From last time…

- Galilean Relativity
  - Laws of mechanics identical in all inertial ref. frames
- Einstein's Relativity
  - All laws of physics identical in inertial ref. frames
  - Speed of light=*c* in all inertial ref. frames
- Consequences
  - *Simultaneity:* events simultaneous in one frame will not be simultaneous in another.
  - *Time dilation:* time interval between events appear different to different observers

Einstein's principle of relativity

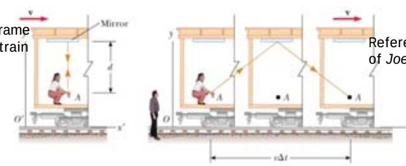
- **Principle of relativity:**
  - *All* the laws of physics are identical in all inertial reference frames.
- **Constancy of speed of light**:
  - Speed of light is same in all inertial frames (*e.g.* independent of velocity of observer, velocity of source emitting light)

(These two postulates are the basis of the special theory of relativity)

Consequences of Einstein's relativity

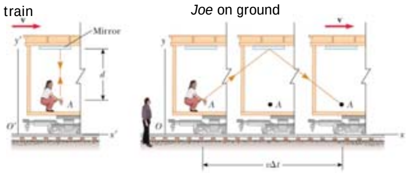
- Many 'common sense' results break down:
  - Events that seem to be simultaneous are not simultaneous in different inertial frames
  - The time interval between events is not absolute. it will be different in different inertial frames
  - The distance between two objects is not absolute. it is different in different inertial frames
  - Velocities don't always add directly

Time dilation



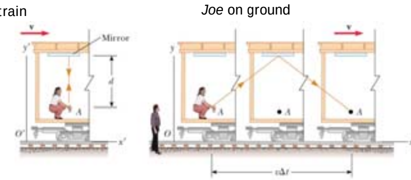
- Laser bounces up and down from mirror on train.
- *Joe* on ground measures time interval w/ his clock.
- *Joe* watches *Jane's* clock on train as she measures the time interval.
- *Joe sees that these two time intervals are different.*

Why is this?



- Jane on train: light pulse travels distance 2*d*.
- Joe on ground: light pulse travels *farther*
- Relativity: both Joe and Jane say light travels at *c*
  - Joe measures longer travel time of light pulse
- This is *time dilation*

Time dilation, continued

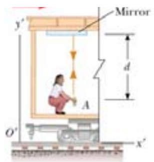


- Observer *Jane* on train: light pulse travels distance 2*d*.
- Time = distance divided by velocity = $2d/c$
- Time in the frame the events occurred at same location called the proper time Δt_p

Time dilation

Time interval in Jane's frame

$$\Delta t_{Jane} = \frac{round\ \ trip\ \ distance}{speed\ \ of\ \ light} = \frac{2d}{c}$$

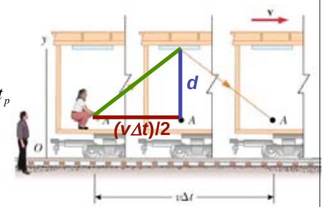


Joe measures a *longer* time

$$\Delta t_{Joe} = \frac{2\sqrt{d^2 + (v\Delta t_{Joe}/2)^2}}{c}$$

$$\Delta t_{Joe} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{2d}{c}\right) = \gamma \Delta t_p$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} > 1$$



The 'proper time'

- We are concerned with two time intervals. Intervals between two **events.**
  - A single observer compares time intervals measured in different reference frames.
- If the events are at the same spatial location in one of the frames…
  - The time interval measured in this frame is called the 'proper time'.
  - The time interval measured in a frame moving with respect to this one will be longer by a factor of γ

$$\Delta t_{other\ \ frame} = \gamma \Delta t_{proper}, \quad \gamma > 1$$

Atomic clocks and relativity

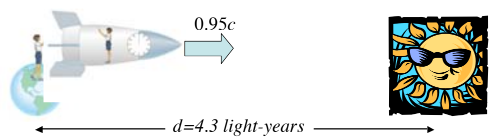
- In 1971, four atomic clocks were flown around the world on commercial jets.
- 2 went east, 2 went west -> a relative speed ~ 1000 mi/hr.
- On return, average time difference was 0.15 microseconds, consistent with relativity.

