Theories with Compact Extra Dimensions

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Why Extra Dimensions?

The Basics of Compact Extra Dimensions

Theories with Extra Dimensions:

Part I
- Large Extra Dimensions
- Universal Extra Dimensions

Part II
- Warped Extra Dimensions
- Back to 4D?
Why Extra Dimensions?

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<th>Unification: Kaluza-Klein (1920’s)</th>
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<td>- Unify gravitation and electromagnetism in a 5D gravity theory.</td>
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<td>- 5D Gravity $\rightarrow$ 4D Gravity + Electromagnetism</td>
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<th>Quantum Gravity: String Theory</th>
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<td>In order for String Theory to be consistent need extra spatial dimensions</td>
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<th>Bottom Up: Tools for Model Building</th>
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<td>- Build models with a lower cutoff than the SM</td>
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<td>- Generate large, stable hierarchies</td>
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Part I

Basics of Extra Dimensional Theories

Flat Extra Dimensions
Extra spatial dimensions with points periodically identified

1 Extra Dimension: equivalent to a circle

with $R = L/2\pi$. We identified the points

\[ x \sim x + L \sim x + 2L \sim x + 3L \sim \cdots \]
Compactification

- When field propagates in one extra dimension
  \[ P_M = P_\mu + P_5 \]
  with \( \mu = 0, 1, 2, 3, M = \mu, 5. \)
- But XD is compact \( \Rightarrow P_5 \) is quantized:
  periodicity \( \Rightarrow \) wavelength has to be integer number of \( 2\pi R. \)
  \[ P_5 = \frac{n}{R} , \quad (n = 0, 1, 2, 3, \cdots) \]
If field has mass $M$

$$P_M P^M = P_\mu P^\mu - P_5^2 = P_\mu P^\mu - \frac{n^2}{R^2}$$

From the 4D point of view:

$$P_\mu P^\mu = M^2 + \frac{n^2}{R^2}$$

E.g. for a photon (or graviton) $M = 0$. There is a “$n = 0$-mode” with zero mass (our photon/graviton), plus infinite excitations with masses $n/R$. 
Compact extra dimensions $\Rightarrow$ particle excitations (Kaluza-Klein tower)

Mass gap $\Delta m \sim 1/R$
Assume space has $3 + n$ dimensions.

The extra $n$ dimensions are compact and with radius $R$.

All particles are confined to a 3-dimensional slice ("brane").

Gravity propagates in all $3 + n$ dimensions.
Gravity appears weak ($M_P \ll M_W$), because it propagates in large extra dimensions... Its strength is diluted by the volume of the $n$ extra dimensions.

Fundamental scale is $M_* \sim M_W$, not $M_P$

$$M_P^2 \sim M_*^{n+2} R^n$$

There is no hierarchy problem:
The fundamental scale of Gravity

$$M_* \sim 1 \text{ TeV}$$
Large Extra Dimensions

If we require $M_* = 1$ TeV:

\[ R \sim 2 \cdot 10^{-17} \ 10^{\frac{32}{n}} \text{ cm} \]

- $n = 1 \implies R = 10^8$ Km. Already excluded!
- $n = 2 \implies R \approx 2$ mm. Barely allowed by current gravity experiments.
- $n > 2 \implies R < 10^{-6}$ mm. This is fine.

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Theories with Compact Extra Dimensions
E.g. for

\[ n = 2 \quad \rightarrow \quad \Delta m = 10^{-3} \text{ eV}. \]
\[ n = 3 \quad \rightarrow \quad \Delta m = 100 \text{ eV}. \]
\[ n = 7 \quad \rightarrow \quad \Delta m = 100 \text{ MeV}. \]
Individual KK graviton couplings gravitationally suppressed \((\sim 1/M_P)\).

But for \(E \gg 1/R\) \(\rightarrow\) sum of KK mode results in

\[
\sigma \sim \frac{E^n}{M_*^{n+2}}.
\]

Collider Processes:

E.g. Graviton production

Individual graviton decay rates \(\sim 1/M_P^2\), \(\Rightarrow\) \(\not{E}_T\) signals at colliders.

Bounds on \(M_*\) from LEP and Tevatron \((1 - 10)\) TeV.
What happens if all fields propagate in the extra dimensions?

