From Last Time...
- Hydrogen atom quantum numbers
- Quantum jumps, tunneling and measurements

Today
- Superposition of wave functions
- Indistinguishability
- Electron spin: a new quantum effect
- The Hydrogen atom and the periodic table

Hydrogen Quantum Numbers
- Quantum numbers, \(n, l, m_l\)
- \(n\): how charge is distributed radially around the nucleus. Average radial distance.
- This determines the energy
- \(l\): how spherical the charge distribution
  - \(l = 0\), spherical, \(l = 1\) less spherical...
- \(m_l\): rotation of the charge around the \(z\) axis
  - Rotation clockwise or counterclockwise and how fast
- Small energy differences for \(l\) and \(m_l\) states

Measuring which slit
- Suppose we measure which slit the particle goes through?
- Interference pattern is destroyed!
- Wavefunction changes instantaneously over entire screen when measurement is made.
- Before superposition of wavefunctions through both slits. After only through one slit.

A superposition state
\[
\frac{1}{\sqrt{2}} \left( \begin{array}{c} \text{Margarita} \\ \text{Beer} \end{array} \right)
\]
- Margarita or Beer?
- This QM state has equal superposition of two.
- Each outcome (drinking margarita, drinking beer) is equally likely.
- Actual outcome not determined until measurement is made (drink is tasted).

What is object before the measurement?
\[
\frac{1}{\sqrt{2}} \left( \begin{array}{c} \text{Margarita} \\ \text{Beer} \end{array} \right)
\]
- What is this new drink?
- Is it really a physical object?
- Exactly how does the transformation from this object to a beer or a margarita take place?
- This is the collapse of the wavefunction.
- Details, probabilities in the collapse, depend on the wavefunction, and sometimes the measurement

Not universally accepted
- Historically, not everyone agreed with this interpretation.
- Einstein was a notable opponent
  - ‘God does not play dice’
- These ideas hotly debated in the early part of the 20th century.
- However, one more set of crazy ideas needed to understand the hydrogen atom and the periodic table.
Spin: An intrinsic property

- Free electron, by itself in space, not only has a charge, but also acts like a bar magnet with a N and S pole.
- Since electron has charge, could explain this if the electron is spinning.
- Then resulting current loops would produce magnetic field just like a bar magnet.
- But as far as we can tell the electron is not spinning.

Electron magnetic moment

- Why does it have a magnetic moment?
- It is a property of the electron in the same way that charge is a property.
- But there are some differences.
- Magnetic moment is a vector: has a size and a direction
- It’s size is intrinsic to the electron
- but the direction is variable.
- The ‘bar magnet’ can point in different directions.

Quantization of the direction

- But like everything in quantum mechanics, this magnitude and direction are quantized.
- And also like other things in quantum mechanics, if magnetic moment is very large, the quantization is not noticeable.
- But for an electron, the moment is very small.
  - The quantization effect is very large.
  - In fact, there is only one magnitude and two possible directions that the bar magnet can point.
  - We call these spin up and spin down.
  - Another quantum number: spin up: +1/2, down -1/2

Electron spin orientations

These are two different quantum states in which an electron can exist.

Other particles

- Other particles also have spin
- The proton is also a spin 1/2 particle.
- The neutron is a spin 1/2 particle.
- The photon is a spin 1 particle.
- The graviton is a spin 2 particle.

Particle in a box

- We labeled the quantum states with an integer
- The lowest energy state was labeled n=1
- This labeled the spatial properties of the wavefunction (wavelength, etc)
- Now we have an additional quantum property, spin.
  - Spin quantum number could be +1/2 or -1/2

There are two quantum states with n=1
Can write them as \( n = 1, \text{ spin} = +1/2 \) \( n = 1, \text{ spin} = -1/2 \)
Spin 1/2 particle in a box

We talked about two quantum states
\[ |n = 1, \text{ spin } = +1/2 \rangle \quad |n = 1, \text{ spin } = -1/2 \rangle \]
In isolated space, which has lower energy?

