An Introduction to Cosmology

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References

• Referencing in these lectures are subjective and minimal to guide students.
• General references not cited explicitly during these lecs:

  Texts  Edward Kolb and Michael Turner, THE EARLY UNIVERSE.
         Scott Dodelson, MODERN COSMOLOGY
         Michel Le Bellac, THERMAL FIELD THEORY
         Steven Weinberg, COSMOLOGY
         Jeremy Bernstein: KINETIC THEORY IN THE EXPANDING UNIVERSE
         Sybren Ruurds Groot, Willem Andries Leeuwen,
         Christianus Gerardus van Weert: RELATIVISTIC KINETIC THEORY

Outline

• Characterize how cosmology differs from particle physics.

• How to find compelling problems in theoretical cosmology.

• Electroweak symmetry breaking related cosmology will be compelling because of the LHC.

• Ingredients of electroweak baryogenesis (including an introduction to thermal field theory)

• What would the recent Higgs rumor do for baryogenesis.
Typical Particle Physicist’s Cartoon of Nature

\[ Z[J] = \int D\phi e^{i \int d^4 x \mathcal{L}[\phi, J]} \]

Minkowski vacuum BC

\[ \frac{\delta \ln Z}{\delta J(x_1) \delta J(x_2) \ldots |_{J=0}} = \langle \phi(x_1) \phi(x_2) \ldots \rangle \]

1) Initial state is known.
2) Poincare group representation. (i.e. highly symmetric)
Cosmology Is Inverse Mapping & Less Symmetric Physics

- “What is the **most likely** history (a set of earlier time conditions) of the universe given all that we know about today?”

\[ Z[J] = \int D\phi e^{i \int d^4x \sqrt{g} \mathcal{L}[\phi, J]} \]

\[ k \rightarrow k/a(t) \]

UV and IR

Less symmetry
Cosmology Is Inverse Mapping & Less Symmetric Physics

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\[ Z[J] = \int D\phi e^{i\int d^4x \sqrt{g}L[\phi, J]} \]

UV and IR

Less symmetry

No longer Minkowski BC: History!

Spectra/interact History!
Cosmology Is Inverse Mapping & Less Symmetric Physics

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\[ Z[J] = \int D\phi e^{i \int d^4x \sqrt{g} \mathcal{L}[\phi, J]} \]

• Just as integrating in degrees of freedom in “backward” RGE requires speculations about the UV, one cannot simply integrate to obtain the past cosmological history, but must make historical speculations.

RGE:
- field boundary conditions:
  - fixed points
  - imagination

IR: EFT \rightarrow averages “predictions”

LHC! lab verified physics

UV and IR

Less symmetry

Spectra/interact History!
Cosmology Is Inverse Mapping & Less Symmetric Physics

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\[ Z[J] = \int D\phi e^{i \int d^4x \sqrt{g} \mathcal{L}[\phi, J]} \]

No longer Minkowski BC: History!

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• Just as integrating in degrees of freedom in “backward” RGE requires speculations about the UV, one cannot simply integrate to obtain the past cosmological history, but must make historical speculations.

Thermodynamics:

Boltzmann description is Irreversible.
Aside: Much of Observation Related Cosmology

**General Theoretical Problem**

- **Einstein equations (Equivalence principle)**
  \[
  S = -\frac{1}{16\pi} \int d^4 x \sqrt{-g} R + \int d^4 x \sqrt{-g} L_M
  \]
  \[
  R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \frac{1}{2} g_{\mu\nu} R[g_{\rho\tau}] - \frac{1}{M_p^2} T_{\mu\nu}[g_{\rho\tau}] \rightarrow \text{Put in known fields (more later \ldots)}
  \]

- **Boltzmann Equations**
  \[
  \left[ p^\alpha \frac{\partial}{\partial x^\alpha} - T^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \right] f(x^\alpha, p^\beta) = C[f]
  \]
  Collision term; Approximation
  \[
  T^{\alpha\nu} = g_{\chi\alpha} \frac{1}{(2\pi)^3} \int \frac{d^3 p}{\sqrt{|g|}} P^\nu P_\nu f_x(t, \vec{x}, \vec{p})
  \]
  \[
  \vec{p} = g_{ij} P^i P^j \vec{p}
  \]

- **Other relevant field equations**
  (e.g. magnetic field, inflaton, axion, quintessence)

**Difficulties/opportunities:**
1) nonlinearity
2) large number of degrees of freedom
3) unknown initial conditions

Key difference w/ other branches of physics
Observations Related to Cosmology

Multi-wavelengths, Multiple probes

1) Best evidence for isotropy
2) Farthest and oldest we can see with a direct probe

Cosmic rays: Protons + (?)

Electromagnetic
- Gamma rays
  - High E
- X-rays
- Neutrinos
- Antimatter

CMB

Radio
- Low E

> 2012: Dark matter probe of the universe? (Selmi's talk)
EM Flux

Electromagnetic flux

Energy (eV) →

1 TeV

Galaxy scale cutoff

\( \gamma + \gamma_{CMB} \rightarrow e^+ + e^- \)
Neutrino Physics

[Graph showing a log-log plot with labeled axes and markers indicating TeV sources and cosmic rays. Credit to F. Halzen.]
FRW Background Geometry Reminder

Homogeneity is mostly supported by the fact that homogeneous assumption fits data well.

