ELECTROWEAK SYMMETRY BREAKING:
Unitarity, Dynamics, and Experimental Prospects

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1. INTRODUCTION

The standard model of elementary particles is a beautiful synthesis that explains many seemingly unrelated and, in some instances, very puzzling phenomena. Like any successful theory, its success allows and encourages us to ask questions at a deeper level than those it answers. There are many such questions, some so far-reaching that generations may pass before the answers are found. Remarkably, we are confident that one fundamental question is sure to be answered with scientific instruments that we now know how to build and hope soon to construct. That is the question of the mechanism that breaks the electroweak symmetry, and thereby gives mass to the W and Z gauge bosons while leaving the photon massless. This review is concerned chiefly with that unknown mechanism—what we already know about it, how we can expect to discover it, and the forms it may take. We also consider a second, related problem, for which there is no similar assurance that the solution is close at hand—the origin of quark and lepton masses.

1.1 Gauge Boson Masses

The forces in the standard model are mediated by gauge bosons associated with the symmetries of the group

$$SU(3)_{\text{color}} \times SU(2)_{L} \times U(1)_{Y}.$$  

The $SU(3)_{\text{color}}$ factor describes the gluonic force of the strong interactions (1). Because the $SU(3)_{\text{color}}$ gauge invariance is unbroken, the eight associated gluons remain massless. The theory of quarks interacting by exchange of massless gluons is called quantum chromodynamics or QCD, to stress the parallel with quantum electrodynamics or QED, which also has an unbroken gauge invariance and an associated massless gauge boson—the photon.

The $SU(2)_{L} \times U(1)_{Y}$ factors describe the unified electroweak interactions (2). The $SU(2)_{L}$ factor is a left-handed isospin, an isospin charge carried by left-chirality fermions (chirality is equivalent to helicity in the ultrarelativistic limit). The subscript $Y$ refers to the "weak hypercharge," determined from the electric charge and left isospin, by the relation

$$Q = T_{3L} + Y.$$  

Unlike the $SU(3)_{\text{color}}$ factor, the $SU(2)_{L} \times U(1)_{Y}$ invariance is broken (3) down to the $U(1)_{\text{EM}}$ subgroup, the unbroken gauge invariance of electromagnetism. As a result, three of the four gauge bosons, $W^{+}$, $W^{-}$,
and Z, of SU(2)_L \times U(1)_Y are massive while the fourth, the photon, \( \gamma \), remains massless.

Experimentally this is not a subtle effect. The W and Z masses \( M_W = 80.9 \pm 1.4 \text{ GeV} \quad M_Z = 92.1 \pm 1.8 \text{ GeV} \)

are responsible for the subnuclear range of the weak force

\[ L_{\text{weak}} \approx \frac{1}{M_W} \approx 10^{-16} \text{ cm.} \]

In contrast, the Maxwell force laws are verified by measurements of the magnetic field of Jupiter to a scale greater than \( L_{\gamma} > 10^{10} \text{ cm.} \)

Therefore the symmetry-breaking mechanism produces a mass hierarchy of at least 26 orders of magnitude,

\[ \frac{M_W}{M_\gamma} > 10^{26}. \]

Although the dynamics of the symmetry-breaking mechanism is unknown, there is good reason to believe that the general framework is spontaneous symmetry breaking. The term “spontaneous” here means the symmetry is not broken explicitly by the interactions but rather by the asymmetry of the state of lowest energy \( (6) \), referred to as the vacuum in quantum field theory. In the absence of an associated gauge symmetry, each spontaneously broken direction in the global (i.e. space-time independent) symmetry space gives rise to a massless, spin-zero particle or Goldstone boson. If the broken direction in symmetry space also corresponds to a gauge symmetry (i.e. a space-time dependent symmetry) then the associated Goldstone boson and the massless gauge boson combine to form a massive gauge boson—the Higgs mechanism. The Higgs mechanism preserves the number of states. A massless gauge boson occurs in two, transverse polarization states, given in four-component notation by

\[ \varepsilon^\mu_{\pm} = \frac{1}{\sqrt{2}} (0, 1, \pm i, 0), \]

where \( \mu = 0, 1, 2, 3 \) and the \( \hat{z} \) or 3-direction is chosen as the direction of motion. Because a massive gauge boson can be brought to rest by a Lorentz transformation, and since there is no preferred direction in the rest frame, spatial isotropy requires three spin states. Returning to a frame in which the massive gauge boson moves in the \( \hat{z} \) direction, the two transverse spins
states are given by Equation 7, while the third spin state is longitudinally polarized

\[ \epsilon_0^\mu = \frac{1}{M} (p, 0, 0, E). \]

(The terms transverse and longitudinal refer to the polarization three-vectors: \( \epsilon_\pm \) is transverse to the momentum \( p \) while \( \epsilon_0 \) is parallel.)

As is made precise by the equivalence theorem (Section 2), at energies large compared to the gauge boson mass the longitudinal mode can be identified with the underlying Goldstone boson from the symmetry-breaking sector. In this sense we can say that three particles from the otherwise unknown symmetry-breaking sector have already been discovered: the longitudinal gauge boson modes \( W_L^\pm \) and \( Z_L \) (here \( L \) denotes longitudinal). Therefore by studying the interactions of longitudinally polarized \( W \)'s and \( Z \)'s in the laboratory we can probe the dynamics of the unknown symmetry-breaking mechanism.

