Practical Applications of Field Theory in Cosmology

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UNITS

• The most natural units to use:
  \( \hbar = c = 1 \)

• Consequence:
  mass = energy = GeV
  length = time = 1/GeV

If \( \hbar = c = \frac{M_{pl}}{\sqrt{8\pi}} = 1 \), dimensionless.

Sometimes normal units will be used in the talk.
Plan

• Lecture 1
  • Basic Intro to cosmology
  • Fundamental problems
  • Inflation

• Lecture 2
  • Baryogenesis/Leptogenesis
  • Dark Energy
  • Outlook
Selected General Background

- **General Cosmology**
  - Scott Dodelson, *MODERN COSMOLOGY*

- **Inflationary references**

- **General Baryogenesis**

- **Cosmology related to supersymmetry**
The Very Basic
What is cosmology?

• Study of the origin and large scale structure of the universe
  ◦ Large scale > 10 kpc (= 30,000 lyr; galaxy size).
  ◦ Largest scale observed (around 10,000 Mpc).

• Traditionally: gravitational and thermal history
  ◦ Far away galaxies seem to receding away from us with a velocity proportional to its distance. (universe is not static or stationary -- history)
  ◦ There is a thermal background radiation at 2.7 degrees Kelvin. (thermal history)
Observational foundations

- Hubble Expansion (redshift of galaxies, quasar, supernovae, etc. as a function of brightness)
- Homogeneous and isotropic $T=2.72^{\circ}K$ background $\gamma$
- Light element abundances (absorption/emission spectra)
- Galaxy surveys (distribution of visible matter)
- Lensing (distribution of invisible clumped matter)
- Temperature fluctuations (primordial, SZ effect, etc.)
- Diffuse gamma ray, X-ray, etc.
- Cosmic rays (neutrino, positron, antiproton, ultra-high energy, etc.)
Where Field Theory Enters

• Einstein equations (Equivalence principle)

\[ S = \frac{-1}{16 \pi} \int d^4 x \sqrt{-g} \left( R + \int d^4 x \sqrt{-g} L_M \right) \]

\[ R_{\mu \nu}[g_{\mu \nu}] = \frac{1}{2} g_{\mu \nu} R[g_{\mu \nu}] = 8 \pi T_{\mu \nu}[g_{\mu \nu}] \]

Put in known fields (more later . . .)

• Boltzmann Equations

\[ [ p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta \gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} ] f (x^\alpha, p^\alpha) = C[f] \]

[De Groot, Van Leeuwen, Van Weert 1980]

Collision term; Approximation
Homogeneity and Isotropy

- “on the average” Homogeneity and isotropy

\[
\begin{aligned}
ds^2 &= dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Theta^2 + r^2 \sin^2 \Theta d\varphi^2 \right] \\
&\uparrow \\
&\text{characterizes the curvature of space at a fixed time}
\end{aligned}
\]

\[
3R = 6k/a^2 < 0.01 \ H^2 \sim (10^{-44} \ \text{GeV})^2
\]

- Stress Tensor: Perfect fluid

\[
T_{\mu\nu} = (\rho(t) + P(t)) u_{\mu} u_{\nu} + P(t) g_{\mu\nu} \quad \text{e.g.} \quad T_{00} = \rho \quad T_{11} = a^2 P
\]

- Open Problem: Is the naïve averaging of the background density correct?
Background

- **Einstein**
  \[ H^2 + \frac{k}{a^2} = \frac{\rho}{3} \]
  \[ H \equiv \frac{a'(t)}{a(t)} \]
  \[\frac{M_{pl}}{\sqrt{8\pi}} = \frac{1}{\sqrt{G_N} \cdot 8\pi} = 1\]
  \[2H'(t) + 3H^2 + \frac{k}{a^2} = -P\]
  expansion rate
  combine: \[\frac{a''(t)}{a(t)} = -\frac{1}{6}(\rho + 3P)\]
  ordinary matter: decelerate
  alternate: \[d(\rho a^3) = -P d(a^3)\]
  perfect fluid energy conservation and adiabatic flow

- **Notation and examples**
  \[w \equiv \frac{P}{\rho} = equation of state\]
  Matter dominated \[\left\{ \rho \propto \frac{1}{a^3}, \ w = 0 \right\} \rightarrow a \propto t^{2/3}\]
  Radiation dominated \[\left\{ \rho \propto \frac{1}{a^4}, \ w = \frac{1}{3} \right\} \rightarrow a \propto t^{1/2}\]
Basic standard picture emerging

- $T_{00}$ contains the following fraction of the total
  - 73 % “dark energy” defined by its negative pressure
  - 22 % cold dark matter
  - 4.4 % in baryons (protons and neutrons)
  - 0.6 % neutrinos
  - 0.005 % in photons
- The universe is spatially flat to about 1 %.
- $a(t)$ is expanding with $H \equiv \frac{d}{dt} \ln a(t) = 70 \text{ km/s/Mpc}$.
- Energy density was homogeneous and isotropic to 1 part in $10^5$ about 15 billion years ago.
Explicit Stress-Energy Components

- Popular Model

\[ \rho_y = \rho_y(t_0) \left( \frac{a_0}{a} \right)^4 \quad P_y = \frac{\rho_y}{3} \]

\[ \rho_{b,c} = \rho_{b,c}(t_0) \left( \frac{a_0}{a} \right)^3 \quad P_{b,c} = 0 \]

\[ \rho_\nu = \rho_\nu(t_0) \left( \frac{a_0}{a} \right)^3 \quad P_\nu = 0 \]

\[ \rho_\Lambda = \rho_\Lambda(t_0) \quad P_\Lambda \approx -\rho_\Lambda \]

\[ \rho_{\text{tot}} = \rho_y + \rho_\nu + \rho_{b,c} + \rho_\Lambda \]

\[ P_{\text{tot}} = P_y + P_\nu + P_{b,c} + P_\Lambda \]

\( \Omega_x = \frac{\rho_X}{\rho_c} \approx 0.044 \)

\( \Omega_c \approx 0.22 \)

\( \Omega_\nu \approx 0.01 \)

\( \Omega_\Lambda \approx 0.73 \)

\( \Omega_{\text{tot}} \approx 1.0 \)

\( \rho_c = 3 H_0^2 \sim 10^{-46} \text{ GeV}^4 \)

\( \Omega_y \sim 10^{-5} \)

noninteracting particle

energy conservation

negative pressure

Daniel Chung

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Temperature of the Universe

- **Equilibrium Thermodynamics**

  \[ n = \frac{g}{(2\pi)^3} \int f(E) d^3p \]

  \[ \rho = \frac{g}{(2\pi)^3} \int f(E) E d^3p \]

  \[ P = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(E) d^3p \]

  \[ E = \sqrt{p^2 + m^2} \]

  \[ f(E) = \frac{1}{\exp\left(\frac{E - \mu}{T}\right) + 1} \]

  Can fall exponentially with temperature

- **Photon Temperature = “temperature of universe”**

  \[ \rho_\gamma = \frac{\pi^2}{30} 2 T^4 \]

  \[ \rho_R = \frac{\pi^2}{30} g_\ast(T) T^4 \]

- **Entropy (conservation gives T history)**

  \[ s = \frac{(\rho + p)}{T} = \frac{2\pi^2}{45} g_\ast(T) T^3 \]

  **Early Universe** (T>1 MeV)

  \[ g_\ast(T) \approx g_\ast(T) \]

  **SM only**

  \[ g_\ast(T > 300 \text{ GeV}) \approx 107 \]

  **today** (for massless neutrinos):

  \[ T \approx 2.34 \times 10^{-4} \text{ eV} \]

  \[ g_\ast \approx 3.9 \]

  \[ g_\ast \approx 3.36 \]

  \[ g_\gamma = 2 \rightarrow \text{photons dominate and can be measured!} \]
Equilibrium?

• Equilibrium conditions: \( \Gamma > H \)
  - Kinetic equilibrium: \( X \{ Y \} \rightarrow X \{ Z \} \) Y and Z in equilib with photon
    - maintains same temperature
    - particle number does not change
  - Chemical equilibrium: \( X \{ Y \} \rightarrow \{ Z \} \) Y and Z in equilib with photon
    - maintains same temperature
    - particle number changes
    - particle number is determined by temperature

• Boltzmann equations govern approach to equilibrium

• Out of equilibrium:
  - Kinetic: decoupled
  - Chemical: freeze out
BBN

(astro-ph/9905320, 0302431)

Start with \{ n, \nu, e, p \} at \( T \geq 10 \, \text{MeV} \).
Produce \{ D, ^3\text{He}, ^4\text{He} \} by the time temperature cooled to \( T \ll 1 \, \text{MeV} \).