Geometry: \( ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Theta^2 + r^2 \sin^2\Theta \, d\phi^2 \right] \) \( \sim \) FLRW metric

\( \rho \approx \begin{cases} \frac{1}{a^4} & \text{photons, neutrinos for a long time} \\ \frac{1}{a^3} & \text{dark matter, baryons today} \\ \text{constant} & \text{cosmological constant} \end{cases} \)

\( \rho \approx \frac{1}{a^2} \) \( \sim \) "dark" energy

Today \( k = \begin{cases} -1 & \text{open} \\ 0 & \text{flat} \\ 1 & \text{closed} \end{cases} \)

\( \frac{3}{2} \frac{\dot{a}^2}{a^2} + \frac{k}{2} = \frac{\rho}{M_p^2} \) \( \rho(\alpha) \)

\( M_p \approx 2.4 \times 10^{18} \text{ GeV} \)
Empirically Universe is Flat

\[(\dot{a}/a)^2 + \frac{k}{a^2} - \frac{1}{3}M_p^2 \rho(a) = 0\]

Note \(k=0\) when \(\rho = \rho_c = 3M_p^2 \left(\frac{\dot{a}}{a}\right)^2\) numerically

\[H_0 = \frac{\dot{a}}{a} \bigg|_{t=t_0}\]

empirically,

\[(\rho_c = 3H_0^2 \sim 10^{-46} \text{GeV}^4)\]

\[\Omega_X = \frac{\rho_X}{\rho_c}\]
Composition of Fluid Recently

What tells us $p \propto \frac{1}{a^4}$ for photon fluid?

Answer: Conservation $\nabla \cdot T^{\mu \nu} = 0$ + Pressure of fluid property of microphysics + statistics
Composition of Fluid Recently

Relationship between temperature and scale factor

Simplest treatment:

Thermodynamics:

\[ S = \frac{\rho + \frac{1}{3} \mu \Omega}{T} \]

Relativistic species with \( \mu = 0 \) dominate \( \Rightarrow \)

\[ \frac{\rho}{\rho_0} = \left( \frac{T}{T_0} \right)^{3/4} \quad \Rightarrow \quad \frac{\rho}{\rho_0} = \frac{4}{3} \frac{T}{T_0} \]

Stat mech:

\[ \rho = \sum_i \sum \frac{d^3 p_i}{(2\pi)^3} \frac{1}{e^\frac{E_i}{T} \pm 1} \]

Collecting counting factor:

\[ L = \frac{2\pi^3}{45} g^* s(T) T^3 \]

Assume total entropy is conserved:

\[ S_0 a^3 = \text{constant} \Rightarrow \]

\[ a \propto \left( \frac{1}{g^*(T)} \right)^{\frac{1}{3}} \]
(Lack of) Rigidity and control over \[ \frac{\delta}{\delta J(x)} \]

- **BBN** $T \sim 1$ MeV
  - Neutrinos decouple

- **T $\sim 1$ eV**
  - Matter dominates

- **T $\sim 0.3$ eV**
  - Neutral H
  - *Clean, linear*
  - CDM clusters

- **T $\sim 10^{-3}$ eV**
  - Stars reionize

- **T $\sim 10^{-4}$ eV**
  - DE dominates
  - Hot baryons, lensing
  - More challenging systematic errors
  - (nonlinearities, plasma physics)
(Lack of) Rigidity and control over \( \frac{\delta}{\delta J(x)} \)

- **inflaton** (solves flatness + horizon + relic; generates density perturbations)
  - [Leonardo's lecs]
  - **(re)heats** (couples to SM)
    - e.g. \( T \sim 10^8 \text{ GeV} \)
    - Moduli oscillate
    - gravitino
    - Leptogenesis
  - e.g. \( T \sim 1 \text{ MeV} \)
  - Neutrinos decouple

- **BBN**
  - \( T \sim 1 \text{ MeV} \)
  - CDM clusters
  - Matter dominate
  - T\( \sim 1 \text{ eV} \)
  - neutral H (Clean, linear)
  - T\( \sim 0.3 \text{ eV} \)
  - T\( \sim 10^{-3} \text{ eV} \)
  - stars reionize

- **LHC**
  - **EWPT**
    - e.g. \( T \sim 100 \text{ GeV} \)
    - Baryon chemical potential freeze in
  - **WIMP freeze out** [Graciela's lecs]
  - Hot baryons, lensing
  - More challenging systematic errors (nonlinearities, plasma physics)
  - T\( \sim 10^{-4} \text{ eV} \)

- **QCD PT**
  - axion oscillates
  - Little probe here.
What is Exciting Now!

Old landmark

- inflaton (solves flatness + horizon + relic; generates density perturbations)
- (re)heats (couples to SM)
  - Moduli oscillate
  - gravitino
  - Leptogenesis

New landmark (energy frontier)

- EWPT
- WIMP freeze out
- Baryon chemical potential freeze in
- QCD PT
- axion oscillate

BBN
- Neutrinos decouple
- matter dominate neutral H
- Clean, linear
- stars reionize
- DE dominates

Hot baryons, lensing
More challenging systematic errors (nonlinearities, plasma physics)

LHC
- Old landmark
- More challenging systematic errors (nonlinearities, plasma physics)
Observationally Motivated Boundary Conditions

Recombination (REC) is a time period of thermal energy of about 0.3 eV when neutral hydrogen formed.

Last scattering surface (LSS) is the 2-sphere surrounding us from which the observed CMB photons last scattered.

- CMB isotropy: Isotropy about us at REC on LSS
- Success of $\Lambda$CDM for structure formation
  - homogeneity and isotropy at REC
  - nearly scale invariant primordial DM power spectrum
- Success of CMB power spectrum theory:
  - thermal equilibrium and RD prior to LSS
  - DM was not scattering appreciably with the photons
  - nearly Gaussian, adiabatic, scale invariant primordial power spectrum
  - charge neutrality at LSS
“Best” Theoretically Motivated BCs dynamical attractors:

- Thermal equilibrium: Boltzmann H Theorem
  $$f = \frac{1}{e^{\frac{E-\mu}{T}} \pm 1}$$
  Short dist. Interactions.
- Non-current carrying cosmic string scaling
- Equation of state dependent scaling laws:
  e.g.
  Rad domination
  $$\frac{\rho_{\text{rad}}}{\rho_{\text{CDM}}} \propto a^{-1}$$
- Thermal phase transitions + horizons

This class = BC “independent”
Less “Generic” Boundary Conditions

- Homogeneous and isotropic
- Moduli displacement

\[ U \supset e^{K/M_p^2} \left( g^{ij} [D_i W] [D_j W]^* - \frac{3}{M_p^2} |W|^2 \right) \]

Suppose positive and large during inflation

Can induce negative mass squares.