The strong theoretical prejudice in favor of spontaneous symmetry breaking despite the absence of any direct experimental support is based on the criterion of renormalizability. Theories of massless gauge bosons, such as quantum electrodynamics and quantum chromodynamics, are renormalizable. A theory is renormalizable if the infinities that appear in intermediate stages of perturbative calculations can be controlled with a finite number of subtractions or redefinitions, and the number of subtractions need not be increased as the calculation is extended to higher orders in the perturbation expansion. The presence of \( p \) and \( E \) in Equation 8 is one of the reasons why theories with longitudinal gauge boson modes are more divergent at high energy than unbroken gauge theories containing only transverse modes. It is then no surprise that introducing gauge boson masses in ad hoc fashion results in a nonrenormalizable (and non-gauge-invariant) theory—one requiring an ever increasing number of subtractions in each higher order of perturbation theory.

The spontaneously broken electroweak theory (3, 7) was motivated by the conjecture that it is renormalizable, because it is an amalgam of two separately renormalizable and gauge-invariant components. The first component is the corresponding unbroken gauge theory with massless gauge bosons; the second, the so-called Higgs sector, gives rise to spontaneous symmetry breaking and to the Goldstone bosons needed to provide longitudinal modes for the gauge bosons. The proposition that the "sum" of two such renormalizable and gauge-invariant components is also renormalizable and gauge invariant has subsequently been verified in detail (8).
The minimal Higgs sector of the model of Weinberg and Salam (3) could be replaced by other physical systems without affecting the interaction of the gauge bosons with ordinary matter (i.e. quarks and leptons) or the renormalizability of the theory. It is only necessary that the alternative physical system do the job of the minimal Higgs sector: induce an SU(2)\(_L\) × U(1)\(_Y\) asymmetric ground state that leaves just the U(1)\(_{\text{EM}}\) symmetry unbroken; this gives rise to three Goldstone bosons that can become the longitudinal modes of W and Z. Other possibilities include nonminimal Higgs boson models, possibly embedded in supersymmetric theories (9), or dynamical models, such as the QCD-like class of theories known as technicolor (10), that do not contain elementary Higgs bosons at all. Though this exhausts the list of currently known possibilities, there is no proof that it exhausts the list of all possibilities: nature may well have chosen symmetry-breaking dynamics that we have not yet imagined.

Since dynamical models embody the "Higgs mechanism" without necessarily containing Higgs bosons, I will in general use the phrase "symmetry-breaking sector" rather than "Higgs sector." The assumed framework of this review is that the electroweak force is described by a spontaneously broken SU(2)\(_L\) × U(1)\(_Y\) gauge theory. Without any further specification of the "Higgs mechanism," this framework alone is sufficient to deduce certain general properties of the symmetry-breaking sector that depend only on the gauge symmetry and unitarity. The approach is very similar to the use of current algebra in the 1960s, when general symmetry arguments were used to follow the trail that led eventually to the underlying dynamics of QCD. In fact certain general properties of the symmetry-breaking sector (11, 12) correspond precisely to properties of low-energy pions that follow from their role in hadron physics as the Goldstone bosons of a spontaneously broken flavor symmetry (13). The "pions" in the present instance are the longitudinal polarization states of the W and Z gauge bosons. If the symmetry-breaking sector is, like QCD, a strong interaction theory, then the analogy to low-energy pion physics is very close. It holds for technicolor models that are based on an unbroken gauge interaction like QCD, but more generally it is true of any theory of symmetry breaking with a strong force.

The search for the symmetry-breaking sector is a search for a new set of particles interacting by a new force. I refer to this unknown physics generically as \(\mathcal{L}_{\text{SB}}\), the Lagrangian of the symmetry-breaking sector. The quanta of \(\mathcal{L}_{\text{SB}}\) are characterized by an unknown mass scale, \(M_{\text{SB}}\), and the force by an unknown coupling strength, \(\lambda_{\text{SB}}\). Using current algebra and unitarity, one can show that the new physics of \(\mathcal{L}_{\text{SB}}\) must produce observable consequences on longitudinal WW scattering, e.g. \(W_+^L W_-^L \rightarrow Z_4 Z_4\), for WW center-of-mass energies at a "cutoff" value \(\Lambda_{\text{SB}}\) bounded by (11)
\[
\Lambda_{SB} \leq \sqrt{\frac{8\sqrt{2\pi}}{G_F}} \approx 1.7 \text{ TeV.}
\]

(One TeV is \(10^{12}\) eV and \(G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2}\) is the Fermi constant of the weak interactions.) The same analysis also shows that the two unknown parameters \(M_{SB}\) and \(\lambda_{SB}\) are generally correlated: the symmetry-breaking force is strong if and only if the mass scale of the quanta is \(\sim 1\) TeV or above. Furthermore, though not rigorously proven, it is true in all known physically plausible models that Equation 9 is a bound on the order of magnitude of \(M_{SB}\).

Equation 9 is the key to the statement that the search for the symmetry-breaking mechanism is not open-ended. An accelerator able to probe \(WW\) scattering up to \(\sim 2\) TeV in the WW center-of-mass system is sufficient to decide whether the symmetry-breaking sector is strongly interacting with quanta at the TeV scale or if it is weakly interacting with a sub-TeV mass spectrum (11, 14). With either a hadron or electron-positron collider, WW fusion, proposed initially as a Higgs boson production mechanism (15, 16) and illustrated in more general form in Figure 1, allows us to measure the strength of the \(W_L W_L\) interaction. The basic idea (11) is that the bosons emitted by the quarks or electrons are off the mass shell \((E^2 - p^2 \neq M_W^2)\) and therefore must rescatter to materialize in the final state as real, on-mass-shell particles. The number of gauge boson pairs then provides a measurement of the strength of the unknown \(W_L W_L\) interaction, denoted by the blob in Figure 1. As discussed in Section 4, studies of the signals and backgrounds suggest that the design parameters of the proposed Superconducting Super Collider (SSC), with proton-proton center-of-mass energy \(\sqrt{s} = 40\) TeV and luminosity \(L = 10^{33} \text{ cm}^{-2} \text{ s}^{-1}\), is a minimal configuration to see a strongly interacting symmetry-breaking signal (11, 14). Proton colliders of one half that energy or one tenth that luminosity would not be sufficient. The corresponding minimal configuration for an electron-positron collider is \(e^+e^-\) center-of-mass energy \(\sqrt{s} = 2-3\) TeV and luminosity in the range \((1-2\frac{1}{2}) \times 10^{33} \text{ cm}^{-2} \text{ s}^{-1}\) (17), as discussed in Section 6.

