initial}}^2 + W_{\text{net}}
\]

- and the Kinetic Energy increases to \( \frac{1}{2} m v_{\text{final}}^2 \)

Gravitational energy

- An object in a gravitational field can do work when it falls.
- We might say that energy is stored in the system.
Ball falls down in gravity

• Ball initially held at rest.
  - $v_{initial} = 0$
  - Kinetic energy = 0
• Ball released.
• Gravitational force = $mg$, falls with acceleration $g$
• Work done by gravitational force in falling distance $h$ is $\text{Force} \times \text{Distance} = mgh$.
• Ball final kinetic energy = $mgh = \frac{1}{2}mv_{final}^2$.

Ball moved up in gravity

• Work done by me on ball
  - Ball initially held at rest by me.
  - I move the ball slowly upward a distance $h$.
  - Force applied by me is $mg$ upward.
• Work done by me on ball is $\text{Force} \times \text{Distance} = mgh$
• Work done by gravity on ball
  - $\text{Force} \times \text{Distance} = -mgh$
• Net (total) work done on ball = $mgh - mgh = 0$
• Consistent with zero change in kinetic energy of ball during this time (i.e. ending velocity is same as starting velocity).

Work Done by Gravity

• Change in gravitational energy,
  $\Delta E = mgh$
  true for any path : $h$, is simply the height difference, $y_{final} - y_{initial}$
• A falling object converts gravitational potential energy to its kinetic energy
• Work needs to be done on an object to move it vertically up - work done is the same no matter what path is taken.

Electrical Energy

• Electricity is the flow of charged particles.
• Charged particles have an electromagnetic force between them similar to the gravitational force.
• This force can do work.
• Doing work against this force can store energy in the system.
• The energy can be removed at any time to do work.

Thermal Energy

• Otherwise known as heat.
• The temperature of an object is related to the amount of energy stored in the object.
• The energy is stored by the microscopic vibratory motion of atoms in the material.
• This energy can be transferred from one object to another by contact.
• It can also be turned into work by contact.

Storing energy

• Energy is neither created nor destroyed, but is just moved around.
• Or more accurately, it changes form.
• I do work by lifting a body against gravity.
• If the body now drops, it can do work when it hits (pounding in a nail, for instance).
Could say that the work I did lifting the body is stored until the body hits the nail and pounds it in.

Potential Energy