

**$WZ$ ,  $W\gamma$ ,  $WW$  and  $ZZ$  pair productions at TeV  $e\gamma$  colliders**

Kingman Cheung

*Dept. of Physics & Astronomy, Northwestern University, Evanston, Illinois 60208, USA***ABSTRACT**

We calculate the gauge-boson pairs  $W^-Z$ ,  $W^- \gamma$ ,  $W^+W^-$ ,  $ZZ$  productions in the  $e^- \gamma$  collisions, where the photon beam is realized by the laser back-scattering method. These processes are important tests for the non-abelian gauge sector of the standard model (SM). Precise calculations of these processes can therefore probe the anomalous gauge-boson interactions. Besides, these processes are important potential backgrounds for the intermediate mass Higgs (IMH) search in the  $e\gamma \rightarrow WH\nu$  production.

## I. INTRODUCTION

The electroweak standard model (SM) has so far been very successful and consistent with experiments. However, it is likely that there exist other models, of which the SM is the effective low energy limit. The future colliding facilities, which will operate at TeV scale, are likely to reveal new physics beyond SM. The symmetry-breaking and the non-abelian gauge-boson sectors are the most peculiar natures of SM. A lot have been discussed on the possibilities of the future hadronic and  $e^+e^-$  colliders to probe the gauge-boson and the symmetry-breaking sectors. With the recent discussion of the physics possibilities at  $e\gamma$  and  $\gamma\gamma$  colliders [1], they might be as important as the hadronic and  $e^+e^-$  colliders. They have backgrounds much cleaner than the hadronic colliders and should be as clean as the  $e^+e^-$  colliders, and also photon has anomalous gluon and quark contents [2] that enable one to study QCD directly.

The  $e^-\gamma$  collisions at  $e^+e^-$  machines can be realized by directing a low energy (a few eV) laser beam almost head-to-head to the incident positron beam. By Compton scattering, there are abundant, hard back-scattered photons in the same direction as the incident positron beam, and carry a substantial fraction of the energy of the incident positrons. Therefore, we have the  $e^-\gamma$  collisions. For details please see Refs. [3]. Other possibilities include the bremsstrahlung and beamstrahlung effects [4] but these methods produce photons mainly in the soft region [3], and beamstrahlung depends critically on the beam structure [4]. Therefore we shall limit all the calculations to  $e\gamma$  collisions produced by the laser back-scattering method.

In recent studies of the Higgs production in  $e\gamma$  collisions [5,6] through

$$e\gamma \rightarrow WH\nu, \tag{1}$$

the cross section is just a factor of 2 or 3 smaller than that of  $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}H$  for  $\sqrt{s} = 1 - 2$  TeV, and so this production might be a possible channel in searching for the Higgs boson. However, the backgrounds have not been fully analysed, therefore we cannot

draw any decisive conclusions. For the Higgs in the intermediate mass range (IMH) the signature, due to the dominate decay of  $H \rightarrow b\bar{b}$  and hadronic decay of  $W$ , will be

$$e^- \gamma \rightarrow W^- H \nu \rightarrow (jj)(b\bar{b})\nu, \quad (2)$$

where there are 4 jets plus missing energy in the final state. Two of the four jets are reconstructed to the  $W$  mass and the other two can be reconstructed as a resonance peak at the Higgs mass. For this signature the backgrounds are the  $W^-Z$ ,  $W^-W^+$  and  $ZZ$  productions when the  $W$  and  $Z$  bosons decay hadronically into four jets. If the Higgs mass falls very close to the  $W$  or  $Z$  masses, the signal is much more difficult to identify and precise calculation of the backgrounds under the  $W$  and  $Z$  peaks is necessary. Therefore, calculations of the  $W^-Z$ ,  $W^-W^+$  and  $ZZ$  pair productions are desirable as important potential backgrounds to the IMH search through  $e^- \gamma \rightarrow W^- H \nu$  production. In addition, it also suffers backgrounds from the  $e^- \gamma \rightarrow \bar{t} b \nu$  production [7] with  $\bar{t} \rightarrow \bar{b} W^-$ , and  $e^- \gamma \rightarrow t \bar{t} e^- \rightarrow b \bar{b} W W e^-$ .