The use of unitarity to deduce the upper limit, Equation 9, on the energy

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{production.png}
\caption{Production of gauge boson pairs by WW fusion. Straight lines represent quarks or leptons and the blob represents the WW scattering amplitude.}
\end{figure}
scale of the symmetry-breaking physics comes from a venerable tradition (18) in weak interaction physics. The original Fermi theory of the weak interaction posited a four-fermion contact interaction to describe fermion-fermion scattering such as $e\nu_\mu \to e\nu_e$. The interaction Lagrangian [Dirac matrices are defined as in (19)],

$$\mathcal{L}_{\text{Fermi}} = \frac{G_F}{\sqrt{2}} [\bar{\epsilon}(1-\gamma_5)\gamma^\mu\nu_e] [\bar{\nu}_\mu(1-\gamma_5)\gamma^\mu\mu],$$  \hspace{1cm} (10)$$

implies a scattering amplitude at high energy (for all helicities negative)

$$M(\nu_\mu e \to \mu\nu_e) = 4\sqrt{2}G_F s$$ \hspace{1cm} (11)$$

that grows quadratically with energy. This can only be valid as a low-energy approximation since continued to arbitrarily high energy it would violate unitarity, the condition that scattering not occur with probability greater than unity. The precise expression of unitarity is that the partial wave amplitudes $a_j$ defined by

$$M(s, t) = 16\pi \sum_j (2j+1)a_j(s) P_j\left(1 + \frac{2t}{s}\right),$$ \hspace{1cm} (12)$$

where $P_j$ is a Legendre polynomial, cannot be greater than one,

$$|a_j(s)| \leq 1.$$ \hspace{1cm} (13)$$

The $j = 0$ partial wave amplitude projected from Equation 11 then implies that

$$\frac{G_F s}{2\sqrt{2}\pi} \leq 1$$ \hspace{1cm} (14)$$

or that "new physics" must intervene to modify the Fermi theory at a scale (18)

$$\Lambda_{\text{Fermi}} \leq \sqrt{\frac{2\sqrt{2}\pi}{G_F}} = 0.9 \text{ TeV}.$$ \hspace{1cm} (15)$$

The bound on $\Lambda_{\text{Fermi}}$ is precisely half the bound on $\Lambda_{\text{SB}}$ given in Equation 9.

The physics that modifies the Fermi theory was established with the discovery of the W boson (20) at the mass predicted by the SU(2)$_L \times$ U(1)$_Y$ theory (3), $M_W = 80$ GeV (4). The fact that $M_W < 0.9$ TeV confirms our faith in the laws of probability. In the SU(2)$_L \times$ U(1)$_Y$ theory the four-fermion interaction is replaced by W boson exchange. Instead of the Fermi Lagrangian of Equation 10 we have the interaction
where $g$ is the SU(2)$_L$ gauge coupling constant. From Equation 16 we compute the scattering amplitude

$$M(\nu_{\mu}e \rightarrow \mu\nu_e) = ig^2 \frac{g^a g^b}{M_W} \frac{g_{a\mu} - g_{a\nu}}{M^2_W} [\bar{\nu}_{\mu}(1 - \gamma^5)\gamma^a \mu] \times q^2 - M^2_W,$$

where $q = \vec{p}_e - \vec{p}_{\nu_e}$ is the momentum of the exchanged W boson and $\nu_e, \ldots$ here denote Dirac spinor wave functions. Comparing Equation 17 at low energy $q \ll M_W$ with the amplitude obtained from Equation 10, we learn that

$$G_F = \frac{g^2}{4\sqrt{2}M^2_W}.$$
$L_m = m\bar{\psi}\psi$  \hspace{1cm} 19.

would explicitly break the SU(2)$_L$ gauge invariance and make the theory nonrenormalizable. The breaking of gauge invariance can be seen by decomposing the Dirac fermion $\psi$ into left- and right-chirality components,

$\psi_L = \frac{1 - \gamma^5}{2} \psi, \quad \psi_R = \frac{1 + \gamma^5}{2} \psi,$  \hspace{1cm} 20.

which are approximately the eigenstates of left and right helicity in the ultrarelativistic limit when the fermion mass is much smaller than the energy. [Here as elsewhere I follow the conventions of (19).] Since $\bar{\psi}_L \psi_L = \bar{\psi}_R \psi_R = 0$, Equation 19 becomes

$L_m = m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L).$  \hspace{1cm} 21.

The term $\psi_L$ transforms like a doublet under SU(2)$_L$, while $\psi_R$ transforms like a singlet (2, 3), so that $L_m$ is clearly not an SU(2)$_L$ singlet.