A. \[ |n = 1, \text{ spin } = +1/2 \rangle \]
B. \[ |n = 1, \text{ spin } = -1/2 \rangle \]
C. Both same

An example of degeneracy: two quantum states that have exactly the same energy.

Indistiguishability

- Another property of quantum particles
  - All electrons are ABSOLUTELY identical.
- Never true at the macroscopic scale.
- On the macroscopic scale, there is always some aspect that distinguishes two objects.
- Perhaps color, or rough or smooth surface
- Maybe a small scratch somewhere.
- Experimentally, no one has ever found any differences between electrons.

Indistiguishability and QM

- Quantum Mechanics says that electrons are absolutely indistinguishable.
  - Treats this as an experimental fact.
    - For instance, it is impossible to follow an electron throughout its orbit in order to identify it later.
- We can still label the particles, for instance
  - Electron #1, electron #2, electron #3
- But the results will be meaningful only if we preserve indistinguishability.
- Find that this leads to some unusual consiquenses

Example: 2 electrons on an atom

- Probability of finding an electron at a location is given by the square of the wavefunction.
  - Probability large here
  - Probability small here
- We have two electrons, so the question we would is ask is
  - How likely is it to find one electron at location \( r_1 \) and the other electron at \( r_2 \)?

- Suppose we want to describe the state with one electron in a 3s state... and one electron in a 3d state

On the atom, they look like this. (Both on the same atom).

- Must describe this with a wavefunction that says
  - We have two electrons
    - One of the electrons is in s-state, one in d-state
  - Also must preserve indistinguishability
**Question**

Which one of these states doesn’t ‘change’ when we switch particle labels.

**Preserves indistinguishability**

Switch particle labels

Wavefunction unchanged

**Physically measurable quantities**

- How can we label particles, but still not distinguish them?
- What is really meant is that no physically measurable results can depend on how we label the particles.
- One physically measurable result is the probability of finding an electron in a particular spatial location.

**Probabilities**

- The probability of finding the particles at particular locations is the square of the wavefunction.
- Indistinguishability says that these probabilities cannot change if we switch the labels on the particles.
- However the wavefunction *could* change, since it is not directly measurable. *(Probability is the square of the wavefunction)*

**Two possible wavefunctions**

- Two possible symmetries of the wavefunction, that keep the probability unchanged when we exchange particle labels:
  - The wavefunction does not change
  - The wavefunction changes sign only

  *In both cases the square is unchanged*
Spin-statistics theorem

- In both cases the probability is preserved, since it is the square of the wavefunction.
- Can be shown that
  - Integer spin particles (e.g. photons) have wavefunctions with ‘+’ sign (symmetric)
    These types of particles are called **Bosons**
  - Half-integer spin particles (e.g. electrons) have wavefunctions with ‘−’ sign (antisymmetric)
    These types of particles are called **Fermions**

So what?

- Fermions - antisymmetric wavefunction:
  
  ![Wavefunction example](image)

Try to put two Fermions in the same quantum state (for instance both in the s-state)

Pauli exclusion principle

- Only wave function permitted by indistinguishability is exactly zero. This means that this never happens.
- Cannot put two Fermions in the same quantum state
- This came entirely from indistinguishability, that electrons are identical.
- Without this,
  - there elements would not have diff. chem. props.,
  - properties of metals would be different,
  - neutron stars would collapse.

Include spin

- We labeled the states by their quantum numbers. One quantum number for each spatial dimension.
- Now there is an extra quantum number: spin.
- A quantum state is specified by it’s space part and also it’s spin part.
- An atom with several electrons filling quantum states starting with the lowest energy, filling quantum states until electrons are used.

Putting electrons on atom

- Electrons are Fermions
- Only one electron per quantum state

  ![Electron occupancy](image)

  - Hydrogen: 1 electron
    - one quantum state occupied
  - Helium: 2 electrons
    - two quantum states occupied

Other elements

- More electrons requires next higher energy states
- Lithium: three electrons
  
  ![Lithium electron occupancy](image)

  - n=2 states
    - higher energy
  - n=1 states
    - lowest energy, fill first
  - Other states empty
  - Elements with more electrons have more complex states occupied
Elements in the same column have similar chemical properties.