Boltzmann equation:

\[
\frac{1}{a^3} \frac{d \left( n_x a^3 \right)}{dt} = -\langle \sigma_{X \rightarrow ij} V \rangle n_{eq}^x n_{eq}^a \left[ \frac{n_x n_a}{n_{eq}^x n_{eq}^a} - \frac{n_i n_j}{n_{eq}^i n_{eq}^j} \right]
\]

main intuition: in equilibrium

\[
n_x = n_{eq}^x \equiv \frac{g_x}{(2\pi)^3} \int \frac{d^3 p}{\exp \left( \frac{E - \mu}{T} \right) \pm 1}
\]

freeze out \( \frac{\Gamma}{H} \) until \( \frac{\Gamma}{H} < 1 \)

Various binding energies important!

\[
e.g. \quad \frac{n_{eq}^n}{n_{eq}^p} \approx e^{(\mu_e - \mu_\nu)/T} e^{-Q/T}
\]

(e.g. \( \Gamma = \langle \sigma_{X \rightarrow ij} V \rangle n_a \))

short distance reaction rate

gravity

\[
Q \equiv m_n - m_p = 1.293 \, \text{MeV}
\]

\( n \nu \leftrightarrow p \, e \)
Chain of events

\[ X_A \equiv \frac{n_A A}{n_N} \]

\[ n_N = n_N + n_p + \sum A A_n \]

A=\# neutron + \# proton

\[ A = n A_n \]

\[ X_A = \frac{n_A A}{n_N} \]

\[ X_n = \frac{1}{2}, \quad X_p = \frac{1}{2} \]

a) \( T \geq 10 \text{ MeV} \)

\[ n \rightarrow p e \]

\[ n \rightarrow p e \bar{\nu} \]

\[ \bar{e} n \rightarrow p \bar{\nu} \]

contains info on gravity

b) \( T \leq 1 \text{ MeV} \)

\[ X_n = \frac{n}{p} = \exp[-Q/T_F] \approx \frac{1}{6} \]

\[ n \rightarrow p e \bar{\nu} \text{ depletes more} \]

\[ X_n \approx \frac{1}{7}, \quad X_p \approx \frac{6}{7} \]

c) \( T \sim 0.1 \text{ MeV} \)

\[ n p \leftrightarrow D \gamma \text{ stops dissociating D} \]

\[ D n \leftrightarrow \gamma ^3 \text{He} \]

\[ p D \leftrightarrow n ^3 \text{He} \]

\[ ^3 \text{He} D \leftrightarrow n ^4 \text{He} \]

\[ Y_p = 4 \frac{n_{^4 \text{He}}}{n_b} = 2 \frac{e^{-2 \Gamma t_D}}{1 + \exp(Q/T_F)} \]

Because \( ^4 \text{He} \) has larger binding energy than other nuclei, most remaining neutrons go into \( ^4 \text{He} \left( B( ^4 \text{He}) = 28.3 \text{ MeV } \right) \)
Observation

$^4$He  
Observe low metallicity HII regions hot enough to photoinize He. Typically dwarf galaxies are favored w/ $z < 0.1$ 
Plot $Y_{^4He}(z)$ as a function of $z$ and look for plateau as $z \to 0$

$D$  
Relatively small binding energy + rate at which out of equilib freeze out occurs faster for BBN than in stars  
more produced in BBN than in stars 
measure high redshift dilute clouds in intergalactic medium (IGM)

Because $n_p \leftrightarrow D \gamma$ determines D abundance while n+p makes up the baryons, D abundance is sensitive to baryon photon ratio:

$$\frac{n_D}{n_b} \sim n_b \left( \frac{T}{m_p} \right)^{3/2} e^{B_D / T}$$
Constraints on High Energy Theory

- Constraints on high energy theory:

Recall that freeze out condition:

\[ \frac{\Gamma}{H} \leq 1 \]

(ref. short distance reaction rate (e.g. \( \Gamma = \langle \sigma_{X \rightarrow ij} v \rangle \))

new physics with light degrees of freedom \( \rightarrow \) increased \( H^2 = \frac{\rho}{3} \)

more neutrons \( \rightarrow \) more \(^4\)He produced \( \rightarrow \) bound on d.o.f.

Bound typically expressed as variation in the number of light \( \nu \)

at 2\( \sigma \),

[astro-ph/0408033]

<table>
<thead>
<tr>
<th>Observations</th>
<th>( \eta_{10} \equiv 10^{10} \eta )</th>
<th>( N_\nu )</th>
<th>( \delta N_{\nu,max} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y_p + \text{D/H}_A )</td>
<td>( 5.94^{+0.58}_{-0.50} )</td>
<td>( 3.14^{+0.70}_{-0.65} )</td>
<td>1.59</td>
</tr>
<tr>
<td>( Y_p + \eta_{CMB} )</td>
<td>( 6.14 \pm 0.25 )</td>
<td>( 3.08^{+0.74}_{-0.68} )</td>
<td>1.63</td>
</tr>
<tr>
<td>( \text{D/H}<em>A + \eta</em>{CMB} )</td>
<td>( 6.16 \pm 0.25 )</td>
<td>( 3.59^{+1.14}_{-1.04} )</td>
<td>2.78</td>
</tr>
<tr>
<td>( Y_p + \text{D/H}<em>A + \eta</em>{CMB} )</td>
<td>( 6.10^{+0.24}_{-0.22} )</td>
<td>( 3.24^{+0.61}_{-0.57} )</td>
<td>1.44</td>
</tr>
</tbody>
</table>

(could be any other thermal light particles)
Basic problems
Problems of cosmology

• What is the composition of dark energy?
• What is the composition of CDM?
• Why more baryons than antibaryons?
• If inflation solves the cosmological initial condition problems, what is the inflaton?
• Classical singularities of general relativity?
• Why is the observed cosmological constant small when SM says it should be big?
• Origin of ultra-high energy cosmic rays?
What is dark energy?

\[ \frac{a}{a} = -\frac{4\pi}{3 M_{pl}^2} (\rho + 3P) > 0 \quad \rightarrow P < -\frac{1}{3} \rho \]

Recall that normal gas of matter has positive pressure.

\[ P(x) = \frac{1}{3} \sum_N \frac{p_N^2}{\sqrt{p_N^2 + m_N^2}} \delta^{(3)}(x - x_N) \]

Field energy can have negative pressure (like inflation).

\[ P = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - V(\phi) \]

- Why is the energy density nearly coincident with the matter density today?
- If a dynamical field explains the coincidence, how can such a small mass scale (cosmological time scale) be protected?
What is **cold** dark matter?

- **Definition:** dark matter that is **nonrelativistic** at the time of matter radiation equality.

- Can it be all in cold baryons not emitting light?
  - BBN (chemistry of producing elements heavier than hydrogen) says no.
  - Lensing (gravitational deflection of light from compact objects) agrees with this picture. (Dodelson's lecture)

- CDM neutrinos would overclose the universe.

- Physics beyond the standard model **necessary**! (Kusenko's lecture)
Why more baryons than antibaryons?

• The absorption spectra measurements, CMB, and BBN agree
  \[ \frac{n_B}{n_\gamma} \sim 10^{-10} \]

• Naturalness of small dimensionless number?

• According to SM, \[ \frac{n_B}{n_\gamma} = 10^{-18} \] at \( T > 100 \text{ GeV} \).

• Probability that the small number is from mere thermal fluctuation is very small.
What is the inflaton?

- CMB data looks like that expected from inflation
  - i.e.
    1. "no" spatial curvature
    2. scale invariant spectra on "superhorizon" scales

- Similar in negative pressure characterization as dark energy; no known particle can produce this
Singularities of GR

- Hawking-Penrose-Geroch theorem: As long as there is nonzero spacetime curvature somewhere and energy is positive, Einstein's theory will develop a singularity. (a classical self-destruction)

- Evidence for black holes exist. Is there a singularity behind the apparent horizon?

- Big bang singularity naively exists: i.e.

\[ R = -6 \left( \frac{a''(t)}{a(t)} + \left[ \frac{a'(t)}{a(t)} \right]^2 + \frac{k}{a^2(t)} \right) \sim \frac{1}{t^2} \rightarrow \infty \]
A small cosmological constant?