The solution, as for the gauge boson masses, is to generate fermion masses by spontaneous symmetry breaking. As in the discussion of the preceding section, the resulting theory can be viewed as the "sum" of gauge-invariant, renormalizable components that "add up" to a gauge-invariant renormalizable result. One component is the unbroken SU(2)$_L \times U(1)_Y$ gauge theory, with massless gauge bosons coupled in an SU(2)$_L \times U(1)_Y$ invariant fashion to massless fermions. The fermion-mass-generating component contains SU(2)$_L \times U(1)_Y$ invariant interactions of the fermions with certain other fields that generate fermion masses in the asymmetric ground state fixed by spontaneous symmetry breakdown. In the simplest version of this mechanism an SU(2)$_L$ doublet Higgs field is given an SU(2)$_L$ invariant coupling to the fermion (3),

$L_{\psi\psi} = f(\phi \bar{\psi}_L \psi_R + \phi^* \bar{\psi}_R \psi_L).$  \hspace{1cm} 22.

In the asymmetric vacuum, the Higgs field acquires a condensate or vacuum expectation value $v$ and the fermion acquires a mass

$m = fv.$  \hspace{1cm} 23.

This mechanism for the generation of fermion masses differs from the mechanism of gauge boson mass generation in two important respects. First, although in the minimal model (3) there is only a single vacuum condensate that generates both fermion and gauge bosons masses, in general there may be several different condensates and the condensate(s) responsible for fermion masses need not make the dominant or even an important contribution to the W and Z boson masses. Second, the means by which the condensates are transmitted to the gauge bosons and fermions
are very different. For the gauge bosons, as reviewed in Section 1.3, the transmission is dictated by the Pauli prescription for gauge invariance that requires gauge bosons to couple with universal strength to all quanta carrying the charge of the gauge group (the so-called minimal coupling ansatz). Thus, for example, in the minimal model with a single vacuum expectation value, \( v \), the W mass is determined by the SU(2)\(_L\) coupling constant \( g \): \( M_W = \frac{1}{2} g v \). In contrast, fermion mass generation requires additional interactions between the fermions and the condensing fields. Since they are not electroweak gauge interactions there is in general no principle that establishes their strength. For instance, in Higgs boson models the new interactions are of the Yukawa form, Equation 22. The result is a gauge-invariant, renormalizable solution to the problem of fermion mass, but a solution that is nonetheless disappointing since the fermion mass values are traded one-to-one for equally mysterious Yukawa coupling constants. In dynamical symmetry-breaking models, the transmission of the vacuum condensate to the fermions is more complicated but no less disappointing—see Sections 3.2 and 3.3.

In analogy to the unitarity upper limit on the mass scale at which the symmetry-breaking effects must appear in scattering of longitudinally polarized W's and Z's, there is also an upper limit due to unitarity on the scale of the fermion mass generation mechanism (21). The bound for a given fermion is inversely proportional to the fermion mass,

\[
\Lambda_f = \frac{16\pi}{\sqrt{2}\xi G_f m_f},
\]

where \( \xi = 1 \) for leptons and \( \sqrt{3} \) for quarks. For the known fermions \( \Lambda_f \) defines a much larger scale than the \( O(2 \text{ TeV}) \) scale of \( \Lambda_{SB} \) given in Equation 9, e.g. for the electron \( \Lambda_e \approx 6 \times 10^6 \text{ TeV} \).

### 1.3 The Minimal Higgs Boson Model

I briefly review the basic structure of the minimal model (3) as an illustration of the preceding discussion and to establish a framework for the sections that follow. As discussed above, the Lagrangian for the complete theory is the sum of three terms,

\[
\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{SB}} + \mathcal{L}_f.
\]

The first term, \( \mathcal{L}_{\text{gauge}} \), describes an unbroken SU(2)\(_L\) \( \times \) U(1)\(_Y\) gauge theory, consisting of massless gauge bosons interacting with massless fermions. The symmetry-breaking component, \( \mathcal{L}_{\text{SB}} \), contains the dynamics that induces an SU(2)\(_L\) \( \times \) U(1)\(_Y\) asymmetric ground state. The third term \( \mathcal{L}_f \) contains the couplings of the fermions to the fields of \( \mathcal{L}_{\text{SB}} \) that acquire
condensates in the asymmetric ground state, thereby generating the fermion masses.

Consider first the gauge interactions, $\mathcal{L}_{\text{gauge}}$. These are four gauge bosons, $W (= W^1, W^2, W^3)$ and $X$ corresponding to the four generators of $SU(2)_L$ and $U(1)_Y$. For simplicity we consider just the first generation of fermions, consisting of the quark and lepton $SU(2)_L$ doublets $(u, d)_L$ and $(e, \nu)_L$ and the right-chirality components $u_R, d_R, \text{and } e_R$, which are $SU(2)_L$ singlets (i.e. do not carry left isospin). The hypercharge $Y$ of the various fermions is fixed by Equation 2 using these $SU(2)_L$ assignments and the known electric charges. Then following the usual prescription for constructing a gauge-invariant theory (e.g. 7) we have in a compact notation

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F_{\mu \nu} \cdot F^{\mu \nu} - \frac{1}{4} F_{\mu \nu}^a F^{a \mu \nu} + i \bar{\psi} D \psi.$$  

The gauge-invariant field strengths for the gauge bosons are

$$F^{\mu \nu} = \partial^\mu X^\nu - \partial^\nu X^\mu - \partial^\mu W^\nu + \partial^\nu W^\mu + g W^\mu \times W^\nu.$$  

The field $\psi$ is a multicomponent spinor, consisting of the $(u, d)_L$ and $(e, \nu)_L$ $SU(2)_L$ doublets and the three $SU(2)_L$ singlets $u_R, d_R, \text{and } e_R$, coupled gauge invariantly to the gauge bosons by the covariant derivative

$$D^\mu = \partial^\mu - ig W^a \cdot T^a - ig' X^a Y$$  

with $g$ and $g'$ the $SU(2)_L$ and $U(1)_Y$ coupling respectively. In Equation 28, $T_L$ and $Y$ are matrices that act on $\psi$ in accordance with the isospin and hypercharge assignments of its components.