\[ \rightarrow \text{Moduli displaced far from the min: init cond} \]

- Quintessence kination phase (kicked by inflation)
- A large flat inflationary patch for low scale inflation
- Initially contracting universe headed for a bounce
- No-boundary proposal for Wheeler-De Witt
- Brane isometries not lifting to the bulk due to sources (i.e. extra D is not just KK states)
Naively, BC “independent” (as opposed to BC sensitive) Histories Are Compelling

- Expansion rate \( H(t) \equiv \frac{\dot{a}}{a} \) is controlled by mass spectrum + interactions

\[
H^2 \propto \sum_i \int \frac{d^3p}{\sqrt{p^2 + m_i^2}} \left[ \exp \left( \frac{\sqrt{p^2 + m_i^2} - \mu_i}{T_i} \right) \pm 1 \right]^{-1} \dot{n} + 3Hn \sim -\langle \sigma v \rangle (n^2 - n_{eq}^2)
\]

\( e.g. \)
1) successful BBN bound on exotic particles
2) DM freeze out

\[
\Omega_x h^2 \propto \left( \frac{T_{today}}{m_x x_F} \right)^3 \left( \frac{m_x H_F}{\langle \sigma_A v \rangle} \right)
\]

3) EW-bgenesis

\[
v_w \partial_z X - D_X \partial_z^2 X = \Gamma[X]
\]

computable from short distance physics
Summary of Compelling Cosmology

1. Short distance physics can be tested in a laboratory (e.g. LHC)

2. BCs are observationally motivated or dynamical attractors (e.g. Boltzmann H-theorem)

3. Probes (e.g. relics) exist to observationally confirm (e.g. dark matter, gravity waves, baryon asymmetry, …)

Exceptions to this may be argued e.g. if probes contained in 3 are numerous and numerically nontrivial. e.g. 

\[ n_T = \frac{-1}{8} \frac{P_T}{P_R} \]

[Leonardo’s lecs]
“Conservative” Expectations

What new physics relevant for cosmology do we expect to learn at the LHC?

1) WIMP physics (dark matter) [Graciela’s lecs]
2) Electroweak symmetry breaking (EWSB) physics (electroweak phase transition + DE + clustering)

There is a no-lose theorem regarding what we are going to learn at the LHC.
No Lose Theorem

\[ \mathcal{M} = c \frac{s}{M_w^2} + \ldots \]

Unitarity from partial waves \( \rightarrow \) unitarizing physics enters at

\[ \sqrt{s} \lesssim 4\pi M_w \approx 1 \text{ TeV} \]

LHC plans to cover the entire energy range.

EWSB physics will be uncovered within the upcoming yrs.
What Does EWSB Physics Do for Cosmology?

Basic questions:

1) Did EWPT take place?
2) How did EWPT influence the cosmological history?
3) What cosmologically measurable correlation information remains?

Regarding 2 + 3:

1) Baryon number freezes in
2) Baryon number may be generated
3) Gravity Waves: PT bubbles stir up the fluid
4) Domain walls/topological defects can be created
5) Dark energy can be affected
6) Dark matter freeze out cosmology can be affected
7) Clustering information on short scales changes before thermalizing

Will now elaborate.
Motivation

What is the baryogenesis question?

“Why are there more baryons than antibaryons?”

\[
6.7 \times 10^{-11} < Y_B < 9.2 \times 10^{-11} \quad (95\% \text{ C.L.}) \quad \text{BBN [1]}
\]
\[
8.36 \times 10^{-11} < Y_B < 9.32 \times 10^{-11} \quad (95\% \text{ C.L.}) \quad \text{CMB [1, 2]}
\]


Why is this particularly interesting?

Baryogenesis requires satisfying 3 Sakharov conditions.
1) Out of equilibrium
2) CP violation
3) Baryon number violation

Although SM generically contains all 3 ingredients qualitatively, it fails quantitatively. \(\longrightarrow\) BSM!
How does SM contain these ingredients?

How SM qualitatively satisfies 3 Sakharov conditions

1) Out of equilibrium: EWPT (connection to EWSB physics)
   quantitatively fails because phase transition is too smooth \(\times\)
   BSM \(\rightarrow\) first order PT

2) CP violation: CKM
   \[
   \delta_{CP} = \left( \frac{g_W}{2m_W} \right)^{12} (m_t^2 - m_u^2)(m_t^2 - m_c^2)(m_c^2 - m_u^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)(m_s^2 - m_d^2) \]
   \(\approx 10^{-22} \quad \times\)
   BSM \(\rightarrow\) more CP violation possibilities

3) Baryon number violation: \(SU(2)_L\)
   \[
   \Delta \mathcal{L} = \frac{g^2}{32\pi^2} \tilde{F}^{(a)}_{\mu \nu} F^{(a)\mu \nu} \quad \checkmark
   \]

\textbf{popular idea: use BSM to make 1 and 2 work}
Electroweak Bgenesis

1) Bubble nucleate
2) CP violating scattering in bubble \(\rightarrow\) source of CP asymmetry
3) CP charge diffuse out in front of bubble generating B through sphalerons
4) True vacuum phase captures the B-asymmetry created

[N.B. There may be other scenarios that people may not have thought about. The tools explained here are more general than just this scenario.]

I will now explain some of the microphysics ingredients.
A lot of people have worked on this over the years...

- Incomplete list of ewbgenesis people:

- Some overview references
  - hep-ph/0609145
  - hep-ph/0312378
  - hep-ph/0303065
  - hep-ph/0208043
  - hep-ph/0009119
  - hep-ph/9901362
  - hep-ph/9901312
  - hep-ph/9802240
Ingredients

1) Symmetry restoration at high temperature
2) Baryon number violation in symmetric phase
3) Baryon number violation in broken phase
4) Phase transition (bubbles versus no bubbles)
5) Charge transport
6) CP violating source

A lot to explain.
1) Symmetry restoration at high temperature
Symmetry restoration begin: thermal effective potential

Consider $\mathcal{L} [\phi]$ where $\phi$ is in thermal equilibrium at temperature $T$. What is $\langle \hat{\phi} \rangle_p = \text{Tr}[\hat{\rho}_T \hat{\phi}]$ as a function of $T$ where

$$\hat{\rho}_T = \frac{e^{-\frac{\hat{H}_T}{kT}}}{\text{Tr}(e^{-\frac{\hat{H}_T}{kT}})}$$

If this is an order parameter, then symmetry is dictated by this.