• Due to SM quantum fluctuations

\[ \cdots \rightarrow \rho_\Lambda \sim M^4 \]

• Possible values of \( M \)
  
  • Planck scale \( 10^{18} \) GeV
  • GUT scale \( 10^{16} \) GeV
  • See-saw scale \( 10^{13} \) GeV

• On the other hand we observe

\[ \rho_\Lambda \sim (10^{-12} \text{ GeV})^4 \]
Ultra-high energy cosmic rays

- There is a GZK cutoff at $10^{19.8}$ eV due to efficient
  \[ \gamma p \rightarrow \pi p \]
- Proton cannot ravel more than 40 Mpc.
- Events above $10^{19.8}$ eV measured (possibly).
- No energetic extragalactic sources within 40 Mpc.
- Primary? Source & acceleration mechanism?
Inflation
Inflation

Motivation: Mostly initial condition problems.

1. Flatness Problem

Friedmann: \( \frac{k}{H^2 a^2} = \Omega - 1 \)  \( \rightarrow \) time dependent

today: \( \frac{k}{H_0^2 a_0^2} = \Omega_0 - 1 < 10^{-2} \)

early universe: \( \frac{k}{H_e^2 a_e^2} = \frac{k}{H_0^2 a_0^2} \left( \frac{a_r}{a_0} \right) \left( \frac{a_e}{a_r} \right)^2 < (10^{-2})(10^{-4})(10^{-6})^2 = 10^{-18} \)

at nucleosynthesis

Why small?

Initial spatial curvature had to be finely tuned for universe to be this old and flat. Why?
More Motivation

2. Horizon/causality problem: Why homogeneous and isotropic on "acausal scales?"

real singularity

 naïve horizon (with \( w > -\frac{1}{3} \)):

\[
d_H = a(t) \int_0^t \frac{dt'}{a(t')} = \frac{t}{2} \left(1 - \frac{2}{3(1+w)}\right) \sim \frac{1}{H}
\]

causal signals travel beyond naïve horizon
Unwanted Relics

3. Unobserved relic problem

e.g. Suppose the SM is embedded in a larger theory with gauge group $G_1$

$$G_1 \rightarrow G_2 \rightarrow ... \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y$$

Monopoles arise whenever $\Pi_2(G_i/G_j) \neq I$

$$n_M \sim H^3 \sim T_c^6 / M_{pl}^3$$

$$n_M / s \sim \frac{T_c^3}{M_{pl}^3} \sim \left( \frac{10^{14}}{10^{19}} \right)^3 \sim 10^{-15}$$

$$m_M \sim 10^{16} \text{GeV} \rightarrow \Omega_M \sim 10^{11} \quad \text{unacceptably large!}$$
**Inflation**

\[ ds^2 = g_{\mu\nu} \, dx^\mu \, dx^\nu = dt^2 - a^2(t) \, dx^i \, dx^j \, \delta_{ij} \]

- **Inflationary solution:** Blow up a small flat patch into the entire universe

  - Flat patch becomes the entire universe (solves flatness)
  - Lengthen the time it takes to reach the singularity (horizon)

\[
\frac{d}{dt} \left[ \frac{x \, a}{(1/H)} \right] > 0 \quad \rightarrow \quad \frac{d^2 a}{dt^2} > 0
\]

causal signals travel beyond naïve horizon

\[ x \equiv \text{comoving coordinate separation} \]

- Dilutes unwanted relics

- **Prediction:** scale invariant density perturbations

\[
\frac{\delta \rho}{\rho} = \frac{1}{(2\pi)^3} \int d^3 k \; \delta_k \, e^{-i \vec{k} \cdot \vec{x}} \]

\[ \text{power:} \quad \left( \frac{\delta \rho}{\rho} \right)_{\text{hor}} \sim k^{3/2} \frac{\left| \delta_k \right|}{\sqrt{2\pi}} \sim 10^{-5} \]
Qualitative description of inflaton $\phi$

single field inflationary models:

$$S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

How to choose the potential and initial conditions?

- $\frac{d^2 a}{dt^2} > 0$ for about 60 e-folds. $\frac{a(t_f)}{a(t_i)} = e^{N_{\text{efold}}}$
- Inflation must end.
- Spatial inhomogeneities of $\phi$ must be sufficiently small to be consistent with cosmology (too big = too many black holes, too small = not enough structure).
- After inflation ends, the universe must reheat to $T > 10 \text{ MeV}$.
- After inflation ends, unwanted relics must not be created (e.g. low enough temperature).
Magic of negative pressure

- Horizon problem

\[
\frac{d^2 a}{dt^2}/a = \frac{-4 \pi}{3 M_{pl}^2} (\rho + 3P) > 0 \quad \Rightarrow P < -1/3 \rho \quad \text{(just like dark energy)}
\]

\[
\rho = \frac{1}{2} \left( \frac{d \phi}{dt} \right)^2 + V(\phi) \quad \quad \quad P = \frac{1}{2} \left( \frac{d \phi}{dt} \right)^2 - V(\phi)
\]

\[
\Rightarrow \left( \frac{d \phi}{dt} \right)^2 < V(\phi)
\]

- 60 efolds desired:

\[
\frac{d a}{dt}/a = H \quad \Rightarrow a \propto e^{\int dt \, H}
\]

\[
\Rightarrow \frac{d^2 \phi}{dt^2} \ll H \frac{d \phi}{dt} \quad \text{slow roll inflation}
\]
Quantitative Single Field

Slow Roll approximation \[ 3H \frac{d \phi}{dt} \approx -V'(\phi) \quad H^2 \approx \frac{V}{3} \]

- Negative Pressure and 60 e-folding

\[ \epsilon \equiv \frac{1}{2} \left( \frac{V'(\phi)}{V} \right)^2 \approx \frac{-dH}{dt} \ll 1 \quad \eta \equiv \frac{V''(\phi)}{V} \ll 1 \quad N(\phi(t_I)) \equiv \left| \int_{\phi(t_I)}^{\phi(t_f)} \frac{d\phi}{\sqrt{2\epsilon}} \right| > 60 \]

- End of inflation: \( \epsilon(\phi(t_f)) \approx 1 \) with \( V(\phi_{\text{min}}) \approx 0 \) at the minimum of the potential

- Density perturbation amplitude: \( \sqrt{P_k} \approx \sqrt{\frac{V}{24\pi^2 \epsilon(\phi_{60})}} \approx 10^{-5} \)

scale invariance nearly automatic! \[ |2\eta(\phi_{60}) - 6\epsilon(\phi_{60})| < 0.2 \]

indicates source of fine tuning

never Planckian
Standard Reheating

• Inflaton field decays: e.g.

\[ L = \frac{1}{2} [ (\partial \phi)^2 + m^2 \phi^2 + L_i ] \]

\[ L_i = -\lambda \phi \bar{\psi} \psi \]

\[ \Gamma_{\text{tot}} = \frac{3 (\Sigma (m^2))}{m} \]

\[ \partial_t^2 \phi + (3H + \Gamma_{\text{tot}}) \partial_t \phi + (m^2 + \frac{\Gamma_{\text{tot}}^2}{4}) \phi \approx 0 \]

use following approx: \( \rho_\phi \approx \frac{1}{2} \left( (\partial_t \phi)^2 + m^2 \phi^2 \right) \)

\[ \frac{(\partial_t \phi)^2}{2} \approx \frac{m^2 \phi^2}{2} \]

\( \Gamma^2 / 4 \ll m^2 \)

\[ \partial_t \rho_R + 4H \rho_R = \Gamma_{\text{tot}} \rho_\phi \]

⇒ reheating temperature as a function of time

\[ \rho_R = \frac{\pi^2}{30} g_* (T) T^4 \]

\[ T_{RH} \approx 0.2 \left( \frac{200}{g_*} \right)^{1/4} \sqrt{\Gamma_{\text{tot}} M_{pl}} \]
Why 60 efolds?