In the minimal model the quanta of $\mathcal{L}_{\text{SB}}$ consists of four spin-zero bosons arranged to form a complex doublet under the $SU(2)_L$ group,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} w_1 + iw_2 \\ h + iw_3 \end{pmatrix}.$$  

Here $w^+ = (1/\sqrt{2})(w_1 + iw_2)$ carries positive electric charge while $h$ and $w_3$ are electrically neutral. Then for this case

$$\mathcal{L}_{\text{SB}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi^\dagger \Phi),$$  

where to insure gauge invariance $D_\mu$ is again of the form of Equation 28 with $T_L$ given by the familiar Pauli matrices appropriate to isospin 1/2, and $Y$ is a diagonal matrix constructed in accordance with Equation 2. The spontaneous symmetry breakdown is induced by the form of the potential $V$,

$$V = \lambda (\Phi^\dagger \Phi - \frac{1}{2} y^2)^2,$$  

where $\lambda$ is a coupling constant.
with a minimum at $\Phi^+\Phi = v^2/2$. The "spontaneity" of the symmetry breaking refers to the property that the minimum could be realized by a condensate $v$ forming along any direction of the four-dimensional space spanned by $w, h$. We choose the components of $\Phi$ so that the desired pattern of symmetry breaking occurs when the field $h$ acquires the condensate $v$, i.e. for $w = 0$ and $h = v$. If we redefine $h$ by $h \rightarrow h + v$ so that it also vanishes at the minimum, then the potential can be rewritten as

$$V = \frac{\lambda}{4} (w^2 + h^2)^2 + \lambda v h (w^2 + h^2) + \lambda v^2 h^2.$$  \hspace{1cm} 32.

Equation 32 illustrates Goldstone's theorem: before breakdown the potential, Equation 31, had a four-dimensional $O(4)$ symmetry in the space spanned by $w, h$ that was reduced to the $O(3)$ symmetry of the space spanned by the triplet $w$ in Equation 32. The number of invariant generators is reduced from the six of $O(4)$ to the three of $O(3)$, each broken generator giving rise to a massless Goldstone boson. In Equation 32 we see that the Higgs field $h$ has a mass

$$m_h^2 = 2\lambda v^2.$$  \hspace{1cm} 33.

while the $w$ are massless—they are the expected Goldstone bosons.

In fact the model defined by $\mathcal{L}_{SB}$ played a venerable role in the physics of the 1960s that led to QCD in the early 1970s: it is precisely the SU(2) sigma model (22) with $w$ replaced by the pions and $h$ replaced by the scalar sigma field. The value of the sigma model is not that it is a comprehensive description of hadron physics—the existence of the sigma meson remains an enigma to this day—but that it correctly embodies the symmetry structure of hadron physics, an SU(2)$_L \times$ SU(2)$_R$ symmetry that breaks spontaneously to SU(2)$_{L+R}$ (ordinary isospin) with three Goldstone bosons, the pions. It therefore embodies the low-energy theorems for pion-pion scattering, which are the counterparts of the low-energy theorems for longitudinally polarized W's and Z's introduced in Section 1.2. According to one of the original practitioners, "We may compare this process to a method sometimes employed in French cuisine: a piece of pheasant meat is cooked between two slices of veal which are then discarded" (23). If $\mathcal{L}_{SB}$ is strongly interacting then the minimal Higgs model may eventually be viewed in just the same way as the sigma model.

The gauge boson mass is "transmitted" to the gauge bosons by coupling the scalars to the gauge bosons in Equation 30, dictated by the gauge-invariant minimal substitution prescription of Equation 28. Since $\Phi$ acquires a condensate $v$, Equation 30 gives rise to mass terms for the gauge...
bosons $W$ and $X$. The charged components $W^\pm = (1/\sqrt{2})(W_1 \pm iW_2)$ acquire a mass
\[ M_w = \frac{1}{2}g \nu \]
while the neutral components $W_3$ and $X$ acquire a mass matrix
\[ M_{W_3-X}^2 = \frac{1}{4} \nu^2 \begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix}. \]
Since the determinant vanishes, the mass of one eigenstate—the photon—vanishes while the mass of the other follows from the trace,
\[ M_Z = \frac{\nu}{2} \sqrt{g^2 + g'^2}. \]
The eigenstates are related to $W_3, X$ by a rotation
\[ \begin{pmatrix} \gamma \\ Z \end{pmatrix} = \begin{pmatrix} \cos \theta_w & \sin \theta_w \\ -\sin \theta_w & \cos \theta_w \end{pmatrix} \begin{pmatrix} X \\ W^3 \end{pmatrix} \]
where the mixing angle is
\[ \sin \theta_w = -\frac{g'}{\sqrt{g^2 + g'^2}}. \]
Substituting Equation 37 into Equation 28 we discover that the electric charge is
\[ e = g \sin \theta_w. \]
From the lowest order relation for the Fermi constant
\[ G_F = \frac{g^2}{4\sqrt{2}M_w^2} \]
and from Equation 34 we deduce the value of the condensate
\[ \nu = (\sqrt{2G_F})^{-1/2} = 246 \text{ GeV}. \]
Comparing the expressions for the $W$ and $Z$ masses and using Equation 38 we learn that the so-called rho parameter (the ratio of the neutral current to the charged current coupling in the low-energy effective four-fermion theory) is equal to unity,
\[ \rho \equiv \frac{M_w^2}{M_Z^2 \cos^2 \theta_w} = 1. \]
in excellent agreement with the most recent global fits (24), which give $1.01 \pm 0.01$.

