QCD,
a background to new physics

PASI 2006

Daniel de Florian
Dpto. de Física- FCEyN- UBA
Purpose(s) of these lectures:

- Introduction to QCD
- Refresh your knowledge on QCD
- Understand the vocabulary!
- New developments in the field
- QCD ~ pQCD
Outline of the lectures

1. Basic QCD
   ✤ Rules
   ✤ QCD at work in $e^+e^-$
   ✤ Parton model and DIS
   ✤ QCD corrections and evolution

2. State of the art
   ✤ QCD at colliders
   ✤ QCD to all orders
   ✤ Compute cross-sections (modern)
   ✤ From strings to QCD
✓ New physics will be studied at hadronic colliders

✓ Precise understanding of QCD is essential

✓ QCD: Production mechanism

✓ QCD: Main background as well

Higgs production at LHC
Assume already some knowledge on QCD: very fast introduction

♦ QCD (Quantum ChromoDynamics) : theory for strong interactions
♦ An essential and established part of the toolkit for discovering physics beyond the standard model

Hadrons (barions and mesons) are the “color singlets” made of “colored” quarks (spin 1/2) interacting with gluons (spin 1)

\[ q = u, d, s, c, b, t \] in 3 colors

 gluons in 8 colors

<table>
<thead>
<tr>
<th>quark</th>
<th>charge</th>
<th>mass (approx.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>2/3</td>
<td>~4 MeV</td>
</tr>
<tr>
<td>d</td>
<td>-1/3</td>
<td>~7 MeV</td>
</tr>
<tr>
<td>c</td>
<td>2/3</td>
<td>~1.4 GeV</td>
</tr>
<tr>
<td>s</td>
<td>-1/3</td>
<td>~150 MeV</td>
</tr>
<tr>
<td>t</td>
<td>2/3</td>
<td>~175 GeV</td>
</tr>
<tr>
<td>b</td>
<td>-1/3</td>
<td>~4.5 GeV</td>
</tr>
</tbody>
</table>

These lectures: neglect quark masses
**QCD: non-abelian gauge theory under SU(3)**

Simple recipe: take free Lagrangian
\[ \mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi \]

Force it to be invariant under non-abelian local transformation
\[ \psi(x) \rightarrow e^{i \alpha_a(x) T^a} \psi(x) \] with 8 generators obeying
\[ [T^a, T^b] = i f^{abc} T^c \]

Change derivative to covariant
\[ (D_\mu)_{ij} = \delta_{ij} \partial_\mu - ig_s T_{ij}^a A_\mu^a \]

Add all gauge invariants!
\[ F_{\mu\nu}^{(a)} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g_s f_{abc} A_\mu^b A_\nu^c \]

\[ \mathcal{L}_{QCD} = -\frac{1}{4} F_{\mu\nu}^{(a)} F^{(a)\mu\nu} + \sum_q \bar{\psi}_i^q (i \gamma^\mu (D_\mu)_{ij} - m_q \delta_{ij}) \psi_j^q \]

(+ gauge fixing terms and eventually ghosts)

one single coupling constant
\[ \alpha_s \equiv \frac{g_s^2}{4\pi} \]
\[ \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{int}} \]

\[ \mathcal{L}_{\text{int}} = \sum_{f=1}^{N_f} \bar{\psi}_f \gamma^\mu t^a_{ij} A^a_{\mu} \psi^i_f \quad q\bar{q}g \text{ vertex} \]

\[ - g f^{abc} \partial^\mu A^a_{\nu} A^b_{\mu} A^c_{\nu} \quad ggg \text{ vertex} \]

\[ - \frac{1}{4} g^2 f^{abc} f^{ade} A^b_{\mu} A^c_{\nu} A^d_{\mu} A^e_{\nu} \quad gggg \text{ vertex} \]