For finite temperature effective potential, consider computing the partition function with IR regulator (box of volume $\Omega$)

$$\mathbb{Z}[\phi, T, \Omega] = \text{Tr} \left[ e^{-\int d^4 x \, i \hat{\phi}} \right]$$

Ordinary path integral:

$$\langle \Phi_2(x), t_1 | e^{i \hat{H} \Delta t} | \Phi_1(x), t_1 \rangle = \langle \Phi_2(x), t_1 + \Delta t | \Phi_1(x), t_1 - \frac{\Delta t}{2} \rangle$$

$$= \int \mathcal{D}\phi \, e^{i \int dx \, [i \frac{\partial \phi}{\partial t} - \frac{\Delta t}{2} \phi^2]} \langle \Phi_2 | \phi \rangle \langle \phi | \Phi_1 \rangle$$
Analytically continue:

\[ \Delta t = \frac{-i}{T} \Rightarrow \langle \phi_2(z) | e^{\frac{-i t}{T}} | \phi_1(z) \rangle = \int \mathcal{D}\phi \ e^{-\frac{1}{2T} \int \Delta t \left[ \phi' \right] \left[ \phi' \right]} \langle \phi_2, \phi \rangle \langle \phi, \phi_1 \rangle \]

\[ \Delta t = i \Delta \tau \]

\[ \Delta \tau \Rightarrow -\Delta \tau \]

\[ \mathcal{Z}[\phi, T, \Omega] = \int \mathcal{D}\phi \ e^{-\frac{i}{2T} \int \Delta t \left[ \phi' \right] \left[ \phi' \right]} \phi(\frac{i}{2T}, \Omega) = \phi(\frac{i}{2T}, \Omega) \]

**Slogan**: Finite $T$ field theory is a Euclidean theory with radius compactified on a circle.

Legendre transform to find the effective potential

\[ \Gamma[\bar{\phi}, T, \Omega] = -\ln \mathcal{Z}[\bar{\phi}^* T, \Omega] - \int d^4x \bar{\phi} \frac{\partial}{\partial \bar{\phi}} \]

where \( \bar{\phi} \) satisfies

\[ \frac{\delta \mathcal{Z}[\bar{\phi}^* T, \Omega]}{\delta \bar{\phi}} \left. \right|_{\bar{\phi} = \bar{\phi}[\phi]} = 0 \]

As usual

\[ \frac{\delta \Gamma}{\delta \bar{\phi}} = \frac{1}{\mathcal{Z}} \left( \frac{\delta \mathcal{Z}}{\delta \bar{\phi}} \right) - \frac{\partial}{\partial \bar{\phi}} \]

\[ \frac{\delta \Gamma}{\delta \bar{\phi}} = -\bar{\phi} \frac{\delta}{\delta \bar{\phi}} - \bar{\phi} \frac{\delta}{\delta \bar{\phi}} = -\bar{\phi} \frac{\delta}{\delta \bar{\phi}} \]
Thermal Effective Potential 3

We want to solve for $\Phi$ in $J[\Phi] = 0$

$$\Rightarrow \frac{\delta \Pi[\Phi, T, \Omega]}{\delta \Phi} = 0$$

Let $V(\Phi, T) \equiv \lim_{n \to \infty} \frac{1}{\Omega} \left[ \Gamma[\Phi, T, \Omega] \right]_{\Phi = \text{homogeneous}}$

$$\frac{\partial V(\Phi)}{\partial \Phi} = 0$$

Let's compute $V(\Phi)$ at one loop (similar to $T=0, V(\Phi)$)

Usual saddle point expansion:

$$S_{\Phi}[\phi, T, \Omega, \hat{\phi}] = -\frac{1}{2\pi i} \oint_{\gamma} d\phi \left( \epsilon \phi + \hat{\phi} \phi \right)$$

$$\frac{\delta S_{\Phi}}{\delta \phi} \bigg|_{\phi = \phi_0} = 0$$

$$Z[\hat{\phi}, T, \Omega] = \exp \left( -S_{\Phi}[\phi_0, T, \Omega, \hat{\phi}] \right) Z_{1L}[\hat{\phi}, T, \Omega]$$

$$Z_{1L}[\hat{\phi}, T, \Omega] = \int D(\phi) \exp \left[ -\left( \frac{d\phi}{\partial x_1} \phi(\bar{z}_1), \phi(\bar{z}_2) \right) \frac{\delta^2 S_{\Phi}}{\delta \phi(\bar{z}_1) \delta \phi(\bar{z}_2)} \bigg|_{\phi = \phi_0} \right]$$

Thermal effective potential
Thermal Effective Potential 4

Legendre transform & restrict to
\[ \ln Z_{11} = -\frac{1}{2} \text{Tr} \left( \ln \frac{S^2 S_E}{S(\phi(z)) S(\phi(z))} \right) \]

\[ V(\phi_e, T) = \lim_{\Omega \to \infty} \frac{1}{\Omega} \sum_{\Omega} S_E[\phi_e, T, M_{ij}\delta(\phi_e)] - i\langle \phi_e \rangle \phi_e + \frac{1}{2\Omega} \text{Tr} \left( \ln \frac{S^2 S_E}{S(\phi(z)) S(\phi(z))} \right) \]

\[ U(\phi_e) \]

\[ \mathcal{L} = \frac{1}{2} (\partial \phi)^2 - U(\phi) \]

\[ S_E[\phi, T, M_{ij}\delta(\phi)] = \int d^4x \left\{ \frac{1}{2} \bar{\phi} \left( \partial \phi \right)^2 + U(\phi) + i\delta \phi \right\} \]