• Largest scale that we see homogeneous and isotropic:

\[ L = \int dx = a_0 \int_{a_{dec}}^{a_0} \frac{da}{H a^2} \approx a_0 \int_{a_{dec}}^{a_0} \frac{da}{H_0 (a_0 / a)^{3/2}} \approx \frac{2}{H_0} \equiv a_0 X \]

• Inflation can take place only if homogeneous (small patch of comoving coordinate size > X became the observable universe):

\[ \frac{1}{H_I} > X a_I \quad \Rightarrow \quad \frac{1}{H_I} > a_0 X \left( \frac{a_I}{a_0} \right) \quad \text{need enough efolds} \]

(Also for curvature) \( a_I = \frac{a_I}{a_e} \equiv e^{-N} \frac{a_e}{a_0} \)

\[ H_I \approx \frac{T_{RH}^2}{\sqrt{3}} \]

\[ \ln \left( \frac{T_{RH}}{T_0} \right) + \frac{1}{2} \ln (\Omega_{R0}) \approx 60 + \ln \left( \frac{T_{RH}}{10^{15} \text{ GeV}} \right) < N \]
Single Scalar Field Computation

Action: \[ S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} R + \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \]

perturb: \[ \phi = \phi_0(\tau) + \delta \phi(x) \]

\[ g_{\mu \nu} = g^{(0)}_{\mu \nu}(\tau) + a^2 \begin{pmatrix} 2 \Phi & -\partial_i B \\ -\partial_i B & 2(\psi \delta_{ij} - \partial_i \partial_j E) \end{pmatrix} \]

gauge degree of freedom: Freedom of slicing the spacetime (splitting perturbed versus unperturbed)

infinitesimally gauge invariant (same as longitudinal gauge)

\[ v = a \left( \delta \phi - \frac{\phi'_0(\tau)}{a'/a} \phi \right) \]

\[ \mathcal{R} = \Phi \quad \text{constant on constant } \phi \quad \text{hypersurface (comoving)} \]

Seed formation of galaxies!
Power Spectrum

quantize: \[ v(\tau, \vec{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left[ a_k v_k(\tau) + a^\dagger_{-k} v_k^*(\tau) \right] e^{i \vec{k} \cdot \vec{x}} \]

\[ [a_k, a^\dagger_{k'}] = \delta^{(3)}(\vec{k} - \vec{k'}) \]

\[ \langle \mathcal{R}(\tau, \vec{x}) \mathcal{R}(\tau, \vec{y}) \rangle = \int \frac{dk}{k} \frac{\sin(k|\vec{x} - \vec{y}|)}{k|\vec{x} - \vec{y}|} P_\mathcal{R} \quad \mathcal{R} \equiv -\left( \frac{a'/a}{a/\phi_0'} \right) v \quad P_\mathcal{R} = \frac{k^3}{2\pi^2} \left( \frac{a'/a}{a/\phi_0'} \right)^2 |V_k|^2 \]

\[ v_k'' + \left[ k^2 - \frac{2+p}{\eta^2} \right] v_k = 0 \quad \text{boundary condition:} \quad v_k \to_{\eta \to -\infty} e^{-ik\eta}/\sqrt{2k} \]

\[ p = 3(3\epsilon - \eta) \]

\[ v_k = \sqrt{\frac{\pi}{2}} \sqrt{-\tau} H^{(1)}_{-2}(\frac{1}{2} + \nu) e^{i \frac{\pi}{2} (\frac{1}{2} + \nu)} \quad \nu \equiv \frac{\sqrt{9 + 4p}}{2} \]

\[ H^{(1)}_{-2}(\frac{1}{2} + \nu) \approx \frac{-i \sqrt{2\pi}}{\pi} \quad \Rightarrow \quad \sqrt{P_k} \approx \sqrt{\frac{V}{24\pi^2 \epsilon(\phi_{60})}} \sim 10^{-5} \]
Quantum to “Classical” Transition

On large scales: \( \frac{k}{aH} \to 0 \)

\[ v_k'' - \frac{a'/a}{a\phi_0'} \partial^2_n \left( \frac{a\phi_0'}{a'/a} \right) v_k \approx 0 \]

Growing mode:

\[ v_k \sim A_k \frac{a\phi_0'}{a'/a} \]

\[ \alpha_k \frac{a\phi_0'}{a'/a} = a_k v_k(\eta) + a_k^\dagger v_k^*(\eta) \]

\[ \mathcal{R}_k = -\alpha_k \]

\[ [\alpha_k, \alpha_l^\dagger] = 0 \]

\[ \lim_{\frac{k}{aH} \to 0} \mathcal{R}_k \] are classical random variables!

Spectral index:

\[ P_{\mathcal{R}} \propto k^{n_s-1} \]

\[ n_s - 1 = 2\eta - 6\epsilon \]

\[ \frac{d n_s}{d \ln k} = 16\epsilon \eta - 24\epsilon^2 - 2\xi \]

\[ \xi = \frac{V'V'''}{V^2} \]

Running of spectral index measurement = measuring potential
Gravity waves

Tensor perturbations: \[
\delta g^{(T)}_{\mu \nu} = \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix}
\]

\[
P_T(k) \equiv \frac{k^3}{\pi^2} \left( |h_+|^2 + |h_\times|^2 \right) \propto k^{n_T-1}
\]

\[
P_T = 8 \frac{H^2}{(2 \pi)^2} = 16 \epsilon P_R \quad r \equiv \frac{P_T}{P_R}
\]

\[n_T - 1 = -16 \epsilon = -r/8\] Consistency relation can rule out single field inflation!

In multifield inflation (more realistic):

\[1 - n_T > r/8\] (n.b. Some people use the convention \(P_T \propto k^{n_T}\))

\[-n_T > r/8\]
What is this good for?

fixes the **boundary condition** to the Boltzmann equation

\[
\Theta(t) = \frac{\Delta T}{T} = \sum_{l=0}^\infty \Theta_l(\eta) e^{i \vec{k} \cdot \vec{x}} P_l(\vec{k} \cdot \vec{y})
\]

\[
\Theta(t) \propto \Phi(t)
\]

\[
\partial_t^2 \Theta_0 + \frac{\partial_t R}{1+R} \partial_t \Theta_0 + k^2 c_s^2 \Theta_0 = F
\]

\[
c_s^2 = \frac{1}{3} \frac{1}{1+R}
\]

\[
R \approx \frac{3 \rho_b}{4 \rho_\gamma}
\]

\[
\frac{\Theta_l(\eta)}{2l+1} \sim (\Theta_0 + \Psi)(\eta_\perp) j_l(k(\eta - \eta_\perp)) + ...
\]

\[
\langle \Theta(\eta_0, \vec{x}, \gamma) \Theta(\eta_0 \vec{x}, \gamma') \rangle = \sum_l C_l P_l(\gamma \cdot \gamma')
\]

\[
(2l+1) C_l = \frac{1}{(2\pi^2)} \int \frac{d k}{k} \frac{k^3 |\Theta_l|^2}{2l+1}
\]

See Dodelson's lecture for more details.
Why Is It Difficult to Test Slow-roll Inflation at Colliders?

Both

\[
\frac{\delta \rho}{\rho + P} \sim \sqrt{P_k^\xi} \approx \sqrt{\frac{V}{24 \pi^2 \epsilon (\phi_{60})}} \sim 10^{-5}
\]

and

\[
N (\phi (t_i)) \equiv \left| \int_{\phi(t_i)}^{\phi(t_f)} \frac{d \phi}{\sqrt{2 M_p \epsilon}} \right| > 50
\]

involve \( M_p \gg \text{TeV} \).

Order parameter is far away from TeV scale vacuum.

e.g. light fields during inflation \(\rightarrow\) heavy fields today
Inflationary Model

\[ W = \lambda_1 \sigma \chi \psi_1 + \lambda_2 \frac{\phi^{n-2}}{M_{pl}^{n-2}} \phi^2 \psi_2 \]

\[ D = \Lambda^2 - |\chi|^2 - |\phi|^2 + |\psi_1|^2 + n |\psi_2|^2 \]

If \( |\chi|^2 + |\phi|^2 \leq \Lambda^2 \) \quad \rightarrow \quad \psi_1 = \psi_2 = 0

\[ U = \lambda_1^2 |\sigma|^2 |\chi|^2 + \lambda_2^2 \frac{\phi^{2n-4}}{M_{pl}^{2n-4}} |\phi|^4 + \frac{g^2}{2} (\Lambda^2 - |\chi| - |\phi|^2)^2 + \text{soft terms} \]

Inflates while \[ |\sigma| \geq \frac{\sqrt{n} \lambda_2}{\lambda_1} \left( \frac{\Lambda}{M_{pl}} \right)^{n-2} \Lambda \]

During inflation, \( \sigma \) mass \( \sim m_{3/2} \sim \text{TeV} \)

During inflation, \( \sigma \) mass \( \sim M_p \)
Non-canonical Kinetic Terms

Sound speed controls most inflationary observables.