Also attention has been focused on the single  $W$  production in the channel  $e^- \gamma \rightarrow W^- \nu$  [8] to probe the  $WW\gamma$  coupling and search for any anomalous gauge-boson interactions. To probe the  $WWZ$  coupling, however, we must go for the boson-pair productions of  $e^- \gamma \rightarrow W^- Z \nu$ ,  $W^+ W^- e^-$ . Another interesting point is that the quartic  $WW\gamma\gamma$  and  $WW\gamma Z$  couplings first come in in the  $e^- \gamma \rightarrow W^- Z \nu$ ,  $W^- \gamma \nu$  and  $W^- W^+ e^-$  productions. The calculation of these processes involves delicate cancellation among the contributions from triple gauge-boson, quartic gauge-boson and the other Feynman diagrams, which consist of well-tested vertices. Therefore, any anomalous interactions of the triple or quartic gauge-boson vertices will result in deviations from SM predictions. It is then favorable to quantify precisely the production of these gauge-boson pairs,  $W^-Z$ ,  $W^- \gamma$  and  $W^-W^+$ , within SM so that any deviations from these predictions will indicate some new physics in the gauge-boson sector. One advantage of these gauge-boson pair productions in  $e\gamma$  collisions over hadronic collisions is that they do not have large QCD background as they do in hadronic collisions. Also these processes as probes to test the triple and quartic gauge couplings should be as

important as the three gauge-boson productions in  $e^+e^-$  colliders [9].

In this paper we calculate the following processes of boson-pair productions in  $e^- \gamma$  collisions,

$$e^- \gamma \rightarrow W^- Z \nu, \quad (3)$$

$$\rightarrow W^- \gamma \nu, \quad (4)$$

$$\rightarrow W^- W^+ e^-, \quad (5)$$

$$\rightarrow Z Z e^-, \quad (6)$$

$$\rightarrow W^- H \nu, \quad (7)$$

$$\rightarrow Z H e^-. \quad (8)$$

The processes in Eqs. (3)–(6) are important because they are the major potential backgrounds to the IMH search in the channels of Eqs. (7) and (8) [5]. Besides, the processes in Eqs. (3)–(5) are important tests for SM because they involve non-abelian gauge couplings. These processes must be quantified precisely within SM before any anomalous triple or quartic gauge-boson interactions can be realized in these channels.

The organization of the paper is as follows: we briefly describe the calculation methods including the photon luminosity function in Sec. II, following which we present the results in Sec. III, and then summarize in Sec. IV. We will also present detail formulas for the matrix elements of the processes involved in the appendix A.

## II. CALCULATIONS

### A. Photon Luminosity

We use the energy spectrum of the back-scattered photon given by [3]

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)} \left[ 1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2} \right], \quad (9)$$

where

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2}\right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}, \quad (10)$$

$\xi = 4E_0\omega_0/m_e^2$ ,  $\omega_0$  is the energy of the incident laser photon,  $x = \omega/E_0$  is the fraction of the incident positron's energy carried by the back-scattered photon, and the maximum value  $x_{\max}$  is given by

$$x_{\max} = \frac{\xi}{1 + \xi}. \quad (11)$$

It is seen from Eq. (9) and (10) that the portion of photons with maximum energy grows with  $E_0$  and  $\omega_0$ . A large  $\omega_0$ , however, should be avoided so that the back-scattered photon does not interact with the incident photon and create unwanted  $e^+e^-$  pairs. The threshold for  $e^+e^-$  pair creation is  $\omega\omega_0 > m_e^2$ , so we require  $\omega_{\max}\omega_0 \lesssim m_e^2$ . Solving  $\omega_{\max}\omega_0 = m_e^2$ , we find

$$\xi = 2(1 + \sqrt{2}) \simeq 4.8. \quad (12)$$

For the choice  $\xi = 4.8$  one finds  $x_{\max} \simeq 0.83$ ,  $D(\xi) \simeq 1.8$ , and  $\omega_0 = 1.25(0.63)$  eV for a 0.5(1) TeV  $e^+e^-$  collider. Here we have assumed that the electron, positron and back-scattered photon beams are unpolarized. We also assume that, on average, the number of back-scattered photons produced per positron is 1 (i.e., the conversion coefficient  $k$  equals 1).

## B. Subprocesses Calculation

The  $W$  and  $Z$  bosons are detected through their leptonic or hadronic decays. So we are not going to impose any acceptance cuts on the  $W$  and  $Z$  bosons for their detections, instead, we assume some detection efficiencies for their decay products to estimate the number of observed events. On the other hand,  $\gamma$  can be observed directly in the final state by imposing a typical experimental acceptance, say,

$$\begin{aligned} p_T(\gamma) &> 15 \text{ GeV}, \\ |\eta(\gamma)| &< 2. \end{aligned} \quad (13)$$

We use the helicity amplitude method of Ref. [10] to evaluate the Feynman amplitudes, and keep the electron mass  $m_e$  finite in all the calculations. There are totally 11 contributing Feynman diagrams in the process  $e^- \gamma \rightarrow W^- Z \nu$ , 9 in  $e^- \gamma \rightarrow W^- \gamma \nu$ , 18 in  $e^- \gamma \rightarrow W^- W^+ e^-$ , and 6 in  $e^- \gamma \rightarrow Z Z e^-$ , in the general  $R_\xi$  gauge. The helicity amplitudes for the processes of Eqs. (3)–(6) are given in Appendix A. The processes of Eqs. (7) and (8) have been calculated in detail in Refs. [5,6]. The total cross-section  $\sigma$  is obtained by folding the subprocess cross-section  $\hat{\sigma}$  in with the photon luminosity function of Eqs. (9) and (10); shown in appendix A.