**Feynman rules**

\[ \begin{array}{cc}
\alpha & \beta \\
\hline
i & j
\end{array} \quad -ig(t^a)_{ij}(\gamma^\mu)_{\alpha\beta} \]

\[ \begin{array}{cc}
\alpha & \beta \\
\hline
i & j
\end{array} \quad -ig(t^a)_{ij}(\gamma^\mu)_{\alpha\beta} \]

**vertex**

\[ -gf^{a_1a_2a_3} \left[ g^\mu_1\mu_2 (p_1 - p_2)^{\mu_3} + g^\mu_2\mu_3 (p_2 - p_3)^{\mu_1} + g^\mu_3\mu_1 (p_3 - p_1)^{\mu_2} \right] \]

\[ -ig^2 \left[ f^{b_1a_1a_2} f^{b_3a_3a_4} (g^\mu_1\mu_3 g^\mu_2\mu_4 - g^\mu_1\mu_4 g^\mu_2\mu_3) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4) \right] \]
Propagators

\[ \alpha \quad p \quad \beta \]
\[ i \quad \overset{\rightarrow}{\longrightarrow} \quad j \]

\[ \frac{i(p + m)_{\alpha \beta}}{p^2 - m^2 + i\epsilon} \delta^{ij} \]

spin polarization tensor

\[ d^{\mu\nu}(p) = \sum_{\lambda} \varepsilon^{\mu}_{(\lambda)}(p) \varepsilon^{\nu*}_{(\lambda)}(p) \]

\[ d^{\mu\nu}(p) = \begin{cases} 
- g^{\mu\nu} + (1 - \alpha) \frac{p^\mu p^\nu}{p^2 + i\epsilon} & \text{covariant gauges} \\
- g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p \cdot n} - n^2 \frac{p^\mu p^\nu}{(p \cdot n)^2} & \text{axial gauges}
\end{cases} \]

In covariant gauges Lorentz invariance is manifest but ghosts must be included to cancel effect of unphysical gluon polarizations
Color algebra

\[ T_R(t^a t^b) = T_R \delta_{ab} \quad T_R = 1/2 \]

\[ (t^a t^a)_{il} = C_F \delta_{il} \quad C_F = \frac{N_c^2 - 1}{2N_c} \]

\[ f^{acd} f^{bde} = C_A \delta^{ab} \quad C_A = N_c \]

\[ T \quad 8 \text{ Gell-Mann (3x3) matrices in fundamental representation} \]

i,j,... quark

a,b,... gluon

\[ T_R(t^a t^b) = T_R \delta_{ab} \]

\[ t^a_{ij} t^a_{jl} = C_F \delta_{il} \]

\[ f^{acd} f^{bde} = C_A \delta^{ab} \]

gluon → quarks  quark → gluons  gluon → gluons
QCD at work

QCD cannot be solved exactly: use perturbation theory

Coupling constant “large”: many orders needed for precision

Several problems appear in the calculation of perturbative corrections
As QED, it has problems with loops: ultraviolet divergences

\[
\sim g^2 \int_{p^2}^{\infty} d^4k \frac{1}{k^2} \frac{1}{(p-k)^2} \to \infty
\]

A manifestation that QFT **FAIL** at very large energies!

To be able to use QFT, search for a procedure to isolate the “large” energy regime **renormalization**

1. Regularize the divergency
2. “Absorb” it by redefinition of “bare” \((g, m, A, \psi)\) parameters in Lagrangian (thanks to gauge symmetry!)
Example