\[ S = \int d^4x \left[ \frac{-1}{2} \sqrt{-g} \left( R - 2g \Phi \right) \right] \]

\[ X \equiv \frac{1}{2} g^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi \]

\[ \rho = 2X \partial_X p - p \]

\[ c_s^2 = \frac{\rho + p}{2X \partial_X \rho} \]

\[ = 1 \text{ for canonical} \]

Inflation condition:

\[ \left| \frac{X \frac{dp}{p}}{\rho \frac{dX}{X}} \right| \ll 1 \]

Observables:

\[ P_k^{\zeta} = \frac{16}{9} \frac{\rho}{64 \pi^2 c_s (1 + p/\rho)} \]

\[ r = -8 c_s (n_T - 1) \]

(Other popular convention:

\[ r = -8 c_s n_T \] )
e.g. DBI Inflation
[hep-th/0404084]

- Probe D3 brane moving in approx. $AdS_3 \times S^5$

DBI

\[
S = \frac{-1}{g_s} \int d^4x \sqrt{-g} \left[ \left( f(\phi) \right)^{-1} \sqrt{1 - f(\phi) \partial^\mu \phi \partial_\nu \phi} + V(\phi) \right]
\]

Approximate $AdS_5$ warp factor:

\[
f(\phi) \approx \frac{\lambda}{\phi^4}, \quad \lambda \approx \frac{R^4}{\alpha'^2}
\]

V from RR and fields associated with compactification.

\[
c_s \sim \frac{1}{\gamma} = \sqrt{1 - f(\phi)(\frac{d\phi}{dt})^2} \ll 1
\]

Allows inflation even when $\frac{d\phi}{dt}$ is reaching the maximum possible "speed."
Non-Gaussianity

Before: computed 2-point function
\[ \langle \zeta(k_1) \zeta(k_2) \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2) P^{(2)}(k_1, k_2) \]
three-point function:
\[ \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) P^{(3)}(k_1, k_2, k_3) \]

Intuition: interactions of the inflaton field.

Estimate: Consider the limit in which \( k_1 \ll k_2, k_3 \)
intuitively, one mode is frozen and sets up background:
\[ ds^2 \approx -dt^2 + a^2(t) e^{2\zeta_B(x)} dx^2 + \ldots \]
three point function must be proportional to
\[ \langle \zeta(x_2) \zeta(x_3) \rangle_B \sim \langle e^{\zeta(x_2)} \zeta(e^{\zeta} x_3) \rangle \sim \langle \zeta(x_2) \zeta(x_3) \rangle + \zeta_B \frac{d \langle \zeta(x_2) \zeta(x_3) \rangle}{d \zeta} + \ldots \]

Fourier transforming, one would expect
\[ P^{(3)}(k_1, k_2, -k_2) \sim P(k_1) \frac{d P^{(2)}(k_2, -k_2)}{d \ln(k_2 + k_3)} \bigg|_{k_3 \to -k_2} \sim O\left(\epsilon\right) P(k_1) P(k_2) \]
canonical
\[ O\left(\frac{d \ln c_s}{dt} \frac{1}{H}\right) P(k_1) \]
noncanon
Future Prospects

- Running of the spectral index will be better known with future experiments such as Planck.

- Polarization:
  - B polarization comes from tensor and lensing (contaminant as far as inflation is concerned).
  - B polarization has no contribution from scalar perturbations.
  - measuring tensor is important for checking consistency condition (to know if it really is inflation!)
  - Unfortunately, typically less than 1% of the scalar spectrum

- Nongaussianities

- Isocurvature

- Theoretical Problems
  - What is the inflaton? Are there truly natural models?
  - Stability of de Sitter space and back reaction.
  - More observables to experimentally ascertain inflation.
Baryogenesis
Observation

• In solar system much more baryons than antibaryons
  explained by \( pp \rightarrow 3 \, p + \bar{p} \)
  \( n_p / n_p < 3 \times 10^{-4} \)

  \( n_{4He} / n_{4He} < 3 \times 10^{-8} (?) \)

• Dominance of matter clear on scales < 10 Mpc:
  bound on \( \gamma \) from \( p \bar{p} \rightarrow \pi^0 \rightarrow 2 \gamma \).

• Other constraints: distortion of CBR, diffuse \( \gamma \)-ray.

[e.g. Cohen and de Rujula 1997] + \( \text{void} \) -
Is there a problem?

- SM contains nonperturbative baryon number violating operators that erase B+L
- These become efficient when $T > T_c \sim 100\text{ GeV}$ erases preexisting B+L
- Otherwise, an aesthetic initial condition problem
- Starting from $n_B = n_b - n_{\bar{B}} = 0$ initial conditions why
  \[ \eta = \frac{n_B}{n_\gamma} \approx 6 \times 10^{-10} \]
  naively, $\eta_{\text{naive}} \sim 10^{-18}$ and not separated.
Illustration of Sakharov Criteria

- Suppose “X” carrying 0 baryon number can decay only into “a” carrying baryon number $b_a$ and “b” carrying baryon number $b_b$.

- Branching ratios:

$$r = \frac{\Gamma(X \rightarrow a)}{\Gamma_X} \quad \text{“CP”:} \quad \bar{r} = \frac{\Gamma(\bar{X} \rightarrow \bar{a})}{\Gamma_X}$$

$$1 - r = \frac{\Gamma(X \rightarrow b)}{\Gamma_X} \quad \text{“CP”:} \quad 1 - \bar{r} = \frac{\Gamma(\bar{X} \rightarrow \bar{b})}{\Gamma_X}$$

- Baryon produced:

$$\Delta B_X = r b_a + (1 - r) b_b$$

$$\Delta B_{\bar{X}} = -\bar{r} b_a - (1 - \bar{r}) b_b$$

$$\Delta B = \Delta B_X + \Delta B_{\bar{X}} = (b_a - b_b) (r - \bar{r})$$

- Out of equilibrium: otherwise, the other direction produces

$$(b_a - b_b)(\bar{r} - r)$$
Boltzmann

• Phase space evolution (useful B-genesis, dark matter, CMB):

\[
\frac{df}{dt} + 3H \int d^3p f = \int \frac{d^3p}{E} C[f] \\
n(t) = \frac{g}{(2\pi)^3} \int d^3p f
\]

\[
\frac{g_X}{(2\pi)^3} \int \frac{d^3p_X}{E_X} C[f] = -\int d\Pi_X d\Pi_a d\Pi_b d\Pi_c (2\pi)^4 \delta^{(4)}(p_X + p_a - p_b - p_c) \times \\
\left[ |M|_{X+a\to b+c}^2 f_X f_a (1 \pm f_b) (1 \pm f_c) - |M|_{b+c\to X+a}^2 f_b f_c (1 \pm f_X) (1 \pm f_a) \right]
\]

d\Pi_X = \frac{g_X}{(2\pi)^3} \frac{d^3p_X}{2E_X}

• Simplification

• Chemical equilibrium of others:  
  e.g.  \( f_b = f_b^{eq}, \quad f_c = f_c^{eq} \)

• Kinetic equilibrium of all states:  
  e.g.  \( f_X = F(t) f_X^{eq}, \quad f_a = A(t) f_a^{eq} \)
Interference

- CP violation involves a complex parameter in the Lagrangian:

\[ L = |m|^2 (|\phi_1|^2 + |\phi_2|^2) - |m|(e^{i\rho} \psi_1 \psi_2 + e^{-i\rho} \overline{\psi}_2 \overline{\psi}_1) + |M_3|(e^{i\theta} \lambda^a \lambda^a + e^{-i\theta} \overline{\lambda}^a \overline{\lambda}^a) + |m_{LR}|^2 (e^{i\phi} \phi_2 \phi_1 + e^{-i\phi} \phi_1^* \phi_2^*) \]

- In this Lagrangian, there is only one physical phase (phase that cannot be removed by field redefinition).

\[ \delta_{\text{phys}} = \phi - \theta - \rho \]

- CP violation = interference of transition amplitudes:

\[ |M|^2 = |M_1 + M_2 e^{i\delta_{\text{phys}}}|^2 = |M_1|^2 + |M_2|^2 + 2 \Re (M_1 M_2^* e^{-i\delta_{\text{phys}}}) \]

\[ |M^{CP}|^2 = |M_1 + M_2 e^{-i\delta_{\text{phys}}}|^2 = |M_1|^2 + |M_2|^2 + 2 \Re (M_1 M_2^* e^{i\delta_{\text{phys}}}) \]

\[ M_1 = \text{real} \]
Cutting

• Recall in the simple example

\[ \Delta B = \Delta B_X + \Delta B_{\bar{X}} = (b_a - b_b)(r - \bar{r}) \]

\[ |M|^2 - |M^{CP}|^2 = \Re (M_1 \bar{M}_2 e^{i\delta_{phys}} - M_1 \bar{M}_2 e^{-i\delta_{phys}}) \]

This is 0 unless \( M_2 \) develops an imaginary part due to virtual states going on shell.