Going beyond the minimal model, the success of Equation 42 is an important clue to the symmetry structure of $\mathcal{L}_{SB}$. Returning to Equations 34 and 35 the reader can see with a little work that $\rho = 1$ follows from the quality of the mass of $W_3$ to that of $W_1$ and $W_2$, which is to say it is a consequence of the unbroken isospin, the vector $\text{SU}(2)_{L+R}$, that survives the spontaneous breaking of $\text{SU}(2)_L \times \text{SU}(2)_R$ in the minimal model. Since this $\text{SU}(2)_{L+R}$ protects Equation 42 from potentially large $O(\lambda_{SB})$ corrections from $\mathcal{L}_{SB}$, it is sometimes called the “custodial SU(2)” (25).

In the first paper cited in (12), it is shown that the validity of $\rho = 1$ in the case of a strongly interacting $\mathcal{L}_{SB}$ implies that the low (but not necessarily the high) energy interactions of the Goldstone bosons $w$ must be $\text{SU}(2)_{L+R}$ symmetric, a kind of converse to the observation of (25).

The third term, $\mathcal{L}_f$ in Equation 25 is needed to transmit the symmetry-breaking condensate to the fermions in order to induce fermion masses. In the minimal model these are the Yukawa couplings, which for the $u$ and $d$ quarks are given by

$$\mathcal{L}_f = \sqrt{2} \kappa_u \bar{\chi}_L \Phi u_R + \sqrt{2} \kappa_d \bar{\chi}_L \Phi^c d_R + \text{h.c.},$$

where h.c. denotes hermitian conjugate, $\chi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$ is the weak isodoublet, $\Phi$ is defined in Equation 29, and $\Phi^c$ is the charge conjugate doublet,

$$\Phi^c = \frac{1}{\sqrt{2}} \begin{pmatrix} h - iw_3 \\ -(w_1 - iw_2) \end{pmatrix}.$$

Shifting $h \rightarrow h + v$ to account for the vacuum condensate, the quarks $q = u, d$ acquire masses

$$m_q = \kappa_q v$$

so that the Yukawa couplings are given by

$$\kappa_q = \frac{m_q}{v} = \frac{g m_q}{2 M_w}$$

and are therefore much smaller than the gauge couplings for light quarks with $q \ll M_w$. The one-generation model is completed by adding a similar coupling of $e_R$ to $\begin{pmatrix} v_e \\ e_L \end{pmatrix}$ (the weak lepton isodoublet), which generates the electron mass. In the absence of a right-handed neutrino no analogous
neutrino coupling is possible. The question of possible neutrino masses involves different physics scales and is not discussed in this review.

The proliferation of Yukawa couplings to fit the fermion masses is among the least appealing features of the minimal model. In the next section we consider another difficulty that is less obvious but equally serious.

1.4 The Naturalness Problem

The potential $V$ contains a wrong-sign (tachyonic) mass term for $w$ and $h$, given by the coefficient of $\frac{1}{2}(w^2 + h^2)$ in Equation 31, equal to $-\lambda v^2$. Because of the tachyonic sign, the state of minimum energy has a condensate $v$, which results in zero mass for the triplet $w$ and a mass $+\sqrt{2\lambda v^2}$ for $h$. The one-loop quantum correction is quadratically divergent,

$$\delta(\lambda v^2) = \frac{9\lambda}{2} \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 + \lambda v^2}. \tag{47}$$

Though expressions like Equation 47 are shocking to novices in field theory, they lose their shock value as the student masters (i.e. is brain-washed by) the renormalization program, which shows that finite predictions can be extracted at the cost of a small number of subtractions or redefinitions. Most notably in quantum electrodynamics, this program has been extraordinarily successful. The divergence in Equation 47 can be removed by introducing a counterterm that in effect shifts the initial value of $\lambda v^2$ by an infinite constant and thereby cancels the divergence generated in Equation 47.

In the renormalization program we renounce any attempt to understand the physical origin of those parameters requiring subtraction (their values are simply fitted to experiment), but we are then able to obtain finite predictions for all other physical quantities in the theory. To understand the naturalness problem it is necessary to go beyond this limited, though powerful, perspective and to ask questions about the origins of the subtracted quantities, assuming they will eventually be understood and calculable in the context of another theory formulated at a deeper level. The expectation is that the deeper theory introduces new physics at high energy that cuts off the divergent behavior of integrals like Equation 47. Denoting the energy scale of the new physics by $\Lambda$, Equation 47 would be replaced by

$$\delta(v^2\lambda) = C \frac{\lambda}{2\pi^2} \Lambda^2, \tag{48}$$

where $C$ is a numerical constant of order unity.
Equation 48 tells us that the parameters of Higgs models are hyper-sensitive to the high-energy scale of the deeper underlying theory. For example, the Higgs boson mass, given in lowest order by $m_h^2 = 2\lambda v^2$, might reasonably range from tens of MeV (26) to perhaps the TeV scale. The scale $\Lambda$ of the deeper theory might be the scale of Grand Unified Theories, $M_{\text{GUT}} = O(10^{14})$ GeV or even the Planck scale suggested by superstring and supergravity models, $M_{\text{Planck}} = O(10^{19})$ GeV. Writing the physical mass as the sum of a bare mass plus the one-loop corrections

$$m_h^2 = m_{h,\text{bare}}^2 + \frac{C\lambda}{\pi^2} \Lambda^2,$$

we see that the bare mass must be tuned with exquisite precision to make the left side much smaller than the two terms on the right side. For instance, if $m_h = 1$ TeV and $\Lambda = M_{\text{Planck}}$ then the cancellation on the right side must work to one part in $10^{17}$! Of course, the renormalization program allows us to arrange the cancellation to any desired precision, but viewed from the perspective of the deeper theory such a cancellation seems extremely unnatural—one might even say, in the absence of any principle requiring or explaining such a cancellation, that is absurdly implausible.

