1. The volume expansion coefficient $\alpha$ and the isothermal compressibility $\kappa_T$ are found experimentally for a certain substance to be

$$\alpha = \frac{1}{V} \frac{\partial V}{\partial T}_P = \frac{1 + 3aN/VT^2}{T}$$

and

$$\kappa_T = -\frac{1}{V} \frac{\partial V}{\partial P}_T = -\frac{1 + aN/VT^2}{P}.$$

(a) Find the volume as a function of pressure along an isotherm.

(b) By taking derivatives of $\alpha$ and $\kappa_T$, show that the forms for these two quantities are thermodynamically consistent, i.e., $V$ is a definite function of $P$ and $T$.

2. An ideal classical monatomic gas is at such a high temperature that the atoms move at ultrarelativistic speeds. Their energies are thus given by $\epsilon = cp$, where $c$ is the speed of light and $p$ is the momentum.

(a) Find the Helmholtz free energy $A$ of the system.

(b) Show that $C_V = 3Nk$.

3. Consider a two-dimensional Fermi gas.

(a) Find the relation between $n$, the number density, and $\mu$, the chemical potential, at zero temperature.

(b) Sketch the dependence of $\mu(T)$, the internal energy $U(T)$, and the specific heat $C_V(T)$ on temperature for all temperatures.

**FORMULA SHEET**

\begin{equation}
\begin{aligned}
dU &= dQ - dW = TdS - PdV; \quad dA = -SdT - PdV; \quad dG = -SdT + VdP \\
C_V &= T \left( \frac{\partial S}{\partial T} \right)_V; \quad \frac{dP}{dT} = \frac{\ell}{T \Delta u} \\
Q &= Tr\rho = \sum_n \exp(-\beta E_n); \quad Q = \sum_{nN} \exp[-\beta(E_{nN} - \mu N)]; \quad A = -kT \ln Q \\
S(U,V) &= k \ln[\Gamma(U)]; \quad \left( \frac{\partial S_r}{\partial E} \right)_{N,V} = \frac{1}{T}; \quad \mu = -\left( \frac{\partial S_r}{\partial N} \right)_{E,V} T
\end{aligned}
\end{equation}

$$\bar{n}_k = \left[ e^{\beta(\epsilon_k - \mu)} \pm 1 \right]^{-1}$$