• Diagrammatically

\[ M_1 \quad + \quad M_2 e^{i\delta_{phys}} \]

Since the real part of this should be taken (which vanishes unless an internal line goes on shell):

[for an example of usage, see hep-ph/0112360 ]
\[ L = \mathcal{L}_{SM} + \bar{N}_1 i \phi N_1 + \lambda_1 N_1 H L + \frac{M_1}{2} N_1^2 + \\
+ \bar{N}_{2,3} i \phi N_{2,3} + \lambda_{2,3} N_{2,3} H L + \frac{M_{2,3}}{2} N_{2,3}^2 \]

\[ + \text{h.c.} \]

\[ \lambda_{2,3} = |\lambda_{2,3}| e^{-i\delta} \]

Since \( N_1 \) is heavier than \( L, H \)

\( \Rightarrow \) can be on shell.

If \( M_{2,3} \gg M_1 \)

\( \Rightarrow \) loop diags suppressed by \( \frac{1}{M_{2,3}} \)
Thermal Leptogenesis

[for review, e.g. hep-ph/0401240]

- Have only perturbatively significant B-L violating operators.
- Generate L as we have been discussing.
- Convert L into B through the B+L violating sphaleron.

\[ B = \left( \frac{8 N_f + 4 N_H}{22 N_f + 13 N_H} \right)(B - L) \]

- Theoretical attractiveness: L-violating operators natural in seesaw neutrino masses
- “uncomfortable” aspect: in gravity mediated SUSY breaking models, gravitino bound strongly constrains it.
Boltzmann Eq.

\[ Y_i = \frac{n_i}{s} \]

\[ \frac{z}{Y_{\psi}^{eq}} \frac{d Y_{\psi}}{dz} = \frac{-1}{H} \sum_{a, i, j, \ldots} \left[ \frac{Y_{\psi}}{Y_{\psi}^{eq}} \frac{Y_a}{Y_a^{eq}} \ldots \bar{Y}_{eq}^{eq} (\psi + a + \ldots \rightarrow i + j + \ldots) \right] \]

\[ z = \frac{m_{\psi}}{T} \]

\[ \text{decay} \quad \bar{Y}_{eq}^{eq} = \frac{K_1(z)}{K_2(z)} \Gamma \]

\[ \text{scatter} \quad \gamma_{eq}^{eq} (\psi + a \rightarrow i + j + \ldots) = \frac{T}{64 \pi^4 n_{\psi}^{eq}} \int_{(m_{\psi} + m_a)^2}^{\infty} ds \hat{\sigma} (s) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right) \]

\[ = \langle \sigma \, v \rangle n_{eq}^{eq} \]

\[ \hat{\sigma} (s) = \frac{2 [s - (m_{\psi} + m_a)^2] [s - (m_{\psi} - m_a)^2]}{s} \sigma (s) \]

(same equation is applicable to dark matter.)
Leptogenesis Estimate

1) Assume temperature of the universe is high enough
   right-handed neutrinos are in equilibrium (fixes initial cond.)
   Typically, CP conserving reactions control this.

2) Temperature falls:
   \[ \langle \sigma v \rangle n_{\nu_R}(T) < \frac{T^2}{M_{pl}} \]
   \[ \rightarrow \text{right handed neutrinos go out of equilibrium} \]
   Lepton number generated

3) When the right handed neutrino abundance falls below L density,
   the lepton number freezes out.
   Yukawa information assuming see-saw.

   \[ \eta \approx \frac{\delta_{CP}}{g_* v^2} \frac{m}{M} \left( \frac{M}{T_c} \right)^{3/2} e^{-M/T_c} \]
   \[ m_\nu \sim 10^{-1} \text{ eV, } M \sim 10^9 \text{ GeV, } g_* \sim 100, \text{ } m_W \sim 100 \text{ GeV} \]
   \[ \sim 10^{-10} \]
   \[ \frac{M}{T_c} \left( \frac{M}{T_c} \right)^{3/2} e^{-M/T_c} \sim 0.1 \] (out of equilibrium temperature)
People and References for EW baryogenesis

- Incomplete list of ewbgenesis people:
  
  Ambjorn, Arnold, Bodeker, Brhlik, 
  
  Carena, Chang, Cline, Cohen, Davoudiasl, 
  de Carlos, Dine, Dolan, Elmfors, Enqvist, 
  Espinosa, Farrar, Gavela, Giudice, Good, 
  Grasso, Hernandez, Huet, Jakiw, Jansen, 
  Joyce, Kane, Kainulainen, Kajantie, Kaplan, 
  Keung, Khlebnikov, Klinkhamer, Kolb, 
  Kuzmin, Laine, Linde, Losada, Moore, 
  Moreno, Multamaki, Murayama, Nelson, 
  Olive, Orloff, Oaknin, Pietroni, Quimbay, 
  Quiros, Pene, Pierce, Prokopec, Rajagopal, 
  Ringwald, Riotto, Rubakov, Rummukainen, 
  Sather, Schmidt, Seco, Servant, 
  Shaposhnikov, Singleton, Thomas, Tkachev, 
  Trodden, Tsypin, Turok, Vilja, Vischer, 
  Wagner, Westphal Weinstock, Worah, 
  Yaffe...

- “Randomly” selected “overview” references
  - hep-ph/0312378
  - hep-ph/0303065
  - hep-ph/0208043
  - hep-ph/0006119
  - hep-ph/9901362
  - hep-ph/9901312
  - hep-ph/9802240
EW Motivation

• In minimal SM, **EW phase transition** is inevitable!

\[ T > T_c \sim m_h \quad \text{EW symmetry restoration} \]

• An exciting era:
  
  probing at LHC and its microphysics

  Nearly everything at \( T_c \) associated with SM measurable

• Almost no cosmological probe to this era
  
  • Explaining the **baryon asymmetry of the universe**
  
  • Establishing thermal equilibrium for WIMPs close
Why worry about electroweak baryogenesis scenario instead of leptogenesis?

• Leptogenesis
  • Computationally simpler: spatially homogeneous
  • Neutrino mass suggests such scenario if see saw invoked (lepton num violation & dim 5 operator suppression scale)
  • May depend on near-future-lab-immeasurable phase:
    • Squeezed by gravitino bound

• EW Baryogenesis is physics at 100 GeV
  • Almost everything about it can be lab probed in principle
  • In SM and MSSM, EW phase transition occurred!
Aspects of MSSM

soft susy breaking (definition: does not introduce quadratic divergence)

\[-L_{\text{soft}} = \frac{1}{2} \left[ M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} \right] \]

\[+ \epsilon_{\alpha \beta} \left[ -b H_d^\alpha H_u^\beta - H_u^\alpha \tilde{Q}_i^{\alpha} \tilde{A}_{u ij} \tilde{U}_j^{\beta} + H_d \tilde{Q}_i^{\alpha} \tilde{A}_d \tilde{D}_j^{\beta} + H_d \tilde{L}_i \tilde{A}_{e ij} \tilde{E}_j^{\beta} + h.c. \right] \]

\[+ \tilde{Q}_i^\alpha m_{Q_j}^2 \tilde{Q}_j^{\alpha*} + \tilde{L}_i^\alpha \tilde{L}_j^{\alpha*} + \tilde{U}_i^{c*} \tilde{U}_j^c + \tilde{D}_i^{c*} \tilde{D}_j^c + \tilde{E}_i^{c*} \tilde{E}_j^c \]

\[+ m_{H_d}^2 |H_d|^2 + m_{H_u} |H_u|^2 \]

supersymmetric Yukawa and mass term

\[W = \epsilon_{\alpha \beta} \left[ -\tilde{H}_u^\alpha \tilde{Q}_i^{\beta} Y_{u ij} \tilde{U}_j^{\beta} + \tilde{H}_d^\alpha \tilde{Q}_i^{\beta} Y_{d ij} \tilde{D}_j^{\beta} + \tilde{H}_d^\alpha \tilde{L}_i^{\beta} Y_{e ij} \tilde{E}_j^{\beta} - \mu \tilde{H}_d^\alpha \tilde{H}_u^\beta \right] \]

\[\tilde{Q}_i = (\tilde{Q}_{L_i} \; \tilde{Q}_{L_i}) \quad \tilde{U}_i = (\tilde{U}_{L_i}^c \; \tilde{U}_{L_i}^c) \quad \tilde{D}_i = (\tilde{D}_{L_i}^c \; \tilde{D}_{L_i}^c) \quad \tilde{L}_i = (\tilde{E}_{L_i}^c \; \tilde{E}_{L_i}^c) \]

\[\tilde{E}_i = (\tilde{E}_{L_i}^c \; \tilde{E}_{L_i}^c) \quad \tilde{H}_u = (\tilde{H}_u \; \tilde{H}_u) \quad \tilde{H}_d = (\tilde{H}_d \; \tilde{H}_d) \]