Though the term is also used in other ways, this is the naturalness problem that uniquely afflicts Higgs boson models. It may be thought of as an instability of the energy scale of the theory against quantum corrections that tend naturally to drive the scale to violently larger values. The problem uniquely affects Higgs models because in $3+1$ dimensions the only renormalizable theories with quadratic divergences are those containing scalar fields. For instance in unbroken gauge theories like QED or QCD, divergences are at most given by powers of logarithms. If instead of the quadratic dependence on $\Lambda$ in Equation 49 there were a logarithmic dependence,

$$m_h^2 = m_{h,\text{bare}}^2 + \frac{C\lambda}{\pi^2} m_{h,\text{bare}} \ln \frac{\Lambda}{m_{h,\text{bare}}},$$

then no fine-tuning would be needed even for $\Lambda$ as large as $M_{\text{Planck}}$.

Two strategies have been proposed to deal with the naturalness problem. One is to suppose that the symmetry-breaking sector, $\mathcal{L}_{\text{SB}}$, does not contain elementary Higgs bosons. In particular, in technicolor models (10) $\mathcal{L}_{\text{SB}}$ is presumed to be a confining gauge theory like QCD at a mass scale roughly $v/F \approx 2700$ times greater than the GeV mass scale of QCD.

\footnote{Above $1.5$ TeV the notion of the minimal model Higgs particle becomes meaningless as the decay width becomes larger than the mass.}
Since QCD is known to undergo spontaneous symmetry breaking, with \( SU(2)_L \times SU(2)_R \) breaking to \( SU(2)_{L+R} \), giving rise to three Goldstone bosons (the pions), it is plausible that a similar theory at a higher mass scale would contain the necessary ingredients for electroweak symmetry breaking.

The second strategy is to provide a principle for the cancellation of the quadratic divergences: supersymmetry (9). In supersymmetric theories the quadratic divergences due to scalar boson loops are precisely cancelled by fermion loop contributions. The remaining finite difference is proportional to the scale of supersymmetry breaking, e.g. the mass differences of the scalar and fermion superpartners. The absence of scalars degenerate with the known leptons and quarks tells us supersymmetry cannot be exact. Naturalness then implies an upper limit on the scale of supersymmetry breaking, since the naturalness problem returns if mass differences of fermion-boson superpartners are too large. To avoid fine-tuning at less than the few percent level, superpartners cannot be heavier than a few TeV.

Supersymmetry and technicolor are discussed in Sections 3 and 4. It is, however, important to recognize that nature may have found a way to solve the naturalness problem that has not yet occurred to us. For that reason we begin in Section 2 with a discussion of what can be learned from general principles—unitarity and symmetry—that hold true in any theory of electroweak symmetry breaking.

2. SYMMETRY AND UNITARITY

General principles that are true of any spontaneously broken gauge theory can be used to deduce the possible range of energy scales and experimental signals associated with the symmetry-breaking sector, \( \mathcal{L}_{SB} \). The essential ingredients are (a) the equivalence theorem, (b) low-energy theorems for Goldstone boson scattering, and (c) partial wave unitarity.

The equivalence theorem is the precise formulation of the statement that the Goldstone bosons \( w^\pm, z (= w_3) \) from \( \mathcal{L}_{SB} \) can be identified with the longitudinal gauge bosons \( W^\pm_L, Z_L \). It says that the identification holds to all orders in the interactions of both the gauge and symmetry-breaking sectors, at energies large compared to the W mass. By measuring the interactions of \( W_L \) and \( Z_L \) at such energies in the laboratory, we are in fact probing the unknown physics of \( \mathcal{L}_{SB} \).

As shown first in the derivation of the pion-pion scattering low-energy theorems (13), scattering of Goldstone bosons near threshold is completely determined by symmetry considerations, independently of unknown or noncalculable dynamical properties. For instance, the pion-pion low-
energy theorems are valid to all orders in the strong interactions provided only that the strong interactions possess an SU(2)L × SU(2)R symmetry that is spontaneously broken to SU(2)L+R (i.e. isospin). The SU(2)L × SU(2)R symmetry of hadron physics was conjectured in the 1960s and later understood as a consequence of the small up and down quark masses in QCD. The symmetry underlying the Goldstone boson low-energy theorems of \( \mathcal{L}_{SB} \) is on an even firmer footing since it follows from the SU(2)L × U(1)y gauge invariance. While the pion low-energy amplitudes depend on \( F_\pi = 92 \text{ MeV} \), the pion decay constant, the w, z low-energy amplitudes depend in general on \( v = (\sqrt{2G_F})^{-1/2} = 246 \text{ GeV} \) and also on the rho parameter, \( \rho \approx 1 \) (Equation 42).

Combining the w, z low-energy theorems and the equivalence theorem, we can deduce low-energy theorems for the longitudinal gauge boson modes \( W_L, Z_L \), perhaps more properly called “intermediate-energy theorems” (a non-euphonious phrase I will not use) since they apply in an interval limited below by the validity of the equivalence theorem and above by the validity of the Goldstone boson low-energy theorems. There is no guarantee that any such window exists in nature; as discussed below, it exists only if \( \mathcal{L}_{SB} \) is strongly interacting and has a typical mass scale \( M_{SB} \approx O(1) \text{ TeV} \).