Universal Extra Dimensions (Appelquist, Cheng, Dobrescu ’01)

- If some SM fields propagate in the bulk ⇒ \( \frac{1}{R} \gtrsim 1 \text{ TeV} \).
- But if we assume all fields can propagate in the extra dimensions. What is the allowed \( R \)?
- Universal propagation → additional symmetries.
For example, a scalar field $\Phi(x, y)$ in one extra dimension:

$$S[\Phi(x, y)] = \frac{1}{2} \int d^4x \, dy \left( \partial_M \Phi \partial^M \Phi - M^2 \Phi^2 \right)$$

- Periodic boundary conditions:

  $$\Phi(y) = \Phi(y + 2\pi R)$$

- Expand in Fourier modes:

  $$\Phi(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0} \left[ \phi_n(x) \cos \left( \frac{ny}{R} \right) + \tilde{\phi}_n(x) \sin \left( \frac{ny}{R} \right) \right]$$

  - $\phi_n(x)$ and $\tilde{\phi}_n(x)$ are 4D fields.
Integrate over the compact dimension:

$$S_{4\text{Deff.}}[\phi, \tilde{\phi}] = \int_0^{2\pi R} dy \, S[\Phi]$$

with

$$S_{4\text{Deff.}} = \sum_{n=0}^{1} \frac{1}{2} \int d^x \left[ \partial_\mu \phi_n \partial^\mu \phi_n - m_n^2 \phi_n^2 \right]$$

$$+ \sum_{n=0}^{1} \frac{1}{2} \int d^x \left[ \partial_\mu \tilde{\phi}_n \partial^\mu \tilde{\phi}_n - m_n^2 \tilde{\phi}_n^2 \right]$$

with

$$m_n^2 = M^2 + \frac{n^2}{R^2}$$
Momentum conservation in the extra dimensions
At any vertex, $P_M$, is conserved.
Then 4D-momentum conservation $\Rightarrow P_5$ is conserved.
E.g. in $(1) + (2) \rightarrow (3)$

$$p_5^{(1)} + p_5^{(2)} = p_5^{(3)}$$

In terms of KK modes, this reads

$$\pm n_1 \pm n_2 = \pm n_3$$

$\Rightarrow$ KK-number conservation
For instance,

$\Rightarrow$ KK excitations must be pair produced

This leads to

- Bounds on $1/R$ are lower / Distinctive phenomenology
The action for a bulk fermion in 5D:

\[ S_{\Psi} = \int d^4x \, dy \, \bar{\Psi}(x, y) \left[ i \partial_M \Gamma^M - M \right] \Psi(x, y) \]
\[ + \int d^4x \, dy \, \bar{\Psi}(x, y) \left[ i \partial_\mu \Gamma^\mu - M \right] \Psi(x, y) - \bar{\Psi}(x, y) \gamma_5 \partial_5 \Psi(x, y) \]

- Clifford algebra in 5D

\[ \{ \Gamma_M, \Gamma_N \} = 2\eta_{MN} \]

with \( \Gamma_\mu = \gamma_\mu \) and \( \Gamma_5 = i\Gamma_5 \).

\( \Rightarrow \) \( \Psi(x, y) \) are 4-component Dirac spinors.
After “dimensional reduction” (integrating in $y$):

$$S_\psi = \sum_{n=0} \int d^4x \left[ \bar{\psi}_n \left( i\partial_\mu \gamma^\mu - M + i \frac{n}{R} \right) \psi_n \right]$$

- Zero mode ($n = 0$), is always a vector-like fermion!
- But in the SM we need chiral fermions!
Chirality: Define

$$\Psi = \Psi_L + \Psi_R$$

And ask properties under $y \rightarrow -y$ reflections ("parity"):

$$\gamma_5 \Psi(-y) = \pm \Psi(y)$$

Given that

$$\gamma_5 \Psi(-y) = -\Psi_L(-y) + \Psi_R(-y)$$

If we have

$$\Psi_R(-y) = \Psi_R(y)$$
$$\Psi_L(-y) = -\Psi_L(y)$$

then $\Psi_L(x, y)$ is odd, $\Psi_R(x, y)$ is even under parity.
In this case, expanding in KK modes:

$$\psi(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=0} \left[ \psi_{nR}(x) \cos \left( \frac{ny}{R} \right) + \tilde{\psi}_{nL}(x) \sin \left( \frac{ny}{R} \right) \right]$$

So that the zero mode is Right-Handed!