Chargino mass matrix

\[L_c = \frac{-1}{2} (\chi^+ \; \chi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} (\chi^+ \; \chi^-) \quad \chi^+ \equiv \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} \quad X \equiv \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \sin \beta & \mu \end{pmatrix} \]
Sakharov conditions

recall: 1) B-violation, 2) CP violation, 3) out of equilibrium

1) Baryon number violation: SU(2) sphaleron

\[ O_{B+L} = C \hbar_L^1 \hbar_L^2 \bar{w}_L^4 \prod_i (q_L, q_L, q_L, l_L, j_L) \int d^4x \text{Tr}[F \bar{F}] + \text{Higgs} \]

e.g. 1 generation \( \bar{u}_L \rightarrow d_L d_L \nu_e \)

unbroken phase: \( \Gamma_{EW} = (k \alpha_w) \alpha_w^4 T^4 \)

\( k \alpha_w \sim O(1) \)

broken phase: \( \Gamma \approx 2.8 \times 10^5 T^4 \left( \frac{\alpha_w}{4\pi} \right)^4 \kappa \zeta^7 \exp(-\zeta) \)

\( 10^{-4} < \kappa < 10^{-1} \)

\( \zeta = E_{sph}(T)/T \)

\( E_{sph} \sim \frac{2 m_w}{\alpha_w} \sim \frac{8 \pi \langle H \rangle}{g} \)
Sakharov conditions

recall: 1) B-violation, 2) CP violation, 3) out of equilibrium

2) CP violation:

In SM:

\[ \delta_{CP} = \left( \frac{g_w}{2m_w} \right)^{12} (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2) j \sim 10^{-22} \]

\[ j = \Im \left[ V_{cs} V_{us}^* V_{ud} V_{cd}^* \right] \sim 10^{-4} \]

Too small.

In MSSM, soft SUSY breaking phases: e.g.

\[ \Im (M_2 \mu) \]

\[ \Delta L = \frac{1}{2} M_2 \bar{W}_R^{\dagger} \bar{W}_L + \mu \bar{h}_R^{\dagger} \tilde{h}_L + h.c. \]
Out of equilibrium

3) Phase transition:

- \( T > 100 \text{ GeV} \), symmetry is restored.
- \( T < 100 \text{ GeV} \), symmetry broken.

\[
V(H) = D (T^2 - T_0^2) H^2 - E T H^3 + \frac{\lambda(T)}{4} H^4
\]

\( T > T_c \)

\( T < T_c \)

\( \langle H \rangle = z \)

\( \langle H \rangle = 0 \)

\( \beta_w \)

(Atractive, because almost no new assumption!)
EW B creation step 1

1. Pick up CP/chiral asymmetry

\[ \langle H \rangle = 0 \]

sphalerons inactive

\[ n^L_b - n^L_{\bar{b}} \neq 0 \]

\[ n_b - n_{\bar{b}} = n^L_b + n^R_b - n^L_{\bar{b}} - n^R_{\bar{b}} = 0 \]

e.g. 1 generation

\[ \bar{u}_L u_R \]

\[ B = 0 \]
EW B creation step 2

\[ \langle H \rangle = z \quad \langle H \rangle = 0 \]

sphalerons active

\[ \Delta (n_b^L - n_{\bar{b}}^L) \]

\[ n_b - n_{\bar{b}} = n_b^L + n_b^R - n_{\bar{b}}^L - n_{\bar{b}}^R \neq 0 \]

e.g. 1 generation

\[ u_R \rightarrow u_R \]

\[ \bar{u}_L \rightarrow d_L \ d_L \ \nu_e \]

\[ B = 1 \]

\[ \bar{u}_L \ d_L \ \bar{\nu}_e \]

\[ B = 0 \]
EW B creation step 3

\[ \langle H \rangle = z \]

sphalerons active

\[ \langle H \rangle = 0 \]

sphalerons inactive

\[ n_b - n_\bar{b} = n_b^L + n_b^R - n_\bar{b}^L - n_\bar{b}^R \neq 0 \]
Computational Steps

- Diffusion equations for (s)quarks and higgsinos: relatively fast process
- Make assumptions about certain processes (Yukawa and strong sphaleron) being in equilibrium due to large interaction rate.
- Solve for SU(2) charged left handed fermions
- Integrate sphaleron transition sourced by above.
Schematically

- One of massaged diffusion equations

\[ v_w \partial_z n_H = D_h \partial_z^2 n_h + \left[ \Gamma_Y \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + \rho n_h}{k_H} \right] - \Gamma_h \frac{n_H}{k_H} \right] + S_H \]

\[ \frac{1}{\Gamma_H} \]

- Source term = CP violating, Higgs field gradient
  - flow of current w/ background force
  - \( S_H > S_Q \)
  - \( n_B \sim \frac{c_1 \Gamma_{EW}}{v_w} \int_{-\infty}^{0} dz \ n_L(z) \exp(c_2 z \Gamma_{EW}/v_w) \)

\[ n_L(z) \sim \frac{\exp(f_1 z)}{f_3} \int_{0}^{\infty} dx \ S_H(x) \exp(-f_2 x) \]
Sufficient CP violation

- **Estimate**

  \[ \eta \sim \frac{(k \alpha_w) \alpha_w^4}{g_s} \delta_{CP} f \sim 10^{-10} \]

  \[ \alpha_w^4 \sim 10^{-6} \quad g_s > 10 \]

  \[ f < v_w \sim 0.1 \rightarrow \delta_{CP} > 10^{-2} \]

  \[ \text{e.g. } \delta_{CP} \sim \frac{3 (M_w \mu)}{T_c^2} \]

- **Importance**

  - Large top Yukawa coupling
    \[ O_{B+L} = h_{L_1} \bar{h}_{L_2} \bar{W}_L^4 \prod_i (q_{L_1} q_{L_2} q_{L_2} l_{L_i}) \]
  
  - Higgs mediated CP asymmetry
    
    Chargino, Higgs(ino), neutralinos, (s)quark
EDM

• experimental EDM bounds

\[ |d_e| < 1.6 \times 10^{-27} \text{ e cm} \quad [\text{Regan et al 2002}] \]
\[ |d_n| < 12 \times 10^{-26} \text{ e cm} \quad [\text{Lamoreaux et al 2002}] \]
\[ |d_{Hg}| < 2.33 \times 10^{-28} \text{ e cm} \quad [\text{Romalis et al 2001}] \]

• Theoretical constraints complicated & uncertain

• e.g. without cancellations,

\[ \text{Arg} \ (M_2 \mu) < 0.05 \quad [\text{Chang et al 2002; Pilaftsis 2002}] \]
sketch of parameter region

- 120 GeV < \( m^{-}_{t} \) < \( m_{t} \)
- 0.2 \( m_{Q} \) \( \leq \) \( A_{t} \) \( \leq \) 0.4 \( m_{Q} \)
- \( m_{h} \) < 115 GeV
- \( \tan \beta > 4 \)
- \( m_{Q} > 1 \) TeV

- \( \mu , M_{1,2} < m_{Q} \)
- \( \Im [\mu M_{1,2}]/T_{c}^{2} > 0.05 \)
- \( \mu , M_{1,2} < 2 T_{c} \)
Dark Energy

\[
\frac{a'''}{a} = -\frac{1}{6} (\rho + 3P) > 0
\]

Cannot be particles:

\[
\rho = \frac{g}{(2\pi)^3} \int f(E) E d^3 p
\]

\[
P = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(E) d^3 p
\]

Can be cosmological constant: \(\rho = \rho_\Lambda = const\) \(P = -\rho_\Lambda\)
Dark Energy Problems

• Why is the cosmological constant small. \( \rho_\Lambda \sim (10^{-12} \text{ GeV})^4 \)

• Why is the dark energy domination time period nearly coinciding with the epoch of structure formation?