\[ \begin{aligned}
\text{Regularization } \Lambda_{cut} & \sim \alpha_B \left\{ 1 + \alpha_B \beta_0 \int_{p^2}^{\Lambda_{cut}^2} \frac{d^4 k}{(k^2)^2} + \mathcal{O}(\alpha_B^2) \right\} \\
\text{Renormalization scale } \mu & \sim \alpha_B \left\{ 1 + \alpha_B \beta_0 \left( \log \frac{\Lambda_{cut}^2}{\mu^2} + \log \frac{\mu^2}{p^2} \right) + \mathcal{O}(\alpha_B^2) \right\} \\
\text{Renormalization} & = \alpha(\mu^2) \left\{ 1 + \beta_0 \alpha(\mu^2) \log \frac{\mu^2}{p^2} + \mathcal{O}(\alpha_B^2) \right\} \\
\alpha(\mu^2) & \equiv \alpha_B \left( 1 + \beta_0 \alpha_B \log \frac{\Lambda_{cut}^2}{\mu^2} + \mathcal{O}(\alpha_B^2) \right)
\end{aligned} \]

Renormalized (running) coupling constant: \( \mu \) dependent (universal)

\[ \frac{d\alpha_s(\mu^2)}{d \log \mu^2} = -\beta(\alpha_s) \quad \beta(\alpha_s) = \beta_0 \alpha_s^2 + ... \]

All order sum of UV logs
OK, I got this one renormalized
QED

\[ \beta_0 = -\frac{1}{3\pi} < 0 \]

**Gross, Wilczek, Politzer**

QCD

\[ \beta_{0}^{\text{quark}} = -\frac{1}{3\pi} T_R N_F < 0 \]

\[ \beta_{0}^{\text{gluon}} = \frac{11}{12\pi} C_A \]

QCD \[ \beta_0 = \frac{11C_A - 2n_F}{12\pi} > 0 \quad (n_F < 16) \]

Coupling constant decreases with energy
The two faces of QCD

$\alpha_s \sim 1$

confinement
large distances $\sim 1$ fermi

asymptotic freedom
short distances

Quarks do not show up as "free particles"

distance $\sim 1/\text{energy}$
The solution of the RGE \[ \frac{d\alpha_s(\mu^2)}{d \log \mu^2} = -\beta(\alpha_s) \] at leading order (LO)

\[ \alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1 + \beta_0 \alpha_s(\mu_0^2) \log \mu^2 / \mu_0^2} \]

\[ \Rightarrow \alpha_s(\mu^2) = \alpha_s(\mu_0^2) - \alpha_s^2(\mu_0^2) \beta_0 \log \frac{\mu^2}{\mu_0^2} + \ldots \]

Better define

\[ \Lambda_{QCD} = \mu_0 \exp \left[ -\frac{1}{2\beta_0 \alpha_s(\mu_0^2)} \right] \]

\[ \Rightarrow \alpha_s(\mu^2) = \frac{1}{\beta_0 \log \frac{\mu^2}{\Lambda_{QCD}}^2} \]

\[ \Lambda_{QCD} \sim 200 \text{ MeV} \]

At the scale of hadron masses pQCD breaks down!

\[ \alpha_s(M_Z^2) = 0.1182 \pm 0.0027 \]

Compare to QED (1/137) \[ \alpha_s > 1/10 \]
In real life:

Dimensional regularization \( \rightarrow \) D dimensions,

“divergences” appear as \( 1/(D-4) \) poles

Finite terms can be subtracted: renormalization scheme

At Next-to-Next-to-Leading Order (NNLO) in \( \overline{\text{MS}} \) scheme

\[
\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln (\mu^2/\Lambda^2)} \left[ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln \left[ \ln (\mu^2/\Lambda^2) \right]}{\ln (\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2 (\mu^2/\Lambda^2)} \right. \\
\times \left. \left( \left( \ln \left[ \ln (\mu^2/\Lambda^2) \right] - \frac{1}{2} \right)^2 + \frac{\beta_2 \beta_0}{8\beta_1^2} - \frac{5}{4} \right) \right].
\]
\[ \beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f \]
\[ \beta_1 = \frac{34}{3} C_A^2 - 4C_FT_F n_f - \frac{20}{3} C_A T_F n_f \]
\[ \beta_2 = \frac{2857}{54} C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9} C_F C_A T_F n_f \]
\[ -\frac{1415}{27} C_A^2 T_F n_f + \frac{44}{9} C_F T_F^2 n_f^2 + \frac{158}{27} C_A T_F^2 n_f^2 \]
\[ \beta_3 = C_A^4 \left( \frac{150653}{486} - \frac{44}{9} \zeta_3 \right) + C_A^3 T_F n_f \left( -\frac{39143}{81} + \frac{136}{3} \zeta_3 \right) \]
\[ + C_A^2 C_F T_F n_f \left( \frac{7073}{243} - \frac{656}{9} \zeta_3 \right) + C_A C_F^2 T_F n_f \left( -\frac{4204}{27} + \frac{352}{9} \zeta_3 \right) \]
\[ + 46C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2 \left( \frac{7930}{81} + \frac{224}{9} \zeta_3 \right) + C_F^2 T_F^2 n_f^2 \left( \frac{1352}{27} - \frac{704}{9} \zeta_3 \right) \]
\[ + C_A C_F T_F^2 n_f^2 \left( \frac{17152}{243} + \frac{448}{9} \zeta_3 \right) + \frac{424}{243} C_A T_F^3 n_f^3 + \frac{1232}{243} C_F T_F^3 n_f^3 \]
\[ + \frac{d_{abcd} d_{abcd}}{N_A} \left( -\frac{80}{9} + \frac{704}{3} \zeta_3 \right) + n_f \frac{d_{abcd} d_{abcd}}{N_A} \left( \frac{512}{9} - \frac{1664}{3} \zeta_3 \right) \]
\[ + n_f^2 \frac{d_{abcd} d_{abcd}}{N_A} \left( -\frac{704}{9} + \frac{512}{3} \zeta_3 \right) \]
QCD at work

Observable computed as an expansion in strong coupling constant

\[ \sigma = \sigma^{(0)} + \alpha_s(\mu) \sigma^{(1)} + \alpha_s^2(\mu) \sigma^{(2)} + \ldots \]

Example: \( e^+e^- \rightarrow \text{hadrons} \)

LO: \( \sigma(e^+e^- \rightarrow \text{hadrons}) \approx \sigma(e^+e^- \rightarrow \text{quarks}) \)

\[ R_{\text{had}} = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \]
\[ R_{\text{had}} \equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_q e_q^2 N_c \]

- $N_c = 3$
- Quark Flavor thresholds
What about the next term in the expansion? $\mathcal{O}(\alpha_s)$

Since coupling constant not so small, can lead to visible effect

Two different type of contributions: real and virtual gluon emission

Real

Virtual
a) Real gluon emission

\[ x_i = \frac{2E_i}{Q} \]

\[ |M_{\text{real}}(x_1, x_2, x_3)|^2 = C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)} \]

Integrate over phase space → real contribution to cross-section

\[ \sigma^R = \int_0^1 dx_1 dx_2 dx_3 \delta(2 - x_1 - x_2 - x_3) |M_{\text{real}}(x_1, x_2, x_3)|^2 \quad \text{singular at } x_i = 1 \]

Origin of singular contributions: soft and collinear emission

\[ \frac{1}{(p + k)^2} = \frac{1}{2 p \cdot k} = \frac{1}{2E_q E_g (1 - \cos \theta_{qg})} \]

soft  \hspace{1cm} \text{collinear}
Looks bad .... we are computing a physical quantity and diverges..

Lets regularize it by introducing a gluon mass $m_g$ to see what happens

$$\sigma^R = \sigma^{(0)} C_F \frac{\alpha_s}{2\pi} \left( \log^2 \frac{m_g^2}{Q^2} + 3 \log \frac{m_g^2}{Q^2} + 7 - \frac{\pi^2}{3} \right)$$

b) Add virtual contribution

$$\sigma^{(NLO)} = \sigma^{(0)} \left( 1 + \frac{\alpha_s}{\pi} \right)$$

$$\sigma^V = \sigma^{(0)} C_F \frac{\alpha_s}{2\pi} \left( - \log^2 \frac{m_g^2}{Q^2} - 3 \log \frac{m_g^2}{Q^2} - \frac{11}{2} + \frac{\pi^2}{3} \right)$$
Cancellation not by miracle

Since (Feynman) we compute virtual and real contributions separately: regularization needed until achieve cancellation

IR much worse than UV! Possible solution: QFT with \(n + \epsilon\) particles?