\[ \frac{1}{\Omega} S_E[\phi_e, T, M_{ij}\delta(\phi_e)] - i\langle \phi_e \rangle \phi_e = U(\phi) \]

\[ \frac{S^2 S_E}{S(\phi(z)) S(\phi(z))} = \left( -\nabla \cdot U^\mu(\phi_e) \right) S^{\mu\nu}(z_i - z_j) \]

\[ \int \frac{d^4p}{(2\pi)^4} \to T \sum_n \frac{d^3p}{(2\pi)^3} \text{Tr} \left[ \sum_{n=-\infty}^{\infty} e^{i2\pi n T(z_i - z_j)} \right] = \delta(z_i - z_j) \]

\[ \text{Tr} \left( \ln \frac{S^2 S_E}{S(\phi(z)) S(\phi(z))} \right) = \Omega \to T \sum_n \int \frac{d^3p}{(2\pi)^3} \ln \left( (2\pi n T)^2 + |p|^2 + U^\mu(\phi_e) \right) \]
Thermal Effective Potential 5

\[ \sum \frac{d}{dy} \left( \sum_{n=0}^{\infty} \ln \left( \frac{y}{\omega_n^2 + y^2} \right) \right) = 2 \sum_{n=0}^{\infty} \frac{y}{\omega_n^2 + y^2} \]

\[ I = \sum_{n=0}^{\infty} \ln \left( (2\pi n T)^2 + |\delta|^2 + \omega^2(d_0) \right) = 2 \sum_{n=0}^{\infty} \int dy \frac{y}{(2\pi n T)^2 + y^2} \bigg|_{y=\sqrt{|\omega^2 + |\delta|^2}}} \]

Note: \( \coth \left( \frac{k_0}{2T} \right) \) has poles at \( \frac{k_0}{2T} = in\pi \)

\( k_0 = 2in\pi T \)

\[ I = \int_{C} \frac{dk_0}{2T} 2 \int dy \frac{y}{-k_0^2 + y^2} \left( \coth \left( \frac{k_0}{2T} \right) \right) \frac{1}{2\pi i} \]

Since there is no contribution to the integral along \( C \), deform the contour.
Thermal Effective Potential 6

\[ I = -\frac{1}{T} \int \delta y \cdot \delta \phi (z) \delta \phi (2\pi T) \left[ \frac{1}{y^2 - k_0^2} \cosh \left( \frac{k_0 \cdot y}{2T} \right) \right]_{y = \sqrt{u'' + i\beta}^2} \]

\[ = -\frac{\sqrt{u'' + i\beta}^2}{T} + 2 \ln \left( \frac{\sqrt{u'' + i\beta}^2}{T} - 1 \right) \]

\[ = \frac{\sqrt{u'' + i\beta}^2}{T} + 2 \ln \left( 1 - e^{-\frac{\sqrt{u'' + i\beta}^2}{T}} \right) \]

\[ \frac{T}{2\pi^2} \text{Tr} \left( \ln \left[ \frac{\delta^2 S_{\phi}}{\delta \phi(z) \delta \phi(z)} \right] \right) = \frac{T}{2\pi^2} \frac{d^3 p}{(2\pi)^3} \left[ \frac{1}{2} \left( \frac{d^3 p}{(2\pi)^3} \sqrt{u'' + i\beta}^2 + T \right) \right. \]

\[ \left. \frac{d^3 p}{(2\pi)^3} \ln \left( 1 - e^{-\frac{\sqrt{u'' + i\beta}^2}{T}} \right) \right] \]

\text{Zero-point energy} \quad \text{thermal corrections}
Thermal Effective Potential 7

\[ V(\phi_c, T) = U(\phi_c) + \frac{1}{2} \left\{ \frac{d^3p}{(2\pi)^3} \sqrt{\omega^2 + |p|^2} + T \left( \frac{d^3p}{(2\pi)^3} \ln \left( 1 - \frac{\sqrt{|p|^2 + \omega^2}}{T} \right) \right) \right\} \]

In the high temperature limit,

\[ \int \frac{d^3q}{(2\pi)^3} \ln \left\{ 1 - \exp \left( \frac{-\sqrt{q^2 + U''(C)}}{T} \right) \right\} = \frac{1}{2} \left[ -\frac{\pi^2 T^3}{45} + \frac{T}{12} U''(C) - \frac{1}{6\pi} (U''(C))^{3/2} + O \left( \frac{M}{T} \right)^4 \right] \]

\[ V(\phi_c, T) \approx U(\phi_c) + \frac{T^2}{24} U''(\phi_c) + \cdots \]

**Example:** Take \( U(\phi) = -\frac{\lambda^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \)

\( Z_2 \): \( \phi \rightarrow -\phi \)

At \( T=0 \), \( \partial_{\phi_c} V(\phi_c, T) = 0 \implies \phi_c = \frac{\lambda}{\sqrt{\lambda}} = \langle \hat{\phi} \rangle \)

\( Z_2 \) spontaneously broken

At \( T >> \mu \), \( V(\phi_c, T) = -\frac{\lambda^2}{2} \phi_c^2 + \frac{\lambda}{4} \phi_c^4 + \frac{T^2}{24} (-M^2 + 3\lambda \phi_c^2) + \cdots \)

\[ = \left( -\frac{\lambda^2}{2} + \frac{\lambda T^2}{8} \right) \phi_c^2 + \frac{\lambda}{4} \phi_c^4 + \cdots \]

\( \partial_{\phi_c} V(\phi_c, T) = 0 \implies \phi_c = 0 = \langle \hat{\phi} \rangle \)

\( Z_2 \) restored

Hence, \( Z_2 \) symmetry has been restored, at high \( T \).
Intuition for Thermal Effective Potential

1) Thermal effective potential is accounting for the energy of 1-particle states.
2) Interactions leading to quadratic divergences generate thermal mass terms for the scalar field.

\[ e^{-\beta V(\phi)} \sim \int D\tilde{\psi} D\psi \exp \left(-S_E[\tilde{\psi}, \psi, \phi]\right) \]

\[ -V \approx P_f \sim \frac{p^3 p^2}{\omega} f \sim \frac{T^5}{\omega} \frac{1}{e^{\omega/T} + 1} \sim \frac{T^5}{\omega} \]

\[ \omega \sim \sqrt{T^2 + y^2 \phi^2} \]

\[ \sim T \left[ 1 + \frac{1}{2} \frac{y^2 \phi^2}{T^2} \right] \]

Prediction: Scalar field VEV varies as a function of T

Positive mass!
At High $T$, $SU(2)_L \otimes U(1)_Y$ is restored in Many BSMs

EW symmetry is restored at high temperature. Broken at low temperature.

