Physics 715 **Sample Midterm Exam** **March 7, 2005**

Best two of three question grades will be counted.

1. The volume expansion coefficient α and the isothermal compressibility κ_T are found experimentally for a certain substance to be

$$\alpha = \frac{1}{V}(\frac{\partial V}{\partial T})_P = \frac{1 + 3aN/VT^2}{T}$$

and

$$\kappa_T = -\frac{1}{V}(\frac{\partial V}{\partial P})_T = -\frac{1 + aN/VT^2}{P}.$$

(a) Find the volume as a function of pressure along an isotherm.

(b) By taking derivatives of α and κ_T, show that the forms for these two quantities are thermodynamically consistent, i. e. , V is a definite function of P and T.

2. An ideal classical monatomic gas is at such a high temperature that the atoms move at ultrarelativistic speeds. Their energies are thus given by $\epsilon = cp$, where c is the speed of light and p is the momentum.

(a) Find the Helmholtz free energy A of the system.

(b) Show that $C_V = 3Nk$.

3. Consider a two-dimensional Fermi gas.

(a) Find the relation between n, the number density, and μ, the chemical potential, at zero temperature.

(b) Sketch the dependence of $\mu(T)$, the internal energy $U(T)$, and the specific heat $C_V(T)$ on temperature for all temperatures.

FORMULA SHEET

$$dU = dQ - dW = TdS - PdV; \quad dA = -SdT - PdV; \quad dG = -SdT + VdP \quad (1)$$

$$C_V = T(\frac{\partial S}{\partial T})_V; \qquad \frac{dP}{dT} = \frac{\ell}{T\Delta v}$$

$$Q = Tr\rho = \sum_n \exp(-\beta E_n); \quad \mathcal{Q} = \sum_{nN} \exp[-\beta(E_{nN} - \mu N)]; \quad A = -kT\ln Q \quad (2)$$

$$S(U,V) = k\log[\Gamma(U)]; \quad \left(\frac{\partial S_r}{\partial E}\right)_{N,V} = \frac{1}{T}; \quad \mu = -\left(\frac{\partial S_r}{\partial N}\right)_{E,V} T \quad (3)$$

$$\overline{n_{\vec{k}}} = [e^{\beta(\epsilon_{\vec{k}} - \mu)} \pm 1]^{-1}$$