PHYSICS 715 PROBLEM SET 3 Due April 4, 2005

1. **Sunlight.** Let \( \rho(T, \lambda) \) be the energy density per unit volume in the wavelength range from \( \lambda \) to \( \lambda + d\lambda \) of black-body radiation at temperature \( T \).
   
   (a) At what wavelength does the spectrum \( \rho(T, \lambda) \) take on its maximum value?
   
   (b) The radiation emitted by the sun follows a blackbody spectrum reasonably well. If the maximum intensity occurs at 480 nm, what is the temperature at the surface of the sun?
   
   (c) In what ways do you expect \( \rho(T, \lambda) \) for the sun to deviate from the blackbody form?

2. **Langevin function.** A paramagnetic salt consists of \( N \) atoms, each with a magnetic moment of fixed length \( \mu \). The Hamiltonian is

   \[ \mathcal{H} = -H\mu \sum_{n=1}^{N} \cos \theta_n, \]

   where \( H \) is the external magnetic field and \( \theta_n \) is the angle of the magnetic moment (treated classically) relative to the field.

   (a) Show that the magnetic moment in equilibrium is

   \[ M = N\mu \left( \coth \frac{\mu H}{kT} - \frac{kT}{\mu H} \right) \]

   (b) Find an expression for \( \chi = \left( \frac{\partial M}{\partial H} \right)_T \), at \( H = 0 \).

   (c) At high temperatures, verify Curie’s law: \( \chi = C/T \), where \( C \) is a constant. Determine \( C \) in terms of the parameters already given.

3. **Classical Molecules as Harmonic Oscillators.** Consider a system of \( N \) noninteracting diatomic molecules. The Hamiltonian is

   \[ \mathcal{H} = \frac{1}{2m} \sum_{n=1}^{N} (\vec{p}_n^2 + \vec{p}'_n^2) + \frac{K}{2} \sum_{n=1}^{N} |\vec{r}_n - \vec{r}'_n|^2, \]

   where \( \vec{p}_n \) and \( \vec{p}'_n \) are the momenta of the atoms in the \( n \)-th molecule and similarly for \( \vec{r}_n \) and \( \vec{r}'_n \).

   (a) Find the Helmholtz free energy.

   (b) Find the specific heat.

   (c) Find the mean square separation of the atoms in a molecule.

4. **Entropy of Mixing.** The entropy of an ideal gas is
\[ S = -\left( \frac{\partial A}{\partial T} \right)_V = Nk_B \ln(V/N) + Nk_B - Nk_Bf'(T), \]

where \( f(T) = -T \ln Q_{\text{int}}(T) - \frac{3}{2} T \ln \left( \frac{m k_B T}{2 \pi h^2} \right). \) Now let us look at two different ideal gases consisting of \( N_1 \) and \( N_2 \) particles. Let us imagine them initially in neighboring containers of volumes \( V_1 \) and \( V_2 \) separated by a conducting partition.

(a) In terms of \( N_1, V_1, N_2, V_2 \) and \( T \), what is the total entropy?

(b) Now remove the partition and allow the gases to mix. After mixing, what is the total entropy?

(c) How much work would be required, at a minimum, to separate the gases?

5. **Phase transition of the ideal Bose system.** The temperature derivative of the specific heat at constant volume of the ideal Bose gas is discontinuous at \( T_c \), the Bose condensation temperature, which makes it technically a third-order transition. Show that the magnitude of the discontinuity is given by:

\[ \left( \frac{d}{dT} C_V \right)_{T-T_c} - \left( \frac{d}{dT} C_V \right)_{T-T_c} = \frac{3.66Nk}{T_c} \]