[Kirzhnits, Linde 72]
2) Baryon number violation in symmetric phase
Baryon # Violation is Anomalous

Change vars: $\psi(x) \rightarrow e^{(a+b\gamma_5)}\theta(x) \psi(x)$

$$\delta S_0 = -\int d^4x \left[ \bar{\psi} m \left( e^{2ib\gamma_5} \theta(x) - 1 \right) \psi + \bar{\psi} \gamma^\mu (a + b\gamma_5) \psi \partial_\mu \theta(x) \right]$$

Fujikawa method:

$$\delta S_1 = i \int d^4x \theta(x) \left[ \frac{(a - b)}{8\pi^2} \text{Tr} F^{(L)\mu\nu} \tilde{F}^{(L)}_{\mu\nu} - \frac{(a + b)}{8\pi^2} \text{Tr} F^{(R)\mu\nu} \tilde{F}^{(R)}_{\mu\nu} \right]$$

Suppose B-number is rotated: $a = \frac{1}{3}$ and $b = 0$

Anomalous current (because L and R couple differently to gauge fields):

$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left( -g_2^2 F^{a\mu\nu} \tilde{F}^{a}_{\mu\nu} + g_1^2 f^{\mu\nu} f_{\mu\nu} \right)$$

$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left(-g_2^2 \partial_\mu K^\mu + g_2^2 \partial_\mu k^\mu\right)$$

$$K^\mu = 2\epsilon^{\mu\nu\rho\sigma} \left( \partial_\nu A^a_\rho A^a_\sigma - \frac{1}{3} g_2 \epsilon_{abc} A^a_\rho A^b_\sigma A^c_\sigma \right),$$

$$k^\mu = 2\epsilon^{\mu\nu\rho\sigma} \left( \partial_\nu B_\rho B_\sigma \right).$$

$$\partial_\mu J_B^\mu \propto \partial_\mu K^\mu_{CS} \rightarrow \Delta N_B \propto \Delta N_{CS}$$
Baryon # Violation is Anomalous

Change vars: \[ \psi(x) \rightarrow e^{i(a+b\gamma_5)\theta(x)} \psi(x) \]

\[ \delta S_0 = -\int d^4x \left[ \bar{\psi} m \left( e^{2ib\gamma_5\theta(x)} - 1 \right) \psi + \bar{\psi} \gamma^\mu (a + b\gamma_5)b \theta(x) \right] \]

Fujikawa method:

\[ \delta S_1 = i \int d^4x \theta(x) \left[ \frac{(a - b)}{8\pi^2} \text{Tr} F^{(L)\mu\nu} \tilde{F}^{(L)}_{\mu\nu} - \frac{(a + b)}{8\pi^2} \text{Tr} F^{(R)\mu\nu} \tilde{F}^{(R)}_{\mu\nu} \right] \]

Suppose B-number is rotated: \[ a = \frac{1}{3} \text{ and } b = 0 \]

Anomalous current (because L and R couple differently to gauge fields):

\[ \partial_\mu J_B^\mu = i \frac{N_f}{32\pi^2} \left( -g_2^2 F^{a\mu\nu} \tilde{F}^a_{\mu\nu} + g_1^2 f^{\mu\nu} \tilde{f}_{\mu\nu} \right) \]

\[ \tilde{F}^{(a)}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\sigma\rho} F^{(a)}_{\sigma\rho} \]

Usually summarized as

\[ \Delta \mathcal{L} = \frac{g^2}{32\pi^2} \tilde{F}^{(a)}_{\mu\nu} F^{(a)\mu\nu} \]
Baryon number violation is unsuppressed in symmetric phase.

\[ \Delta \mathcal{L} = \frac{g^2}{32 \pi^2} \tilde{F}^{(a)}_{\mu \nu} F^{(a) \mu \nu} \]

unsuppressed in false vacuum (electroweak symmetry is unbroken)

\[ \Gamma = \kappa' \left( \frac{g^2 T^2}{m_D^2} \right) \alpha_w^5 T^4 \]

\[ F = T \int \frac{d^3 k}{(2\pi)^3} \left[ \log \left( 1 + e^{-(E_k - \mu_{B+L})/T} \right) + (\mu_{B+L} \rightarrow -\mu_{B+L}) \right] \]

\[ F \sim \mu_{B+L}^2 T^2 + \mathcal{O}(T^4) \sim \frac{n_{B+L}^2}{T^2} + \mathcal{O}(T^4) \]

\[ \frac{dn_B}{dt} \approx -\mathcal{O}(5) N_F \Gamma \frac{\Gamma}{T^3} \]

Relaxes to zero in equilibrium.
Baryon # Violation is Unsuppressed in Symmetric Phase

\[ \Delta \mathcal{L} = \frac{g^2}{32\pi^2} \bar{F}_{\mu
u}^{(a)} F^{(a)\mu\nu} \]

unsuppressed in false vacuum (electroweak symmetry is unbroken)