### III. RESULTS AND DISCUSSION

We show the dependence of the cross sections for all the processes of Eqs. (3)–(8), together with  $e^+ e^- \rightarrow \nu \bar{\nu} W^* W^* \rightarrow \nu \bar{\nu} H$ , in Fig. 1. We typically choose  $m_H = 100$  GeV in the intermediate mass range, and impose the acceptance cuts of Eq. (13) on the photon that occurs in the final state. The cross section of  $W^- W^+$  is overwhelming due to a high contribution from the Feynman diagrams with an almost on-shell  $t$ -channel  $\gamma$  propagator. This high cross section, of order 10 pb in the energy range of 0.5–2 TeV, is an advantage to probe the triple or quartic gauge-boson couplings. The  $WZ$  and  $W\gamma$  cross section is of order 0.5 and 1 pb in the same energy range, respectively. For a yearly luminosity of  $10 \text{ fb}^{-1}$ , we have about 5000  $WZ$  and 10000  $W\gamma$  events. The hadronic branching fraction of both  $W$  and  $Z$  is  $\sim 0.7$  and assuming 50% hadronic detection efficiency, we still have 625 observed events for  $WZ$  and 3500 events for  $W\gamma$ , which are numerous enough to observe any anomalous gauge-boson interactions. Therefore, even with 1% anomalous gauge coupling it could result in about 6 and 35 events in  $WZ$  and  $W\gamma$  productions respectively, and in the order of hundreds of events for  $WW$  production. Therefore, as mentioned above these gauge-boson pair productions as probes to test the triple and quartic gauge couplings are as important as the three gauge-boson productions in  $e^+ e^-$  collisions, which are of order 0.1 pb for  $\sqrt{s_{e^+e^-}} = 0.5\text{--}2$  TeV [9].  $ZZ$  production is insignificant at all for the energy range that we are considering. The  $WH$  production is of order 0.1–0.2 pb for  $\sqrt{s} = 1\text{--}2$  TeV and

$m_H = 100$  GeV, and  $ZH$  production is so much smaller that it will never be discovered. In comparison we also show the cross section of  $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}H$ , which is dominant over the  $e^+e^- \rightarrow ZH$  production for  $\sqrt{s} > 0.5$  TeV. We can see that at  $\sqrt{s} = 1(2)$  TeV the  $e^-\gamma \rightarrow W^-H\nu$  cross section is only about a factor of 2.5 (2) smaller than that of  $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}H$ .

For the IMH search in  $WH$  production total background from  $WW$ ,  $WZ$  and  $ZZ$  is about two order of magnitudes larger (see Fig. 1). But from Fig. 2 we can see that the high cross section of  $WW$  can go down sharply by requiring a moderate transverse momentum  $p_T(VV)$  cut, say  $p_T(VV) > 25(50)$  GeV at  $\sqrt{s} = 0.5(2)$  TeV, on the  $WW$  system to keep the  $\gamma$ -propagator far from being on-shell. Further reduction of the  $WW$  cross section can be achieved by central electron vetoing method, i.e., rejecting events with electrons detected in the central rapidity region ( $|\eta| < 3$ ). Then the total background from  $WZ$ ,  $ZZ$  and  $WW$  is only a few times larger than the IMH signal in  $e^-\gamma \rightarrow W^-H\nu$  production. Furthermore, if  $b$ -tagging has a high efficiency and the invariant mass reconstruction has a good resolution these backgrounds can be substantially reduced, so  $WH$  production remains a possible channel to search for the IMH. However, a more detail analysis taking into account the other backgrounds from  $e^-\gamma \rightarrow \bar{t}b\nu$ ,  $t\bar{t}e^-$  and detector resolutions is necessary to establish the Higgs-boson signal.