Real and Virtual diagrams have very similar structure: cuts

In the infrared region: virtual and real are kinematically equivalent (-1) from Unitarity
Cancellation is a general feature: Kinoshita-Lee-Nauenberg (KLN)

**KLN Theorem**

Infrared singularities in massless theory cancel out after a sum over degenerate (initial and final) states

- hard
- hard + soft gluon
- 2 collinear partons

Physically a hard parton can not be distinguish from a parton plus a soft gluon or two collinear partons : degenerate states

In QED: Bloch-Nordsieck (sum over final states only needed)

We can use QCD to compute observables corresponding to processes inclusive enough → InfraRed safe

\[ e^+ e^- \rightarrow q\bar{q} \] is not IR

while \[ e^+ e^- \rightarrow 2 \text{ jets} \] is
3 jet production

IR safe: KLN works
cancellation not as complete as for fully inclusive:
some logs remain

$\alpha_s \log R$
Scale dependence: at which scale evaluate the coupling?

Scale is unphysical, in principle any value possible, but...

According to RGE, dependence cancels if observable computed to all orders in perturbation theory

In “real life” we have access only to the first orders in the expansion

\[ \sigma^{(NLO)}(\mu) = \sigma^{(0)} \left( 1 + \frac{\alpha_s(\mu)}{\pi} \right) \]

residual dependence due to truncation of expansion: higher order needed!

\[
\alpha_s(\mu^2) = \alpha_s(\mu_0^2) - \alpha_s^2(\mu_0^2) \beta_0 \log \frac{\mu^2}{\mu_0^2} + O(\alpha_s^3)
\]

If higher orders included, scale dependence is reduced

\[
\sigma^{(NNLO)}(\mu) = \sigma^{(0)} \left[ 1 + \frac{\alpha_s(\mu)}{\pi} + O^{(2)} \left( \log \frac{\mu^2}{Q^2} \right) \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 \right]
\]

In general coefficients depend on logs of ratios between the typical energy scale of the process and renormalization scale.
For single scale problems \((Q)\), \(\mu \sim Q\) is a reasonable choice to avoid large logarithms.

Use scale dependence to set (lower) limit for unknown terms: higher order contributions should be as big as scale dependence.

\[
\mu = \frac{1}{2} Q^{1/2}
\]
to evaluate TH “uncertainty”
Lot of evidence about QCD from lepton colliders: precision EXP & TH

QCD is non-abelian

Gluon has spin 1

![Graph showing data and theoretical predictions]

- **ALEPH Data**
- **QCD SU(3)**
- **abelian model**

- Vector Gluon, LO + Fragment.
- Scalar Gluon, LO + Fragment.
Multiple jet production

number of jets $\sim \alpha_S^{n-1}$

See what happens in a more complicated “environment”
Deep inelastic scattering

\[ ep \rightarrow eX \quad Q \gg \Lambda_{QCD} \]

If \( Q^2 < M_Z^2 \) the cross section is dominated by one-photon exchange

\[
k'_0 \frac{d\sigma}{d^3k'} = \frac{1}{k \cdot p} \left( \frac{\alpha}{q^2} \right)^2 L^{\mu\nu} W_{\mu\nu}
\]

