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Ferromagnetism–The Curie Temperature of Gadolinium

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Abstract

The Curie temperature of Gadolinium is determined by measuring the magnetic susceptibility of a Gadolinium sample as a function of temperature. The data are analyzed using the Curie-Weiss law which contains the Curie temperature as a parameter. Gadolinium is unusual in that the Curie temperature is very close to room temperature.

1 Introduction

Ferromagnetic materials show ferromagnetic behavior only below a critical temperature called the Curie temperature, above which the material has normal paramagnetic behavior. The approach to ferromagnetism as a function of temperature from above is described by the Curie-Weiss Law which gives the magnetic susceptibility as a function of temperature.

$$\chi = \mu - 1 = \frac{C}{T - T_C} \tag{1}$$

where χ and μ are the magnetic susceptibility and and relative magnetic permeability of the material respectively. C is a constant characteristic for a given substance and T_C is the Curie temperature. Eqn. 1 is only valid **above** the Curie temperature.

The relative magnetic susceptibility of a material is readily determined by placing a sample of the material inside a small coil and measuring the inductance of the coil with and without the sample. If the inductance is measured as a function of temperature from above to below the Curie temperature, the Curie-Weiss law, Equ. 1, can be used to determine the Curie temperature.

The relative magnetic permeability of the sample can be written as

$$\mu = \frac{L(T)}{L_0} \tag{2}$$

where $L(T)$ is the inductance of the coil at temperature T and L_0 is the inductance of the coil without the sample. This is not exactly the relative permeability since not all the magnetic flux will couple to the sample. From Eqn. 1 we can write an equation linear in T as

$$\left(\frac{L(T)}{L_0} - 1\right)^{-1} = \frac{T - T_C}{C}. \tag{3}$$

The left hand side of this equation is zero when $T = T_C$ so a plot of the left hand side vs temperature extrapolated to zero will intersect the x-axis at $T = T_C$. The lack of a 100% fill factor for the coil will not affect this result.

2 Theory

Paramagnetism is a property exhibited by substances which, when placed in a magnetic field, are magnetised parallel to the field to an extent proportional to the field (except at very low temperatures or very large magnetic fields). Paramagnetic materials always have permeabilities greater than 1, but the values are in general not as great as those of ferromagnetic materials. Ferromagnetic materials have the property, that below a certain temperature called the Curie temperature, the atomic magnetic moments tend to line up in a common direction.

The classical theory of paramagnetism treats the substance as a collection of magnetic dipoles with no interactions between them. In an external magnetic field, each magnetic dipole has a potential energy given by:

$$E = \, -\mu \cdot \mathbf{H} = -\mu H \cos\theta \tag{4}$$

where μ is the magnetic moment of the dipole, H is the applied magnetic field and θ is the angle between the dipole and the direction of H. If there are N dipoles per unit volume the magnetization would be given by $\mathbf{M} = \mathbf{N}\mu$ where the direction of the magnetization would be that of the applied field. However in the presence of thermal agitation it is necessary to use the Boltzmann distribution to average over the dipole distribution in thermal equlibrium at temperature T. We then have:

$$M = N\mu\overline{\cos\theta} = N\mu \int e^{-E/kT} \cos\theta d\Omega / \int e^{-E/kT} d\Omega \tag{5}$$

where $d\Omega$ is the element of solid angle and $e^{-E/kT}$ is the Boltzmann distibution of a dipole at angle θ with respect to the applied field at absolute temperature T. The integration yields a result in terms of the Langevin function $L(x) = \coth x - 1/x$ with $x = \mu H/kT$ giving:

$$M = N\mu L(x). \tag{6}$$

When $x \ll 1$ the Langevin function $L(x)$ approaches $x/3$ so that Equ. 6 becomes:

$$M \cong N\mu^2 H/3kT. \tag{7}$$

This is a very good approximation except at low temperature or extremely high magnetic fields and gives us the Curie law $\chi \equiv M/H = C/T$.

The theory outlined above neglects the interaction between the magnetic moments. For certain metals this additional interaction is very important and leads to ferromagnetism. Ferromagnetism is chacterized by a crtical temperature called the Curie temperature above which the substance behaves as a paramagnet and below the Curie temperature the substance possesses a spontaneous magnetization in the absense of a magnetic field. Upon application of a weak magnetic field, the magnetization increases rapidly to a high value called the saturation magnetization, which is in general a function of temperature.