In Fig. 3, we show the dependence of the differential cross section  $d\sigma/dM(VV)$  on the invariant mass  $M(VV)$  of the boson pair at  $\sqrt{s} = 0.5$  and 2 TeV. As expected, these curves rise a little bit above their corresponding  $M(VV)$  threshold and then fall gradually as  $M(VV)$  increases further. However, the presence of any anomalous triple or quartic gauge-boson interactions can alter the  $WZ$ ,  $W\gamma$ ,  $WW$  and  $WH$  curves to some extent. These spectra can therefore serve as SM predictions to probe the anomalous gauge-boson sector.

## IV. CONCLUSIONS

We have quantified the productions of  $e^- \gamma \rightarrow W^- Z \nu$ ,  $W^- \gamma \nu$ ,  $W^- W^+ e^-$ ,  $ZZ e^-$  within SM, and presented the helicity amplitudes for these processes. These processes can probe the non-abelian gauge sector of the SM, and should be as good as the three gauge-boson pair productions in  $e^+ e^-$  collisions and better than those in hadronic collisions. The production rate of  $W^- W^+$  pair is high, and that of  $W^- Z$  and  $W^- \gamma$  are large enough that a percent-level anomalous gauge-boson interactions can be detected if they exist. On the other hand, the IMH search in the  $e \gamma \rightarrow WH \nu$  channel seems impossible due to high background from  $WW$  and  $WZ$ . However, we have shown in Fig. 2 that a  $p_T(VV)$  cut can substantially reduce the  $WW$  background, together with central electron vetoing method and  $b$ -tagging the total background from boson-pair productions is only a few times larger than the IMH signal. Nevertheless, a more detail signal-background analysis is needed.

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## APPENDIX A

In this appendix we present the matrix elements for processes  $e^- \gamma \rightarrow W^- Z \nu, W^- \gamma \nu, W^- W^+ e^-, ZZ e^-$ , from which explicit helicity amplitudes can be directly computed. To start with, we introduce some general notation:

$$g_a^W(f) = -g_b^W(f) = \frac{g}{2\sqrt{2}}, \quad (\text{A1})$$

$$g_a^Z(f) = g_Z \left( \frac{T_{3f}}{2} - Q_f x_w \right), \quad (\text{A2})$$

$$g_b^Z(f) = -g_Z \frac{T_{3f}}{2}, \quad (\text{A3})$$

$$g_a^\gamma(f) = e Q_f, \quad (\text{A4})$$

$$g_b^\gamma(f) = 0, \quad (\text{A5})$$

$$g^V(f) = g_a^V(f) + g_b^V(f) \gamma^5 \quad (V = \gamma, W, Z), \quad (\text{A6})$$

$$D^X(k) = \frac{1}{k^2 - M_X^2 + i\Gamma_X(k^2)m_X}, \quad \Gamma_X(k^2) = \Gamma_X \theta(k^2) \\ (\text{with } X = \gamma, W, Z, H), \quad (\text{A7})$$

$$P_V^{\alpha\beta}(k) = \left[ g^{\alpha\beta} + \frac{(1-\xi)k^\alpha k^\beta}{\xi k^2 - m_V^2} \right] D^V(k), \quad (\text{A8})$$

$$\Gamma^\alpha(k_1, k_2; \epsilon_1, \epsilon_2) = (k_1 - k_2)^\alpha \epsilon_1 \cdot \epsilon_2 + (2k_2 + k_1) \cdot \epsilon_1 \epsilon_2^\alpha - (2k_1 + k_2) \cdot \epsilon_2 \epsilon_1^\alpha, \quad (\text{A9})$$

$$g_{VWW} = \begin{cases} e \cot \theta_w & \text{for } V = Z \\ e & \text{for } V = \gamma. \end{cases} \quad (\text{A10})$$

Here  $Q_f$  and  $T_{3f}$  are the electric charge (in units of the positron charge) and the third component of weak isospin of the fermion  $f$ ,  $g$  is the SU(2) gauge coupling, and  $g_Z = g/\cos\theta_w$ ,  $x_w = \sin^2\theta_w$ , with  $\theta_w$  being the weak mixing angle in the Standard Model. Dots between 4-vectors denote scalar products and  $g_{\alpha\beta}$  is the Minkowskian metric tensor with  $g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$ ;  $\xi$  is a gauge-fixing parameter.

In Figs. 4 and 5, the momentum-labels  $p_i$  denote the momenta flowing along the corresponding fermion lines in the direction of the arrows. We shall always denote the associated spinors by the symbols  $u(p_i)$  and  $\bar{u}(p_i)$  for the ingoing and outgoing arrows, which is usual for the annihilation and creation of fermions, respectively.