Leptonic tensor: computable QED

Hadronic tensor

Construct the most general tensor: parity, current conservation

\[
W_{\mu\nu} = \frac{1}{2\pi} \int d^4y \ e^{iqy} \langle p | J_{\mu}(y) J_{\nu}(0) | p \rangle = F_1 \left( \frac{q_{\mu} q_{\nu}}{q^2} - g_{\mu\nu} \right) + F_2 \frac{1}{p \cdot q} \left( p_{\mu} - \frac{p \cdot q}{q^2} q_{\mu} \right) \left( p_{\nu} - \frac{p \cdot q}{q^2} q_{\nu} \right)
\]

Structure functions \( F_i(x, Q^2) \)

\[
Q^2 = -q^2 \quad x = \frac{Q^2}{2p \cdot q}
\]
Parton Model

Proton made up of pointlike particles: partons

★ Photon virtuality sets resolution $\lambda \sim 1/Q$

★ Photon-quark Interaction $t_{hard} \sim 1/Q$

★ Interaction between partons $t \sim 1/\Lambda_{QCD}$

As $Q >> \Lambda_{QCD}$

During “hard interaction”, partons don’t have time to interact among them

Scattering is incoherent on the single partons
Parton Model

Factorization

\[ \sigma(ep \rightarrow eX) = \int_0^1 dz \sum_i f_i(z) \hat{\sigma}(eq_i \rightarrow eX) \]

- Probability to find parton “i” with momentum fraction \(z\) in proton
- \(\hat{\sigma}\) = “partonic” cross section computed perturbatively
- Parton distributions (PDF) from experiments: \textit{universal}
At lowest order

\[(p')^2 = (zp + q)^2 = 2zp \cdot q - Q^2 = 0 \quad \rightarrow \quad z = x\]

- Point-like interaction → scaling \(F_2(x, Q^2) = \sum_q e_q^2 x f_q(x)\)
- Quarks are fermions → \(F_L(x, Q^2) = F_2(x, Q^2) - 2xF_1(x, Q^2) = 0\)

Polarized cross section (spin dependent)

\[g_1 = \sum_q f_q(x, \uparrow) - f_q(x, \downarrow)\]

\[F_L(x, Q^2) = F_2(x, Q^2) - 2xF_1(x, Q^2) = 0\]
When weak interactions considered (parity violation) \( \Rightarrow \mathbf{F}_3 \)

\[
\frac{d^2\sigma(\nu + p)}{dx \, dQ^2} = \frac{G_F^2}{4\pi x} \left( \frac{M_w^2}{Q^2 + M_w^2} \right)^2 \left[ (1 + (1 - y)^2) \mathbf{F}_2^{\nu} - y^2 \mathbf{F}_L^{\nu} \pm (1 - (1 - y)^2) x \mathbf{F}_3^{\nu} \right]
\]

Using strong isospin symmetry (\(\mathbf{p} \leftrightarrow \mathbf{n}\))

\[
\begin{align*}
f_{u/n}(x) &= f_{d/p}(x) \equiv d(x) \\
f_{\bar{u}/n}(x) &= f_{\bar{d}/p}(x) \equiv \bar{d}(x) \\
f_{d/n}(x) &= f_{u/p}(x) \equiv u(x) \\
f_{s/n}(x) &= f_{s/p}(x) \equiv s(x)
\end{align*}
\]

and measuring several DIS cross-sections

\[
\begin{align*}
\mathbf{F}_{2ep}^e/x &= \frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{4}{9} \bar{u}(x) + \frac{1}{9} \bar{d}(x) + \frac{1}{9} s(x) + \frac{1}{9} \bar{s}(x) + \frac{4}{9} c(x) + \frac{4}{9} \bar{c}(x) \\
\mathbf{F}_{2en}^e/x &= \frac{1}{9} u(x) + \frac{4}{9} d(x) + \frac{1}{9} \bar{u}(x) + \frac{4}{9} \bar{d}(x) + \frac{1}{9} s(x) + \frac{1}{9} \bar{s}(x) + \frac{4}{9} c(x) + \frac{4}{9} \bar{c}(x) \\
\mathbf{F}_{2\nu p}^\nu/x &= 2d(x) + 2\bar{u}(x) + 2s(x) + 2\bar{c}(x) \\
\mathbf{F}_{3\nu p}^\nu &= 2d(x) - 2\bar{u}(x) + 2s(x) - 2\bar{c}(x) \\
\mathbf{F}_{2\bar{\nu} p}^{\bar{\nu}}/x &= 2u(x) + 2\bar{d}(x) + 2c(x) + 2\bar{s}(x) \\
\mathbf{F}_{3\bar{\nu} p}^{\bar{\nu}} &= 2u(x) - 2\bar{d}(x) + 2c(x) - 2\bar{s}(x)
\end{align*}
\]