• Is there a light \( (10^{-42} \text{ GeV}) \) scalar field associated with dark energy? (e.g. another inflaton, just much lighter)

• What are prospects for measuring dark energy?
Popular Proposals for CC solution

• Back reaction due to particle creation
  • Recent progress in resummation techniques, but outcome still unclear.

• Bottom-up stable tuning approach
  • Abott's solution to CC: washboard potential protected by a noncompact global symmetry
    • Problem with empty/cold universe may be fixed using supersymmetry and assumptions about nonrenormalizable operators
    • Predictions may include modified gravity and absence of axion detection

• Landscape/anthropic arguments
Example

[hep-th/0604190]

Improve Abbott's “solution”:

Itzhaki's ingredients:

a) Dilaton (scalar-tensor gravity) – natural in string theory
b) Non-compact global symmetry (e.g. shift symmetry)
c) Supersymmetry
d) Assumption about non-renormalizable operators

idea:
Use Dilatonic coupling of the form $e^{-\phi^2} R$ to obtain

$$ U \sim \frac{1}{2} (R - m^2) \phi^2 $$

This leads to a precise relationship between the cosmological constant energy density and $m^2$ at the onset of reheating phase transition.

$$ R = 4 V(\chi) $$

precision determined by slope
Example continued

\[ U \sim \frac{1}{2} (R - m^2) \phi^2 \]
\[ R = 4 V(\chi) \]

Energy available for reheating? \[ \rho_{RH} \leq \frac{1}{4} m^2 M_p^2 \]

What about the rest of the cosmological constant?

Tune \( m \) and coupling to ensure acceptable CC at minimum of the phase transition vacuum.

i.e. if \( g \phi^4 \), a relationship between \( m \) and \( g \).

Can be protected by SUSY up to SUSY breaking effects of \( \frac{M_s^2}{M_p} \)

Isn't this tuning the same as the original CC tuning?

No, since now, we know exactly what the value of the CC to cancel! (This knowledge was provided to us by the Abott's mechanism.)

Isn't this tuning the same as the usual N=1 SUSY protection of CC? \( \frac{M_s^2}{M_p^4} \)

No, SUSY breaking generically leads to \( M_s^4 \) energy densities and not \( \frac{M_s^2}{M_p} \) densities.

Itzhaki's Point: Tuning for reheating DIFFERENT from CC problem
Problems with Proposal

- Sensitive to assumptions about nonrenormalizable operators:
  
  tree level $\rightarrow$ loop corrections $\rightarrow$ stable and predictive CC without UV completion

  However, does there exist any top don mod which give desired restrictions on the nonrenormalizable operators?

- Coupling to SM and reheating scenario not completed (in progress)

- Implications:
  
  - Dilaton $\rightarrow$ modifications of gravity near millimeter.
  - Low scale scale SUSY breaking.
  - Low scale inflation
  - axion vev may be fixed (allowing larger Peccei Quinn breaking scales).
Tracker Model

Why is the dark energy domination time period nearly coinciding with the epoch of structure formation?

- Energy density follows the radiation energy density until a transition to matter domination occurs

\[ V = \lambda \frac{\Lambda^6}{\phi^2} \]
\[ \frac{d^2 \phi}{dt^2} + 3 H \phi - 2 \lambda \frac{\Lambda^6}{\phi^3} = 0 \]
\[ H^2 = (\rho_B + \rho_Q)/(3 M_p^2) \]

Start with \( \rho_B \propto \left( \frac{a}{a_Q} \right)^{-3(1 + w_B)} \) dominant

\[ \rightarrow \rho_\phi \sim \lambda \frac{\Lambda}{M_p^2} \left( \frac{a}{a_Q} \right)^{-3(1 + w_B)/2} \]

decreases more slowly than \( \rho_B \)

large field var:
\[ \phi = M_p \sqrt{\frac{8}{3(1 + w_B)}} \left( \frac{a}{a_Q} \right)^{3(1 + w_B)/4} \]
\[ w_\phi = -1 + \frac{2(1 + w_B)}{4} \]

\( \Lambda \sim 10 \text{ MeV} \)

Controls the time of onset of quint domination.
Problems

- Light Mass: otherwise Q sits at the min of potential

  Form of the potential is difficult to protect in QFT
  All operators not protected by symmetries should appear in the EFT.

  e.g.
  \[
  W = (N_c - N_f) \frac{\Lambda^{(3N_c - N_f)l(N_c - N_f)}}{(\det Q^i Q^j)^{(l(N_c - N_f))}}
  \]

  \[
  Q^i Q^j = \phi^2 \delta^i_j
  \]

  rad. mass terms

  \[
  m^2 \phi^2
  \]

- pheno. problems with Planck mass field variation

  e.g.
  \[
  \phi = M_p \sqrt{\frac{8}{3(1 + w_B)}} \left( \frac{a}{a_Q} \right)^{3(1 + w_B)/4}
  \]

  \[
  c \left( \frac{\Phi}{M_p} \right)^n \text{Tr} \left[ F^{\mu \nu} F_{\mu \nu} \right] \rightarrow \text{gauge coupling time variation}
  \]

  Also, large vev variation → cutoff physics is important → lack of predictibility from theory
Tracking Problems

- Tracking requires some tuning of initial conditions

\[ V = \lambda \frac{\Lambda^{4+\alpha}}{\phi^\alpha} \]

phase portrait of attractors

\[ y = \frac{\sqrt{V}}{\rho} \]

\( x \equiv \frac{1}{\sqrt{6}} \frac{M_p}{\rho} \frac{d \phi}{d \ln a} \)

Other Properties Quintessence

- Lightness of quintessence: like photons, do not cluster appreciably
- CMB sensitivity requires at least few percent level contribution of quintessence at $z=1000$. Certainly not true for cosmo constant.
- Just like inflatons, non-canonical kinetic terms can change $c_s$.
- Suppression of CMB multipoles on long wavelength is possible with quintessence with $c_s \ll 1$. (e.g. astro-ph/0301284)
- Because quintessence needs to be weakly coupled (gravitational strength) to protect its light mass, it cannot be produced at colliders.
Distinguishing CC and Quintessence

- Equation of state
  - Clustering and lensing
  - Supernovae or any other standard candles
  - Integrated Sachs-Wolfe effect (ISW)
  - Last scattering surface CMB effects
- Sound speed effects for non-canonical Q [e.g. astro-ph/0301284]
- Interactions of Q with dark matter (DM)
  - DM at colliders + DM direct detection + effects of Q during DM freeze out [e.g. astro-ph/0207396]
Collaborative score card

- ✓ Why is the Higgs field light?
- ✓ What is the origin of electroweak symmetry breaking?
- ✓ Is it simply an accident that the gauge couplings seem to meet?
- ✓ How is gravity incorporated into the SM?
- ✓ Why is the CP violation from QCD small?
- ✓ What is the dark energy?
- ✓ ✓ What is the CDM?
- ✓ Why more baryons than antibaryons?
- ✓ If inflation solves the cosmological initial condition problems, what is the inflaton?
- ✓ Classical singularities of general relativity?
- ✓ Why is the observed cosmological constant small when SM says it should be big?
- ✓ Origin of ultra-high energy cosmic rays?
- ✓ ✓ with SUSY
- ✓ ✓ with PQ

Many other speculative connections exist. Not very convincing yet, unfortunately. Restricting to particle physics.
Outlook

• Inflation non-trivial tests: correlation of observables
  • B-modes: gravity waves
  • running spectral index: measurement of CMB down to short scales
  • non-gaussianities: 21-cm line
• LHC is turning on: Higgs most likely to be measured
  • unprecedented opportunity for testing EW bgenesis
  • leptogenesis is comparatively more difficult to test
• DE being non-zero opens up new views
  • e.g. axion PQ scale can be lowered – connection with string theory?
  • e.g. inflationary model must be low scale model
  • e.g. implications for near-millimeter scale modification of gravity
  • e.g. lowscale SUSY breakng
• Exciting opportunities to connect HE theory with cosmology sure to be enhanced with LHC results!