1.  $e^- \gamma \rightarrow W^- Z \nu$

The contributing Feynman diagrams for  $e^-(p_1)\gamma(p_2) \rightarrow W^-(k_1)Z(k_2)\nu(q_1)$  are given in Fig. 4. We define a shorthand notation

$$J_1^\mu = \bar{u}(q_1)\gamma^\mu g^W(e)u(p_1) \times D^W(p_1 - q_1), \quad (\text{A11})$$

then the helicity amplitudes are given by

$$\begin{aligned} \mathcal{M}^{(a)} &= -g_{ZWW}g_{\gamma WW} \Gamma_\alpha(-k_1, p_2; \epsilon(k_1), \epsilon(p_2)) P_W^{\alpha\beta}(p_2 - k_1) \\ &\quad \times \Gamma_\beta(-k_2, p_1 - q_1; \epsilon(k_2), J_1), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned} \mathcal{M}^{(b)} &= -g_{ZWW}g_{\gamma WW} \Gamma_\alpha(k_2, k_1; \epsilon(k_2), \epsilon(k_1)) P_W^{\alpha\beta}(k_1 + k_2) \\ &\quad \times \Gamma_\beta(p_2, p_1 - q_1; \epsilon(p_2), J_1), \end{aligned} \quad (\text{A13})$$

$$\begin{aligned} \mathcal{M}^{(c)} &= g_{ZWW}g_{\gamma WW} [2\epsilon(p_2) \cdot \epsilon(k_2)\epsilon(k_1) \cdot J_1 - \epsilon(p_2) \cdot J_1\epsilon(k_1) \cdot \epsilon(k_2) \\ &\quad - \epsilon(p_2) \cdot \epsilon(k_1)\epsilon(k_2) \cdot J_1], \end{aligned} \quad (\text{A14})$$

$$\begin{aligned} \mathcal{M}^{(d,e)} &= g_{\gamma WW}\Gamma_\alpha(-k_1, p_2; \epsilon(k_1), \epsilon(p_2))P_W^{\alpha\beta}(p_2 - k_1) \\ &\quad \times \left[ \bar{u}(q_1)\gamma_\beta g^W(e) \frac{\not{p}_1 - \not{k}_2 + m_e}{(p_1 - k_2)^2 - m_e^2} \not{\epsilon}(k_2)g^Z(e)u(p_1) \right. \\ &\quad \left. + \bar{u}(q_1)\not{\epsilon}(k_2)g^Z(\nu) \frac{\not{q}_1 + \not{k}_2}{(q_1 + k_2)^2} \gamma_\beta g^W(e)u(p_1) \right], \end{aligned} \quad (\text{A15})$$

$$\begin{aligned} \mathcal{M}^{(f)} &= g_{ZWW}\Gamma_\alpha(k_2, k_1; \epsilon(k_2), \epsilon(k_1))P_W^{\alpha\beta}(k_1 + k_2) \\ &\quad \times \bar{u}(q_1)\gamma_\beta g^W(e) \frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2} \not{\epsilon}(p_2)g^\gamma(e)u(p_1), \end{aligned} \quad (\text{A16})$$

$$\begin{aligned} \mathcal{M}^{(g)} &= -\bar{u}(q_1)\not{\epsilon}(k_1)g^W(e) \frac{\not{q}_1 + \not{k}_1 + m_e}{(q_1 + k_1)^2 - m_e^2} \not{\epsilon}(k_2)g^Z(e) \frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2} \\ &\quad \not{\epsilon}(p_2)g^\gamma(e)u(p_1), \end{aligned} \quad (\text{A17})$$

$$\mathcal{M}^{(h)} = -\bar{u}(q_1)\not{\epsilon}(k_1)g^W(e)\frac{\not{q}_1 + \not{k}_1 + m_e}{(q_1 + k_1)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)\frac{\not{p}_1 - \not{k}_2 + m_e}{(p_1 - k_2)^2 - m_e^2}\not{\epsilon}(k_2)g^Z(e)u(p_1), \quad (\text{A18})$$

$$\mathcal{M}^{(i)} = -\bar{u}(q_1)\not{\epsilon}(k_2)g^Z(\nu)\frac{\not{q}_1 + \not{k}_2}{(q_1 + k_2)^2}\not{\epsilon}(k_1)g^W(e)\frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)u(p_1), \quad (\text{A19})$$

$$\mathcal{M}^{(j)} = -g^2m_W^2x_w \tan\theta_w \frac{\xi}{\xi(p_2 - k_1)^2 - m_W^2} \epsilon(k_1) \cdot \epsilon(p_2) \epsilon(k_2) \cdot J_1 \quad (\text{A20})$$

$$\mathcal{M}^{(k)} = -g^2m_W^2x_w \tan\theta_w \frac{\xi}{\xi(k_1 + k_2)^2 - m_W^2} \epsilon(k_1) \cdot \epsilon(k_2) \epsilon(p_2) \cdot J_1. \quad (\text{A21})$$