Extraction of quark distributions possible
QCD corrections and scaling violation

Does simple parton model survive at higher orders?

Quarks can radiate gluons: real corrections

\[ \frac{1}{(p - k)^2} = \frac{1}{2p \cdot q} = \frac{1}{2E_q E_g (1 - \cos \theta)} \]

Divergences again...

when gluon has no transverse momentum \( k_T \rightarrow 0 \)

Will virtual contributions solve the problem again?

No (not all of them)!!!
Contribute to different kinematics

\[ (p')^2 = (zp + q)^2 = 2zp \cdot q - Q^2 = 0 \rightarrow z = x \]

\[ (p')^2 = (zp + q - k)^2 \sim 2zyp \cdot q - Q^2 = 0 \rightarrow zy = x \]

Sum of real + virtual: soft singularities cancelled \((y=1)\) collinear remain!
Why cancellation does not occur?

Feynman diagrams are the same as in

\[ e^+ e^- \rightarrow \text{hadrons} \]

KLN: Infrared singularities in massless theory cancel out after a sum over degenerate (initial and final) states

PDF spoils sum!
Parton model: separation between soft and hard physics

Introduce new factorization scale $\mu_F$

Separate soft and hard contributions
(virtuality of quark/ transverse momentum of gluon)

$k_T > \mu_F$

$k_T < \mu_F$
Real contribution

\[ F_{2}^{\text{cor}}(x, Q^2) = \sum_{q} e_q^2 x \frac{\alpha_s}{2\pi} \log \left( \frac{Q^2}{\mu_0^2} \right) \int_{x}^{1} \frac{dy}{y} P_{qq}(y) q \left( \frac{x}{y} \right) + \text{finite} \]

First thing to notice: scaling broken due to gluon radiation
\[ F_{2}^{\text{cor}}(x, Q^2) = \sum_q e_q^2 x \frac{\alpha_s}{2\pi} \log \left( \frac{Q^2}{\mu_0^2} \right) \int_x^1 \frac{dy}{y} P_{qq}(y) q\left( \frac{x}{y} \right) + \text{finite} \]

\[
\log \left( \frac{Q^2}{\mu_0^2} \right) = \log \left( \frac{Q^2}{\mu_F^2} \right) + \log \left( \frac{\mu_F^2}{\mu_0^2} \right)
\]

Hard (and finite) \hspace{2cm} \text{soft (and divergent) to PDF}

Factorization IR equivalent to UV renormalization (DR and fact. scheme)

\[ q(x, \mu_F^2) = q(x) + \frac{\alpha_s}{2\pi} \log \left( \frac{\mu_F^2}{\mu_0^2} \right) \int_x^1 \frac{dy}{y} P_{qq}(y) q\left( \frac{x}{y} \right) \]

Factorization scale unphysical (situation similar to renormalization)

In DIS one typically chooses \( \mu_F = \mu_R = Q \), but could be different

Fixed order calculations show “spurious” factorization scale dependence
\[
q(x, \mu_F^2) = q(x) + \frac{\alpha_s}{2\pi} \log \left( \frac{\mu_F^2}{\mu_0^2} \right) \int_x^1 \frac{dy}{y} P_{qq}(y) q \left( \frac{x}{y} \right)
\]