The basis of the Weiss molecular theory of ferromagnetism is that below the Curie temperature, a ferromagnet is composed of small, spontaneously magnetized regions called domains and the total magnetic moment of the material is the vector sum of the magnetic moments of the individual domains. Each domain is magnetized due to the strong magnetic interaction within the domain which tends to align the individual magnetic moments within the domain. The spontaneous magnetization below the Curie temperature comes about from an internal magnetic field called the Weiss molecular field which is proportional to the magnetization of the domain. Thus the effective field acting on any magnetic moment within the domain may be written as:

$$H = H_0 + \lambda M \tag{8}$$

where H_0 is an externally applied field and λM is the Weiss molecular field whose order of magnitude in iron is 10^7 oersteds. Above the Curie temperature, the Curie law now becomes:

$$M/(H_0 + \lambda M) = C/T. \tag{9}$$

Solving for $\chi = M/H_0$ we obtain:

$$\chi = M/H_0 = C/(T - C\lambda) = C/(T - T_C) \tag{10}$$

which defines the Curie temperature as $T_C = C\lambda$ giving us the Curie-Weiss law for the paramagnetic behavior of ferromagnetic substances above the Curie temperature. We see that the Curie-Weiss law leads to a nonzero magnetization when $H_0 = 0$ at $T = T_C$.

A quantitative description of ferromagnetism requires a quantum theory treatment. The Heisenberg theory is based on an effective interaction between the electron spins, the exchange interaction energy, given by:

$$ExchangeEnergy = -2J_{ij}\mathbf{S_i} \cdot \mathbf{S_j} \tag{11}$$

where $\mathbf{S_i}$ and $\mathbf{S_j}$ are the spin angular momentum vectors of two electrons i and j, and J_{ij} is the exchange integral between the electrons. Depending on the balance between the Pauli Principle and the electrostatic interaction energy of the electrons, the exchange integral may be positive or negative corresponding to parallel spins (ferromagnetism) or antiparallel spins (antiferromagnetism or ferrimagnetism). The order of magnitude of J is $kT_C \sim 0.025$ eV for Gadolinium. Shown below is a plot of the magnetization at temperature T relative to the magnetization at at $T = 0$ as obtained from the theory. The plot is from *Statistical and Thermal Physics* by F. Reif, Sec. 10.7.

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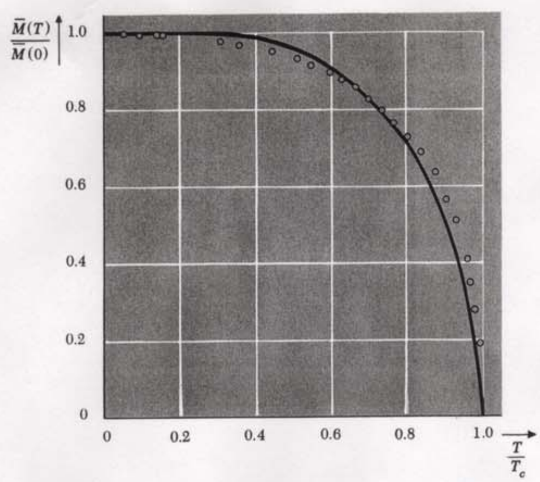

***Fig. 10·7·2** Spontaneous magnetization $\bar{M}$ of a ferromagnet as a function of temperature T in zero external magnetic field. The curve is based on the molecular field theory of (10·7·10) and (10·7·9) with $S = \frac{1}{2}$. The points indicate experimental values for nickel (measured by P. Weiss and R. Forrer, Ann. Phys., vol. 5, p. 153 (1926)).*

The following table gives Curie temperatures for common ferromagnetic materials.

Table 16.1 *Some ferromagnetic materials and their Curie temperatures.*

Material	T_c (K)
Iron	1,043
Nickel	633
Fe & Ni alloy (50% each)	803
Gadolinium	293
Gadolinium chloride ($GdCl_3$)	2.2
Chromium bromide ($CrBr_3$)	37
Europium oxide (EuO)	77
Europium sulfide (EuS)	16.5

Source: D. H. Martin, *Magnetism in Solids* (MIT Press, Cambridge, MA, 1967).

3 Apparatus

3.1 Coil

The coil is from a commercial relay and has an inductance of about 2.25 mH.

3.2 Gadolinium Sample

The Gadolinium sample is in the form of a rod 6.35 mm in diameter and length 10 mm. The Gadolinium is 99.9% pure.

3.3 Thermocouple Thermometer

The thermometer is a Fluke Model 51 type K thermocouple thermometer. with a range of −200°C to 1370°C. The rated accuracy is $\pm(0.1\%$ of reading $+ 0.7°C)$.