Had we chosen $\gamma_5 \psi(-y) = -\psi(y)$, i.e.

$$\psi_R(-y) = -\psi_R(y)$$
$$\psi_L(-y) = \psi_L(y)$$

Then the zero mode would be Left-Handed.
But how do we define “parity” in a circle?

- **Orbifold Compactification:**
  Identify points opposite in the circle ($y \sim -y$).

Circle now reduced to segment, with “fixed points” at 0 and $\pi R$.

- Fields can be even or odd under $y \rightarrow -y$.
- Bulk fermions have chiral zero modes (either LH or RH).
Disclaimer: Theories with more than 4D are non-renormalizable.

\[ \Lambda \]

\[ \text{Enp} \]

\[ \frac{1}{R} \]

5D Eff. Theory (KK)

4D Eff. Theory (~SM)

\[ \Rightarrow \Lambda R = N_{\text{max}}. \]
But Orbifolding breaks KK-number conservation! Translation invariance broken in the $y$ direction $\Rightarrow p_5$ not conserved!

The presence of fixed points breaks KK number. By how much?

Localized 4D operators at $y = 0$ and $y = \pi R$ generate KK-number-violating interactions. E.g:

$$S_{\text{loc.}} = \int d^4x \int_0^{\pi R} dy \ i\bar{\Psi}(x, y) \gamma_\mu D^\mu \Psi(x, y) \left( \delta(y) \frac{c_1}{\Lambda} + \delta(y - \pi R) \frac{c_2}{\Lambda} \right)$$
• UV physics might not operate differently in $y = 0$ and $y = \pi R$.

If $c_1 = c_2 \Rightarrow$ KK-number violating interactions still respect KK-parity.
E.g. in $(1) + (2) \leftrightarrow (3)$

$$(-1)^{n_1+n_2+n_3} = 1$$
Conservation of KK parity $\Rightarrow$

- Can produce *level* 2 KK modes in s-channel.

![Diagram showing KK modes](attachment:diagram.png)

- Lightest KK Particle of level 1 (LKP) is stable

$\Rightarrow$ LKP is Dark Matter candidate
Electroweak precision constraints:

\[ \frac{1}{R} \gtrsim 300 \text{ GeV for 5D} \]
\[ \frac{1}{R} \gtrsim (400 - 600) \text{ GeV for 6D} \]

- KK excitations for fermions are vector-like $\Rightarrow$ do not add to $S$ as new chiral matter.

- Direct bounds are already more constraining
Spectrum at each KK level is degenerate at tree level. Localized operators split the masses (one-loop generated).

First KK mode in 5D model, with $c_i$’s computed at one-loop.

For $\Lambda R = 20$:

$\gamma^{(1)}$ is the LKP. Actually is $B^{(1)}$ (effective $\theta_W$ for the KK modes is small !)
- Light KK modes $\Rightarrow$ large cross sections.
- But, almost degenerate KK levels $\Rightarrow$ little energy release.

Best mode $q\bar{q} \rightarrow Q_1 Q_1 \rightarrow Z_1 Z_1 + E_T \rightarrow 4\ell + E_T$ (Cheng, Matchev, Schmaltz '02).
Reach using this golden mode $q\bar{q} \rightarrow 4\ell + \not{E}_T$ for $\sqrt{s} = 14$ TeV
Direct Limits on UED

But not the best for $\sqrt{s} = 7$ TeV and a few fb$^{-1}$. In this case best bound is from multijet + missing $E_T$ (Murayama et al, 2011)

![Graph showing limits on 1/R vs. ΛR](image)

- Current bound is $1/R > 600$ GeV
- Bounds above 1 TeV only with higher energy.
Production and Decay of Second KK Level:

They couple to 2 zero modes through brane couplings (loop generated). (Datta, Kong, Matchev '05)

\[ \Lambda R \gg 1 \quad \text{and} \quad c_i \sim O(1). \]

But has to compete with \[ 2 \rightarrow 1 + 1 \]
• Signals very different in 6D (Burdman, Dobrescu, Pontón '06).