Computed using latticized numerical simulation

\[ \Gamma = \kappa' \left( \frac{g^2 T^2}{m_D^2} \right) \alpha_w^5 T^4 \]

\[ \kappa' = \frac{3\sigma}{2\pi} \lim_{V,\tau \to \infty} \frac{\langle (N_{CS}(\tau) - N_{CS}(0))^2 \rangle}{V \tau} \]

\[ n_B(t) \sim -O(5) N_F \frac{\Gamma}{T^3} \]

\[ \sigma^{-1} = \frac{3}{m_D^2 \gamma}, \quad \gamma = \frac{N_c g^2 T}{4\pi} \left[ \ln \frac{m_D}{\gamma} + 3.041 \right] \]

\[ m_D(T) = \frac{g^2 T^2}{12} \left( 4N_c + N_f + 2N_s \right) \]

\[ \kappa' = (10.0 \pm 0.2) \left[ \ln \frac{m_D}{\gamma} + 3.041 \right] \]


Baryon # Violation is Unsuppressed in Symmetric Phase

N.B. Confusingly, people often also call this sphaleron transitions even though this is not. We will see later what sphaleron transitions correspond to.
3) Baryon number violation in broken phase
Sphalerons Are Saddle Point Approximations

\[ \Delta \mathcal{L} = \frac{g^2}{32\pi^2} \tilde{F}_{\mu\nu}^{(a)} F^{(a)\mu\nu} + \text{Higgs} \]

Broken phase B violating transition amplitude in saddle point approximation (unlike symmetric phase, can be treated analytically)


e.g. A MSSM extension containing 3 singlets \( \{\tilde{\nu}_i^c\} \) called \( \mu\nu\text{SSM} \) [Lopez-Fogliani, Munoz 05]

\[ \{H_1, H_2, \tilde{L}_i\} = \{v_1 h_1(\xi), v_2 h_2(\xi), v_3 h_3(\xi)\} U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix} \]

\[ \tilde{\nu}_i^c = v_i h_4(\xi) \]

\[ W_i^a \sigma^a dx^i = -\frac{2i}{g} f(\xi) dU^\infty (U^\infty)^{-1} \]

\[ U^\infty = \frac{1}{r} \begin{pmatrix} z & x + iy \\ -x + iy & z \end{pmatrix} \]

\[ \xi = \rho \nu \]

\[ f(\xi) \xrightarrow{\xi \to 0} \alpha \xi^2, \quad h_i(\xi) \xrightarrow{\xi \to 0} \beta_i \xi \quad i \in \{1, 2, 3\}, \quad h_4 \xrightarrow{\xi \to 0} c + \beta_4 \xi^2 \]

\[ f(\xi), h_i(\xi), h_4(\xi) \xrightarrow{\xi \to \infty} 1 \]

Interpolates between

\[ N_{CS} = 0 \quad \& \quad N_{CS} = 1 \]
Larger $\frac{E_{sp}(T_c)}{T_c}$ Suppresses Transitions

\[ \Delta \mathcal{L} = \frac{g^2}{32\pi^2} \tilde{F}_{\mu\nu}^{(a)} F^{(a)\mu\nu} + \text{Higgs} \]

Broken phase B violating transition amplitude in saddle point approximation (unlike symmetric phase, can be treated analytically)

\[ \text{solution} \quad \Rightarrow \quad E_{sp}(T_c) \]


\[ \Gamma_{sp} \approx 2.8 \times 10^5 T^4 \left( \frac{\alpha W}{4\pi} \right)^4 \kappa \left[ \frac{E_{sp}(T)}{B} \right]^7 e^{-E_{sp}(T)/T} \]

\[ 10^{-4} \lesssim \kappa \lesssim 10^{-1} \]

\[ B(x) = 1.58 + 0.32x - 0.05x^2 \]


If B number is to be preserved in true vacuum, $\frac{E_{sp}(T_c)}{T_c}$ must be large.
4) Phase Transition: Bubbles versus no bubbles
Bubbles versus no Bubbles

EWSB can proceed smoothly or through bubble nucleation

[Kirzhnits, Linde 72]

Without the bump, no coexistence phase

$$V_T(\phi) = \left[ -\mu^2 + c_1(T)T^2 \right] \frac{\phi^2}{2} - E\phi^3 + \frac{\lambda}{4}\phi^4 + \rho_\Lambda$$

Will refer to 1st order PT as those forming bubbles
Bubbles are useful for the EW-bgenesis scenarios
Sources of Bumps

Sources of Bumps separating minima:
1) Tree level terms
2) Thermal corrections – suppressed (MSSM – fine tuned)

Bosons can give non-analytic contributions.

\[-V \approx P_b \sim p^3 p^2 f(\omega) \sim \omega_0^4 f(\omega_0) + \frac{T^5}{\omega_T}\]

\[f(\omega_0) \sim \frac{1}{e^{\omega_0/T} - 1} \sim \frac{T}{\omega_0}\]

\[\omega_0 = \sqrt{0 + y^2 \phi^2}\]

\[P_b \sim T \omega_0^3 + \frac{T^5}{\omega_T}\]

\[\sim T(y^2 \phi^2)^{3/2} + T^4 - 2y^2 \phi^2 T^2\]

Can be important for 1st order PT, but usually difficult.
Completion of the Phase Transition

\[ \Gamma_{\text{bubble nucleation}} \sim \Theta(T_4) e^{-\frac{S_3}{T}} \]

\[ S_3 = \int 4\pi r^2 dr \left[ \frac{1}{2} \left( \frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right] \]

\[ \frac{d^2 \phi_b}{dr^2} + \frac{2 d\phi_b}{r dr} - \frac{\partial V}{\partial \phi_b} = 0, \quad \text{with} \quad \left. \frac{d\phi_b}{dr} \right|_{r=0} = 0 \quad \text{and} \quad \left. \phi_b \right|_{r=\infty} = 0 \]

\[ \langle H \rangle = 0 \]

\[ \langle H \rangle \neq 0 \]

\[ \left( \frac{\Gamma_{\text{bubble nucleation}}}{H^4} \right) = \Theta(T_0^4) e^{-\frac{S_3[T_0]}{T}} \]

\[ \gtrsim \frac{H}{M_p} \sim \frac{T}{T_n}^2 \]

\[ \Rightarrow \frac{S_3}{T_n} \left|_{T_n} \right| \lesssim \left| 4 \right| \]


Although we need to talk about charge transport, delay that until lecture 2.