The contributing Feynman diagrams for  $e^- \gamma \rightarrow W^- \gamma \nu$  are obtained from those in Fig. 4 by replacing the  $Z$  by  $\gamma$ . The helicity amplitudes for  $e^-(p_1)\gamma(p_2) \rightarrow W^-(k_1)\gamma(k_2)\nu(q_1)$  can be obtained from the above expressions by replacing the corresponding  $g_{ZW}$  and  $g^Z(e$  or  $\nu)$  by  $g_{\gamma W}$  and  $g^\gamma(e$  or  $\nu)$  respectively, and substituting the  $\tan\theta_w$  in  $\mathcal{M}^{(j)}$  and  $\mathcal{M}^{(k)}$  by  $-1$ . Since  $g^\gamma(\nu) = 0$ , diagrams 4(e) and (i) do not contribute to the  $W^- \gamma$  production.

## 2. $e^- \gamma \rightarrow W^- W^+ e^-$

The contributing Feynman diagrams for the process  $e^-(p_1)\gamma(p_2) \rightarrow W^-(k_1)W^+(k_2)e^-(q_1)$  are shown in Fig. 5. We can also define a shorthand notation

$$J_V^\mu = \bar{u}(q_1)\gamma^\mu g^V(e)u(p_1) \times D^V(p_1 - q_1), \quad \text{where } V = \gamma, Z \quad (\text{A22})$$

then the helicity amplitudes are given by

$$\mathcal{M}^{(a)} = \sum_{V=\gamma, Z} -g_{VWW} g_{\gamma WW} P_W^{\alpha\beta}(p_2 - k_2) \times \Gamma_\alpha(-k_1, p_1 - q_1; \epsilon(k_1), J_V) \Gamma_\beta(p_2, -k_2; \epsilon(p_2), \epsilon(k_2)), \quad (\text{A23})$$

$$\mathcal{M}^{(b)} = \sum_{V=\gamma, Z} -g_{VWW} g_{\gamma WW} P_W^{\alpha\beta}(p_2 - k_1) \times \Gamma_\alpha(p_1 - q_1, -k_2; J_V, \epsilon(k_2)) \Gamma_\beta(-k_1, p_2; \epsilon(k_1), \epsilon(p_2)), \quad (\text{A24})$$

$$\mathcal{M}^{(c)} = \sum_{V=\gamma, Z} g_{VWW} g_{\gamma WW} [2\epsilon(k_1) \cdot \epsilon(k_2) \epsilon(p_2) \cdot J_V - \epsilon(k_1) \cdot J_V \epsilon(k_2) \cdot \epsilon(p_2) - \epsilon(k_2) \cdot J_V \epsilon(k_1) \cdot \epsilon(p_2)], \quad (\text{A25})$$

$$\mathcal{M}^{(d)} = -\bar{u}(q_1)\not{\epsilon}(k_2)g^W(e)\frac{\not{q}_1 + \not{k}_2}{(q_1 + k_2)^2}\not{\epsilon}(k_1)g^W(e)\frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)u(p_1), \quad (\text{A26})$$

$$\mathcal{M}^{(e)} = -\bar{u}(q_1)\not{\epsilon}(p_2)g^\gamma(e)\frac{\not{q}_1 - \not{p}_2 + m_e}{(q_1 - p_2)^2 - m_e^2}\not{\epsilon}(k_2)g^W(e)\frac{\not{p}_1 - \not{k}_1}{(p_1 - k_1)^2}\not{\epsilon}(k_1)g^W(e)u(p_1), \quad (\text{A27})$$

$$\begin{aligned} \mathcal{M}^{(f)} &= \sum_{V=\gamma,Z} g_{VWW}D^V(k_1 + k_2)\Gamma_\alpha(k_1, k_2; \epsilon(k_1), \epsilon(k_2)) \\ &\quad \times \bar{u}(q_1)\gamma^\alpha g^V(e)\frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)u(p_1), \end{aligned} \quad (\text{A28})$$

$$\begin{aligned} \mathcal{M}^{(g)} &= \sum_{V=\gamma,Z} g_{VWW}D^V(k_1 + k_2)\Gamma_\alpha(k_1, k_2; \epsilon(k_1), \epsilon(k_2)) \\ &\quad \times \bar{u}(q_1)\not{\epsilon}(p_2)g^\gamma(e)\frac{\not{q}_1 - \not{p}_2 + m_e}{(q_1 - p_2)^2 - m_e^2}\gamma^\alpha g^V(e)u(p_1), \end{aligned} \quad (\text{A29})$$