3.4 Thermoelectric Cooler

The Thermoelecric Cooler is an Advanced Thermoelectric model TCP-30. The cooler looks like a hot plate but can go up or down in temperature. The device is a solid state heat pump based on the Peltier Effect by which DC current applied across two dissimilar materials causes a temperature differential.

The two materials are p-type and n-type bismuth-telluride semiconductors. The cooler has a control temperature range of $-20°$ C to $100°$ C, the actual operational temperature range being set by the heat load on the cooler, and is controlled by a separate temperature controller which supplies the power to the thermoelectric cooler. The maximum cooler power rating is 12V at 6A.

3.5 Programmable Temperature Controller

The Temperature Controller is an Oven Industries model 5C7-362. The controller is computer controlled and supplies power to the Thermoelectric Cooler from a 12 V DC power supply. The controller output signal to the cooler is Pulse Width Modulated at 2700 Hz and sets the average power output to maintain a set-point temperature that has been entered into the control software. The sample temperature is measured by a thermistor mounted inside the coil together with the sample. The thermistor signal is fed back to the temperature controller and the software compares the thermistor temperature to the set-point temperature, signaling the controller to supply just enough power to make the thermistor and set-point temperatures equal. This control scheme can control the temperature to $\pm 0.1°$C at the control sensor.

3.6 Thermistor Sensor

The thermistor is an Oven Industries model TS67 with a base resistance of 15,000 ohms at 25° C. The temperature accuracy as determined from the look-up table in the computer program is $\pm 1.0°$ C over a 0° C to 100° C temperature range. However the temperature scale can be independently calibrated and the result programmed into the controller. The reference temperature is taken to be 25° C since the thermistor input for the reference temperature is replaced by a 15k resistor. Since the temperature of this resistor is not necessarily at 25° C, an INPUT 1 OFFSET temperature must be entered.

3.7 Control Circuit

The power supply and temperature controller are mounted in a control circuit box which has an ON-OFF switch and an ammeter that is zero centered and reads the current to the temperature controller. A negative current cools and a positive current heats the Cooler. This current starts out high and gradually drops to zero as the temperature set-point is reached.

3.8 Inductance Meter

The Inductance Meter is a Metric model DLM-240 3-1/2 digit digital LCR meter. The inductance range is from 200 μH to 200 H.

4 Experiment

1. Calibrate the Fluke thermometer in an ice-water equilibrium mixture.

2. Calibrate the Temperature Controller by placing the thermistor and the thermometer in the oil bath at room temperature. You will have to read the temperature reported by the thermistor using the software described below. If the thermometer and thermistor readings do not agree, an offset temperature parameter can be entered into the software.

3. Place the Gadolinium sample and the thermistor inside the coil together with a small piece of sponge rubber to hold everything in place. The whole assembly is placed in a small beaker filled with mineral oil which seves as a heat sink. Place the beaker on the cooler plate.

4. Open the controller program MC363.exe located as an icon on the desktop. Initialize the connection to the controller by clicking on **INITIALIZE** having selected the appropriate serial port. The Instructor will guide you through the configuration parameters and show you how to set the desired temperature. The set temperature is entered into the upper left-hand box labeled **FIXED SET TEMPERATURE**.

5. Measure the inductance of the coil as a function of temperature in steps of 0.1° C from 17° to 27° C. It is important to change the temperature slowly enough so that the sample temperature and thermistor temperature stay in equilibrium. Changing no faster than 0.1° per min. seems to be adequate. The inductance values should be somewhere between 2 and 4 mH. Make sure that the temperature of the sample has stabilized before you record the inductance value.

6. Eqn. 3 is a linear relation (y vs x)beweeen measured quantities and temperature with the x intercept giving $T = T_C$ when y = 0. Plot the left hand side vs temperature and the result will show a linear $T - T_C$ behavior above the Curie temperature. Extrapolate the linear portion of the plot to y = 0 and determine the temperature intercept. You will have to use some judgement as to how much data to include in the linear extrapolation. Remember that your result for the Curie temperature is no better than your temperature calibration of the sample temperature. A computer least squares fit of the data is appropriate. Your result should include your result for the Curie temperature with error and a value for the chi-square of the fit. Determine the chi-square probability for your fit.

References

[1] Charles Kittel. *Introduction to Solid State Physics 6th ed. 1986.* John Wiley & Sons, Inc. See the sections on Paramagnetism and Ferromagnetism.

[2] F. Reif. *Fundamentals of Statistical and Thermal Physics.* McGraw-Hill Book Company. Secs. 6.3, 7.8, 10.6, 10.7.