Altarelli-Parisi equation  (RGE like: resummation of collinear logs)

Increase “resolution” scale: resolve more details of “partonic structure”

\[
\frac{\partial q(x, \mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(y) q \left( \frac{x}{y}, \mu_F^2 \right) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{qq}(y) g \left( \frac{x}{y}, \mu_F^2 \right)
\]

Probabilistic interpretation

\[
\frac{\partial g(x, \mu_F^2)}{\partial \log(\mu_F^2)} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{gg}(y) \sum_q q \left( \frac{x}{y}, \mu_F^2 \right) + \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} P_{gg}(y) g \left( \frac{x}{y}, \mu_F^2 \right)
\]
\[ P^{(0)}_{qq}(z) = C_F \left[ \frac{1 + z^2}{(1 - z)^+} + \frac{3}{2} \delta(1 - z) \right] \]
\[ \int_0^1 \frac{f(z)}{(1 - z)^+} \equiv \int_0^1 \frac{f(z) - f(1)}{1 - z} \]
\[ P^{(0)}_{gg}(z) = 2 C_A \left[ \frac{z}{(1 - z)^+} + \frac{1 - z}{z} + z(1 - z) \right] \]
\[ + \left( \frac{11}{6} C_A - \frac{2}{3} T_R n_F \right) \delta(1 - z) \]
\[ P^{(0)}_{gq}(z) = C_F \left[ \frac{1 + (1 - z)^2}{z} \right] \]
\[ P^{(0)\bar{q}g}(z) = T_R \left[ \bar{z}^2 + (1 - z)^2 \right] \]
Scaling violations are:

- Positive at small $x$
- Slightly negative at large $x$

Main effect of increase in $Q^2$ is shift of partons from larger to smaller $x$

Resolve shorter distances in the proton: quark with fraction $x$ can be resolved as a $qg$ pair (quark with smaller momentum)
AP Evolution equations allow to predict the $Q^2$ dependence of DIS data

And very well!

Region studied to find scaling!
pQCD vocabulary: LO-NLO-NNLO-...

Improved (factorized) Parton Model

$$\sigma(ep \rightarrow eX) = \int_0^1 dz \sum_{i=q,\bar{q},g} f_i(z, \mu_F^2) \hat{\sigma}^{\text{hard}}(ei \rightarrow eX)$$

Factorized

LO Leading Order: Born partonic cross-section

+ LO evolution of pdfs

$$F_2(x, Q^2) = \sum_q e_q^2 x f_q(x, Q^2)$$
NLO  
Next-to-Leading Order:  
Born $+\mathcal{O}(\alpha_s)$ (finite) cross-section 
$+ \text{ NLO evolution of pdfs}$

\[ F_2(x, Q^2) = \sum_q e_q^2 x f_q(x, Q^2) + \alpha_s \sum_q e_q^2 \int_{x/y}^1 \frac{dy}{y} C_q^{(1)}(y) f_q(x/y, Q^2) + \alpha_s \sum_q e_q^2 \int_{x/y}^1 \frac{dy}{y} C_g^{(1)}(y) f_g(x/y, Q^2) \]

NNLO  
Next-to-Next-to-Leading Order:  
$\ldots + \mathcal{O}(\alpha_s^2)$ (finite) cross-section 
$+ \text{ NNLO evolution of pdfs}$ 
$+ \alpha_s^2 C_i^{(2)}(y)$
Higher order Altarelli-Parisi kernels known → three-loop


9607 (3-loop) Feynman diagrams: 20 man-year work !!

And you just want to go to the beach!
Summary of first Lecture

- Rich UV structure: Confinement and asymptotic freedom
- Infrared structure also complicated: KLN Theorem
- DIS and the parton model: parton distributions in the nucleon
- QCD predicts scale dependence of PDFs

Tomorrow

- State of the art
- Use these tools at hadronic colliders
- (very) New ideas to compute amplitudes