$$\begin{aligned} \mathcal{M}^{(h)} &= g_{\gamma WW}P_W^{\alpha\beta}(p_2 - k_2)\Gamma_\alpha(p_2, -k_2; \epsilon(p_2), \epsilon(k_2)) \\ &\quad \times \bar{u}(q_1)\gamma_\beta g^W(e)\frac{\not{p}_1 - \not{k}_1}{(p_1 - k_1)^2}\not{\epsilon}(k_1)g^W(e)u(p_1), \end{aligned} \quad (\text{A30})$$

$$\begin{aligned} \mathcal{M}^{(i)} &= g_{\gamma WW}P_W^{\alpha\beta}(p_2 - k_1)\Gamma_\alpha(-k_1, p_2; \epsilon(k_1), \epsilon(p_2)) \\ &\quad \times \bar{u}(q_1)\not{\epsilon}(k_2)g^W(e)\frac{\not{q}_1 + \not{k}_2}{(q_1 + k_2)^2}\gamma_\beta g^W(e)u(p_1), \end{aligned} \quad (\text{A31})$$

$$\begin{aligned} \mathcal{M}^{(j)} &= \sum_{V=\gamma,Z} g^2 m_W^2 x_w \frac{\xi}{\xi(p_2 - k_2)^2 - m_W^2} \epsilon(p_2) \cdot \epsilon(k_2) \epsilon(k_1) \cdot J_V \\ &\quad \times \begin{cases} -\tan\theta_w & \text{for } V = Z \\ 1 & \text{for } V = \gamma \end{cases}, \end{aligned} \quad (\text{A32})$$

$$\begin{aligned} \mathcal{M}^{(k)} &= \sum_{V=\gamma,Z} g^2 m_W^2 x_w \frac{\xi}{\xi(p_2 - k_1)^2 - m_W^2} \epsilon(p_2) \cdot \epsilon(k_1) \epsilon(k_2) \cdot J_V \\ &\quad \times \begin{cases} -\tan\theta_w & \text{for } V = Z \\ 1 & \text{for } V = \gamma \end{cases}. \end{aligned} \quad (\text{A33})$$

### 3. $e^- \gamma \rightarrow ZZ e^-$

The contributing Feynman diagrams for the process  $e^-(p_1)\gamma(p_2) \rightarrow Z(k_1)Z(k_2)e^-(q_1)$  are the same as in Fig. 5(d) with the  $W$ -bosons replaced by  $Z$ -bosons plus all possible permutations. Totally it has six contributing Feynman diagrams. They are given by

$$\mathcal{M}^{(a)} = -\bar{u}(q_1)\not{\epsilon}(k_1)g^Z(e)\frac{\not{q}_1 + \not{k}_1 + m_e}{(q_1 + k_1)^2 - m_e^2}\not{\epsilon}(k_2)g^Z(e)\frac{\not{p}_1 + \not{p}_2 + m_e}{(p_1 + p_2)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)u(p_1), \quad (\text{A34})$$

$$\mathcal{M}^{(b)} = -\bar{u}(q_1)\not{\epsilon}(k_1)g^Z(e)\frac{\not{q}_1 + \not{k}_1 + m_e}{(q_1 + k_1)^2 - m_e^2}\not{\epsilon}(p_2)g^\gamma(e)\frac{\not{p}_1 - \not{k}_2 + m_e}{(p_1 - k_2)^2 - m_e^2}\not{\epsilon}(k_2)g^Z(e)u(p_1), \quad (\text{A35})$$

$$\mathcal{M}^{(c)} = -\bar{u}(q_1)\not{\epsilon}(p_2)g^\gamma(e)\frac{\not{q}_1 - \not{p}_2 + m_e}{(q_1 - p_2)^2 - m_e^2}\not{\epsilon}(k_1)g^Z(e)\frac{\not{p}_1 - \not{k}_2 + m_e}{(p_1 - k_2)^2 - m_e^2}\not{\epsilon}(k_2)g^Z(e)u(p_1), \quad (\text{A36})$$

plus those terms with  $(k_1 \leftrightarrow k_2)$ .

These matrix elements are to be summed over polarizations and spins of the final state gauge-bosons and fermions respectively, and average over the polarizations of the incoming photon and spins of the initial state electron. Then the cross section  $\sigma$  is obtained by folding the subprocess cross-section  $\hat{\sigma}$  in with the photon luminosity function as

$$\sigma(s) = \int_{M_{\text{final}}/s}^{x_{\text{max}}} dx F_{\gamma/e}(x) \hat{\sigma}(\hat{s} = xs), \quad (\text{A37})$$

where

$$\hat{\sigma}(\hat{s}) = \frac{1}{2(\hat{s}-m_e^2)} \int \frac{d^3k_1}{(2\pi)^3 k_1^0} \frac{d^3k_2}{(2\pi)^3 k_2^0} \frac{d^3q_1}{(2\pi)^3 q_1^0} \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - k_1 - k_2 - q_1) \sum |\mathcal{M}|^2 \quad (\text{A38})$$

and  $M_{\text{final}}$  is the sum of the masses of the final state particles.