Let’s recap the picture of electroweak baryogenesis and pictorially illustrate the charge transport.
A Cartoon

0. nucleate a bubble.

1. Pick up CP/chiral asymmetry through mixing or interactions involving Higgs.

\[ \langle H \rangle = z \]

\[ \langle H \rangle = 0 \]

\[ n_b^L - n_b^R \neq 0 \]

\[ n_b - n_b = n_b^L + n_b^R - n_b^L - n_b^R = 0 \]

e.g. 1 generation

\[ \bar{u}_L u_R \quad B=0 \]
2. Diffuse out to front of bubble wall.

$\langle H \rangle = z$

$sphalerons$

$\langle H \rangle = 0$

$\Delta (n_b^L - n_b^R) = n_b^L + n_b^R - n_b^L - n_b^R \neq 0$

\[ n_b - n_b = n_b^L + n_b^R - n_b^L - n_b^R \neq 0 \]

\[ \text{e.g. 1 generation} \quad u_R \rightarrow u_R \]

\[ \bar{u}_L \rightarrow d_L d_L \nu_e \]

$B = 1$
A Cartoon

3. Capture and preserve the baryon number.

\[ \langle H \rangle = z \quad \text{or} \quad \langle H \rangle = 0 \]

\[ n_b - n_{\bar{b}} = n_b^L + n_b^R - n_{\bar{b}}^L - n_{\bar{b}}^R \neq 0 \]

e.g. 1 generation

\[ \begin{array}{c}
    u_R \\
    d_L \\
    d_L \nu_e
\end{array} \quad \text{with} \quad B = 1

\text{BUT, weak sphalerons are still active}

\[ \Gamma_{sp} \sim 2.8 \times 10^5 \, T^4 \left( \frac{\alpha_W}{4\pi} \right)^4 \kappa \left[ \frac{E_{sp}(T)}{B} \right]^7 e^{-E_{sp}(T)/T} \]

\[ \frac{dn_B}{dt} \approx -O(5)N_F \frac{\Gamma_{sp}}{T^3} \]
To Preserve Baryon Number

3. Capture and preserve the baryon number.

\[ \langle H \rangle = z \]

\[ \langle H \rangle = 0 \]

sphalerons

\[ n_b - n_b^L = n_b^L + n_b^R - n_b - n_b^R \neq 0 \]

e.g. 1 generation

\[ \frac{d n_R}{d t} \approx -O(5) N_F \frac{\Gamma_{sp}}{T^3} \]

\[ \Gamma_{sp} \approx 2.8 \times 10^5 T^4 \left( \frac{\alpha W}{4\pi} \right)^4 \kappa \left[ \frac{E_{sp}(T)}{B} \right]^7 e^{-E_{sp}(T)/T} \]

BUT, weak sphalerons are still active

To allow baryon asymmetry to survive,

\[ \frac{E_{sp}(T_c)}{T_c} \geq 45 \]

e.g.

B Conservation

- Strong enough phase transition to prevent wash-out

\[ \Gamma = A(T) \exp \left[ -E_{sp}^h(T)/T \right] \]

\[ \frac{E_{sp}(T_c)}{T_c} \gtrsim 45 \rightarrow \frac{v(T_c)}{T_c} \gtrsim 1 \]

SM cannot accommodate this. This (and CP viol which I will not discuss) are interesting BSM motivations.

Requires O(0.1) parametric tuning/hierarchy in BSM parameters
B Conservation

Typically, complicated parameter space $\rightarrow$ numerical scan

Can utilize approximate discrete symmetry to identify parametric region.

[Huber, Schmidt 00]

[Barger, DC, Long, Wang 11]
Discrete Symmetry Limit

$T_c \to 0$

Discrete symmetry of T=0 theory!

$\phi \to -\phi + \frac{2E}{\lambda}$

[fold about the bump]

"Fixed" by $T=0$ VEV

Arbitrarily strong PT can be found near an enhanced discrete symmetry point if the condition for its spontaneous symmetry breaking is met at T=0 and if the coset representation containing an electroweak singlet element does not commute with the electroweak group.

[Barger, DC, Long, Wang 11]
Z2 SM Example

SM + real singlet
[e.g. Ham, Jeong, Oh 04; Espinosa, Konstandin, Riva 11]

\[ a_2 = 2 \lambda \]

\[ \text{EW} \times \mathbb{Z}_2 \to \text{EW} \times \mathbb{Z}_2 \to \text{EW} \times \mathbb{Z}_2 \]

[Barger, DC, Long, Wang 11]
LHC Dec 2011 Hints (> 2 sigma level)

$2.8\sigma$

$m_h \approx 125\text{GeV}$

Production and decay rates are within an order of magnitude of the SM. 

**SM:** $\Gamma(H) \approx 0.004 - 0.005 \text{ GeV}$ at $M_H \approx 126 \text{ GeV}$
LHC Hints (> 2 sigma level)

\[
\frac{v(T_c)}{T_c} \gtrsim 1
\]

\[
U(h, s) = \left[ \frac{\lambda}{4} (h^4 + s^4) - \frac{\lambda v^2}{2} (h^2 + s^2) + \frac{a_2}{4} h^2 s^2 \right] + \left[ \frac{\Delta b_4}{4} s^4 + \frac{\Delta b_2}{2} s^2 \right]
\]

Higgs total decay width $\rightarrow$ block invis. decay

1) Upper bound on $\Delta b_4$
2) Excludes continuous symmetry limits
Lec 1 summary

1) Compelling cosmology involves the following:
   a) solid short distance physics tests in lab
   b) observationally motivated/dynamical attractor BCs
   c) probes exist to observationally confirm

2) Electroweak symmetry breaking related cosmology will be compelling at the LHC

3) Many interesting ingredients compose electroweak baryogenesis illustrating the richness of the physics implications of LHC to cosmology.

4) Even the limited Higgs information gives interesting qualitative constraint on BSM from the perspective of electroweak baryogenesis.
Lec 2 plan

1) Transport in electroweak baryogenesis
2) Other topics at the energy/intensity frontier if time permits
3) Big questions in cosmology