• KK not separated by $1/R$:

$$M_{j,K}^2 = \frac{1}{R^2} (j^2 + k^2)$$

$\Rightarrow$ 2nd KK mode $(1,1)$ must decay to zero modes via loops/localized interactions.

E.g.:
More scalar degrees of freedom: E.g. $A_M$, with $M = 0, 1, 2, 3, 5, 6$

In 5D $A_5$ not physical. Eaten by KK modes to get their masses (NGB of breaking of translation invariance).

In 6D one linear combination of $A_5$ and $A_6$ is eaten, but one remains in the spectrum

LKP is a scalar: $B_{5,6}$, extra component of 6D hypercharge gauge boson $\Rightarrow B_H$
Level (1, 0)

The mass spectrum of the (1, 0) level. The lightest KK particle is the \( G_{\mu}^{(1,0)} \) with mass 450 GeV.

The mass spectrum includes
- \( G_{\mu}^{(1,0)} \)
- \( W_{\mu}^{(1,0)} \)
- \( B_{\mu}^{(1,0)} \)
- \( W_{H}^{(1,0)} \)
- \( B_{H}^{(1,0)} \)
- \( H^{(1,0)} \)
- \( G_{H}^{(1,0)} \)
- \( L_{\mu}^{(1,0)} \)
- \( E_{\mu}^{(1,0)} \)
- \( Q_{+}^{(1,0)} \)
- \( Q_{-}^{(1,0)} \)
- \( T_{\mu}^{(1,0)} \)
- \( D_{\mu}^{(1,0)} \)

with masses ranging from 400 GeV to 700 GeV, and the scale \( 1/R = 500 \text{ GeV} \).
Level (1, 1)

The mass spectrum of the (1, 0) level is shown in Figure 2 for $1/R = 500$ GeV. The lightest KK particle is predicted to be the spinless hypercharge gauge boson, $B^W_{(1,0)}$. Higher-loop contributions in effects are small, and have multi-loop contributions are expected to be important.

The mass spectrum of the (1, 1) modes is shown in Figure 2 for $1/R = 500$ GeV. The mass spectrum of the (1, 0) level is shown in Figure 2 for $1/R = 500$ GeV. Higher-loop contributions in effects are small, and have multi-loop contributions are expected to be important.

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5D Case

- LKP is $B^{(1)}$
- Annihilation determines abundance:

$$\sigma(\nu^{(1)}\nu^{(1)} \to f\bar{f}) = \frac{N_c g_Z^2}{24\pi \beta s^2},$$

where $g_Z = \frac{2s_W c_W}{\sqrt{2}}$, and

$$\bar{g}_L(R) = e s_W \left[T_3 - Q_f s^2_W\right].$$

With no helicity suppression for annihilation, the LKP realizes the correct relic density for larger WIMP masses. The 6d curve is for a 2-torus with equal radii (2 LKPs): $\Omega h^2 = 0.1$.

(Servant-Tait '03, Tait '11)
Co-Annihilation

Since mass splittings are small, co-annihilation important

Co-annihilation leads to an increase in the number of LKPs after freeze-out. To compensate, we dial down the mass of the LKP so that the correct energy density results.
LKP is the scalar KK mode of the hypercharge gauge boson $B_\mu$, $B_H$.

Annihilation through Higgs dominates:

$\Rightarrow$ relic abundance sensitive to $m_h$.

Co-annihilation effect not important since splitting with closest fermion is larger than in 5D.
Figure 5: The region (shaded) of the $m_h$ vs. $M_B$ plane in which the $B_H$ thermal relic abundance is within the range measured by WMAP ($0.096 < \Omega h^2 < 0.122$).

1. $B_H$ masses need to be smaller since annihilation cross sections are larger.
2. $m_{B_H} < 500$ GeV. $m_h = 125$ GeV appears not viable!
Some remarks:

- Standard UED models do not solve the hierarchy problem
- Theories with compact extra dimensions can be viewed as (dual to) 4D strongly coupled theories:
  - KK modes $\leftrightarrow$ hadron excitation spectrum
  - Inverse of compactification radius $R^{-1} \leftrightarrow$ Excitation gap in strong interaction: $\Lambda$ (Eg. $\Lambda_{QCD} \simeq 1$ GeV )
- Finding ED theory with large scale separation $\leftrightarrow$ hierarchy problem
- Resulting models dual to strong dynamics solutions to the HP.