## REFERENCES

- [1] D. Borden, D. Bauer and D. Caldwell, SLAC preprint, SLAC-PUB-5715, (1992).
- [2] E. Witten, Nucl. Phys. **B120**,189 (1977).
- [3] V. Telnov, Nucl. Instr. & Methods **A294**, 72 (1990); I. Ginzburg, G. Kotkin, V. Serbo and V. Telnov, Nucl. Instr. & Methods **205**, 47 (1983); *idem* **219**, 5 (1984).
- [4] V. N. Baier and V. M. Katkov, Phys. Lett. **25A**, 492 (1967); R. J. Noble, Nucl. Instr. & Meth. **A256**, 427 (1987); R. B. Palmer, Ann. Rev. Nucl. Part. Sci. **40**, 529 (1990).
- [5] E. Boos, *et al.*, Phys. Lett. **B273**, 173 (1991).
- [6] K. Hagiwara, I. Watanabe, and P. Zerwas, Phys. Lett. **B278**, 187 (1992).
- [7] G. Jikia, Nucl. Phys. **B374**, 83 (1992).
- [8] I. Ginzburg, *et al.*, Nucl. Phys. **B228**, 258 (1983); K. Mikaelian, Phys. Rev. **D30**, 1115 (1984); J. Robinson and T. Rizzo, Phys. Rev. **D33**, 2608 (1986); G. Couture, S. Godfrey and P. Kalyniak, Phys. Lett. **B218**, 361 (1989); E. Yehudai, Phys. Rev. **D41**, 33 (1990); S. Choi and F. Schrempp, DESY Report 91-155, to appear in *Proc. of EE500 European Working Groups,  $e^+e^-$  Collision at 500 GeV: The Physics Potential*, ed P. Zerwas, (DESY, Hamburg, 1992); O. Philipsen, DESY Report 92-004; A. Denner and S. Dittmaier, CERN Report CERN-TH.6585/92.
- [9] V. Barger, T. Han and R. N. J. Phillips, Phys. Rev. **D39**, 146 (1989); A. Tofighi-Niaki and J. F. Gunion, Phys. Rev. **D39**, 720 (1989).
- [10] V. Barger, A. Stange, and R. J. N. Phillips, Phys. Rev. **D44**, 1987 (1991).

## FIGURES

FIG. 1. The production cross sections for the processes in Eqs. (3)–(8), and  $e^+e^- \rightarrow \nu\bar{\nu}W^*W^* \rightarrow \nu\bar{\nu}H$  for  $m_H = 100$  GeV versus  $\sqrt{s}$  of the parent  $e^+e^-$  collider. The acceptance cuts on the final state photon are  $p_T(\gamma) > 15$  GeV and  $|\eta(\gamma)| < 2$ .

FIG. 2. The differential cross sections  $d\sigma/dp_T(VV)$  for the processes in Eqs. (3)–(8) for  $m_H = 100$  GeV versus the transverse momentum  $p_T(VV)$  of the boson pair at  $\sqrt{s} =$  (a) 0.5 and (b) 2 TeV. The acceptance cuts on the final state photon are  $p_T(\gamma) > 15$  GeV and  $|\eta(\gamma)| < 2$ .

FIG. 3. The differential cross sections  $d\sigma/dM(VV)$  for the processes in Eqs. (3)–(8) for  $m_H = 100$  GeV versus the invariant mass  $M(VV)$  of the boson pair at  $\sqrt{s} =$  (a) 0.5 and (b) 2 TeV. The acceptance cuts on the final state photon are  $p_T(\gamma) > 15$  GeV and  $|\eta(\gamma)| < 2$ .

FIG. 4. Contributing Feynman diagrams for the process  $e^-\gamma \rightarrow W^-Z\nu$ . Those for  $e^-\gamma \rightarrow W^-\gamma\nu$  are the same with ( $Z \leftrightarrow \gamma$ ), except it does not have contributions from (e) and (i).

FIG. 5. Contributing Feynman diagrams for the process  $e^-\gamma \rightarrow W^-W^+e^-$ . Those for  $e^-\gamma \rightarrow ZZe^-$  are as in (d) plus five other permutations