First atomic clock: 1949

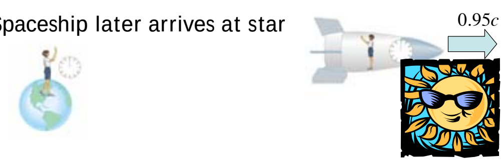
Miniature atomic clock: 2003

Traveling to the stars

Spaceship leaves Earth, travels at 0.95c

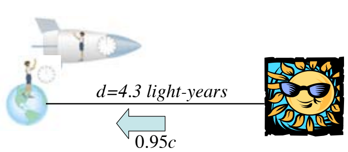


Spaceship later arrives at star

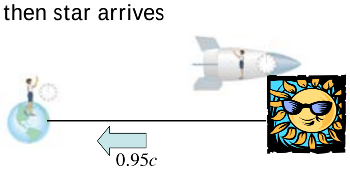


The ship observer's frame

Earth leaves…



..then star arrives



Comparing the measurements

- The ship observer measures 'proper time'
  - Heartbeats occur at the same spatial location (in the astronaut's chest).
- On his own clock, astronaut measures his normal heart-rate of 1 second between each beat.
- Earth observer measures, with his earth clock, a time much longer than the astronaut's ($\Delta t_{earth} = \gamma\, \Delta t_{astronaut}$)

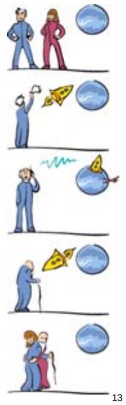
$$\Delta t_{earth} = \gamma \Delta t_{astronaut} = \frac{\Delta t_{astronaut}}{\sqrt{1 - v^2/c^2}} = 3.2 \times \Delta t_{astronaut} = 3.2\ \ \text{sec}$$

Earth observer sees astronaut's heart beating slow, and the astronaut's clock running slow.
Earth observer measures 3.2 sec between heartbeats of astronaut.

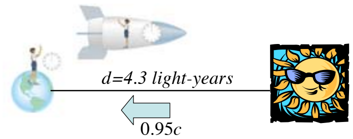
The twin 'paradox'

The Earth observer sees the astronaut age more slowly than himself.

- On returning, the astronaut would be younger than the earthling.
- And the effect gets more dramatic with increasing speed!
- All this has been verified - the 'paradox' arises when we take the astronaut's point of view.



- Special relativity predicts that astronaut would disagree, saying earthling is younger!
- Why?



If both measure the time interval between heartbeats of the earthling, the earthling measures the proper time.

Any other measurement of the time interval is longer!

The astronaut says the earthling's heart beats more slowly.

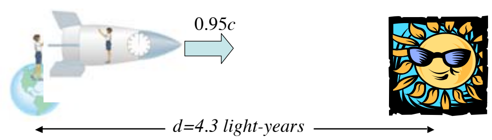
Apparently a direct contradiction.

Resolution

- Special relativity applies only to reference frames moving at constant speed.
- To turn around and come back, the astronaut must accelerate over a short interval.
- Only the Earthling's determination of the time intervals using special relativity are correct.
- General relativity applies to accelerating reference frames, and will make the measurements agree.

Total trip time

Spaceship leaves Earth, travels at 0.95c



$$\Delta t_{earth} = \frac{d}{v} = \frac{4.3\ \text{light-years}}{0.95\ c} = 4.5\ \text{years}$$

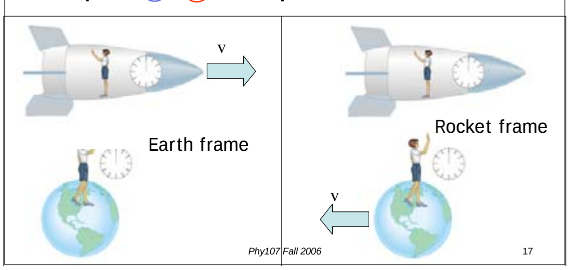
Time for astronaut passes more slowly by a factor gamma.
Trip time for astronaut is 4.5 yrs/3.2 = **1.4 years**

Relative velocity of reference frames

Both observers agree on relative speed, hence also gamma.

Speed of light

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.95)^2}} = 3.203$$



Are there other 'paradoxes'?

- Both observer's agree on the speed (0.95*c*)
  - Earth observer: ship moving
  - Ship observer: earth and star moving
  - They both agree on the speed
- But they disagree about the total trip time.
- If the time intervals are different, and speed is the same, how can distances be the same?

- **The distances are not the same!**
  Length contraction

Length Contraction

- People on ship and on earth agree on relative velocity v = 0.95 c.
- But they disagree on the time (4.5 vs 1.4 years).
- What about the distance between the planets?

Earth frame $d_{earth} = v\, t_{Earth}$ = .95 (3x10^8 m/s) (4.5 years)
= 4x10^{16}m **(4.3 light years)**

Ship frame $d_{ship} = v\, t_{ship}$ = .95 (3x10^8 m/s) (1.4 years)
= 1.25x10^{16}m **(1.3 light years)**

Length contraction and proper length

- Which one is correct?
  - Just like time intervals, distances are different in different frames.
  - There is no preferred frame, so one is no more correct than the other.
- The 'proper length' L_p is the length measured in a frame at rest with respect to objects
  - Here the objects are Earth and star.

Length in moving frame

$$L = \frac{L_p}{\gamma} = L_p\sqrt{1-\frac{v^2}{c^2}}$$

Length in object's rest frame

Is any measurement the same for all observers?

The real 'distance' between events

- Need a quantity that is the same for all observers
- A quantity all observers agree on is

$$x^2 - c^2t^2 \equiv (\text{separation})^2 - c^2(\text{time interval})^2$$

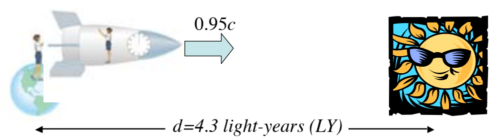
- Need to look at separation both in space and time to get the full 'distance' between events.
- In 4D: 3 space + 1 time

$$x^2 + y^2 + z^2 - c^2t^2$$

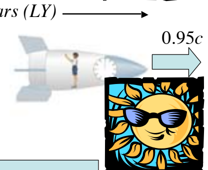
- The same or 'invariant' in any inertial frame

Events in the Earth Frame

- Event #1: leave earth



- Event #2: arrive star



$$\Delta t_{earth} = \frac{d}{v} = \frac{4.3\ \text{light-years}}{0.95\ \text{c}} = 4.5\ \text{years}$$

A relativistic invariant quantity

Earth Frame	Ship Frame
Event separation = 4.3 LY	Event separation = 0 LY
Time interval = 4.526 yrs	Time interval = 1.413 yrs
$(\text{separation})^2 - c^2(\text{time interval})^2 = (4.3)^2 - (c(4.526\,yrs))^2 = -2.0\ LY^2$	$(\text{separation})^2 - c^2(\text{time interval})^2 = 0 - (c(1.413\,yrs))^2 = -2.0\ LY^2$

- The quantity (separation)^2-c^2(time interval)^2 is the same for all observers
- It mixes the space and time coordinates

Time dilation, length contraction

- $t = \gamma\, t_{proper}$
  - *t_{proper} measured in frame where events occur at same spatial location*
- $L = L_{proper} / \gamma$
  - *L_{proper} measured in frame where events are simultaneous*

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

γ always bigger than 1

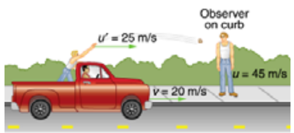
γ increases as v increases

γ would be infinite for v=c

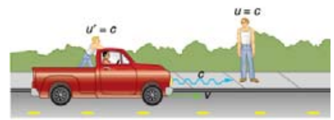
Suggests some limitation on velocity as we approach speed of light

Addition of Velocities (Non-relativistic)

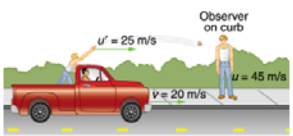
- Could try to reach higher velocity by throwing object from moving platform.
- Works well for non-relativistic objects.



Addition of Velocities (Relativistic)

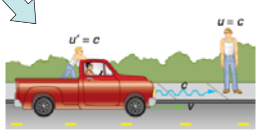


Relativistic Addition of Velocities



Very low velocity: Nonrelativistic

What about intermediate velocities?



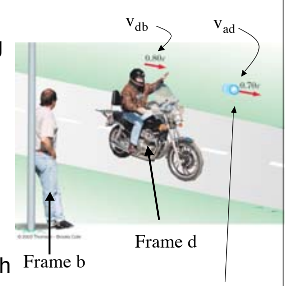
Very high velocity: Extreme relativistic

Relativistic Addition of Velocities

- Galilean addition of velocities can not be applied to objects moving near the speed of light
- Einstein's modification is

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad} v_{db}}{c^2}}$$

- The denominator is a correction based on length contraction and time dilation



Relativistic Addition of Velocities

- As motorcycle velocity approaches *c*, v_{ab} also gets closer and closer to *c*
- End result: nothing exceeds the speed of light

$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad} v_{db}}{c^2}}$$