The threshold behavior implied by the \( W_L, Z_L \) low-energy theorems would violate unitarity if continued to arbitrarily high energy. In particular, partial wave unitarity for \( W_L^+ W_L^- \rightarrow Z_L Z_L \) scattering in the angular momentum \( J = 0 \) channel shows that dynamics from \( \mathcal{L}_{SB} \) must intervene at a scale \( \Lambda_{SB} \leq 1.7 \text{ TeV} \) (Equation 9). The most probable interpretation is that the scale of the cutoff is just the typical mass scale of the quanta of \( \mathcal{L}_{SB} \), that is, \( \Lambda_{SB} \approx M_{SB} \). In any case, the analysis teaches us that \( M_{SB} \leq 1 \text{ TeV} \) if and only if \( \mathcal{L}_{SB} \) is strongly interacting, and it allows us to estimate the expected gauge boson pair production cross sections.

2.1 The Equivalence Theorem

The equivalence theorem was proved initially in tree approximation (27) and was later shown to be valid to all orders both in the gauge interactions, \( g \) and \( g' \), and in the interaction of the symmetry-breaking sector, \( \lambda_{SB} \) (11, 28). The validity of the theorem to all orders in \( \lambda_{SB} \) is crucial since we wish to apply it when \( \mathcal{L}_{SB} \) is strongly interacting and perturbation theory in \( \lambda_{SB} \) fails. Intuitively the theorem is a plausible consequence of the Higgs mechanism that transmutes the Goldstone bosons w and z into the longitudinal gauge boson modes \( W_L \) and \( Z_L \). This is seen explicitly by the gauge transformation from a renormalizable gauge (in which the Goldstone boson fields appear in the Lagrangian) to the unitary gauge (in which the Goldstone fields do not appear) (29). Nevertheless the only known proof
to all orders (11) is lengthy and complicated, making use of the Bechi-Rouet-Stora (BRS) identities that embody the full content of gauge invariance in spontaneously broken gauge theories. Here I only state the theorem and illustrate it with a simple example.

According to the theorem, physical S-matrix amplitudes involving external longitudinally polarized gauge bosons $W_L(p_1), W_L(p_2), \ldots$ as well as any other external particles may be evaluated approximately in renormalizable gauges by substituting the corresponding Goldstone bosons $w(p_1), w(p_2), \ldots$ as external particles, that is,

$$M[W_L(p_1), W_L(p_2), \ldots] = M[w(p_1), w(p_2), \ldots] + O\left(\frac{M_w}{E_i}\right).$$

As indicated, the equality holds up to corrections of order $M_w/E_i$. In the generalized $R$ gauge (30) the dependence of the gauge-variant Goldstone boson scattering amplitudes on the gauge parameter $\zeta$ is also of the order of the $M_w/E_i$ corrections.

In addition to being essential to the derivation of the $W_LW_L$ low-energy theorems, Equation 51 greatly simplifies perturbative calculations for heavy—and therefore strongly coupled—Higgs systems (see 31; also 11, 32–34). For instance, to evaluate correctly heavy Higgs production and decay by $WW$ fusion in unitary gauge requires evaluation of many diagrams with “bad” high-energy behavior that cancel to give the final result (35–37). If only the $s$-channel Higgs boson pole is retained in unitary gauge (38), the result has bad high-energy behavior and seriously overestimates the yield. But to leading order in the strong coupling $\lambda = m_h^2/2v^2$ it suffices using Equation 51 to compute just a few simple diagrams using the interactions of Equation 32. The result embodies the cancellations of many diagrams in unitary gauge and trivially has the correct high-energy behavior.

As a simple example, consider the decay of a heavy Higgs boson to $W_L^+W_L^-$. In unitary gauge the $hW_L^+W_L^-$ amplitude is

$$M(h \rightarrow W_L^+W_L^-) = gM_w\varepsilon_L(p_1) \cdot \varepsilon_L(p_2).$$

For $m_h \gg M_w$ we neglect terms of order $M_w/m_h$, so that Equation 8 implies $\varepsilon_L(p) \approx p/M_w$ and similarly from $m_h^2 = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2$ we find

$$M(h \rightarrow W_L^+W_L^-) = g \frac{m_h^2}{2M_w} + O\left(\frac{M_w}{m_h}\right).$$

In a renormalizable gauge the corresponding amplitude can be read off (taking care with factors of 2) from the $hww$ vertex in the potential, Equation 32,
From Equations 33 and 34 it is easy to see that Equations 53 and 54 are indeed equal up to $M_w/m_h$ corrections.

2.2 Low-Energy Theorems

The pion scattering low-energy theorems, derived originally by current algebra methods (13), follow from the properties of pions as Goldstone bosons generated by the spontaneous breaking of the chiral symmetry group $SU(2)_L \times SU(2)_R$ to the isospin group $SU(2)_{L+R}$. There are theorems for three independent amplitudes, one being for instance

$$M(w^+w^- \rightarrow z\bar{z}) = 2\rho v^2.$$  \hspace{1cm} 56.

Equation 55 holds in the chiral symmetry limit, $m_\pi = 0$, at center-of-mass energies smaller than the typical hadron mass scale and smaller than the scale $4\pi F_\pi \approx 1$ GeV associated with higher order effects in the low-energy chiral symmetric expansion (39), $s \ll \min \{m_\pi^2, (4\pi F_\pi)^2\}$.

To derive the analogous low-energy theorems for the Goldstone bosons $w^\pm, z$ of $\mathcal{L}_{SB}$, we must know the global symmetry group $G$ of $\mathcal{L}_{SB}$ and the remaining symmetry group $H$ of the ground state after spontaneous breaking, the counterparts of $SU(2)_L \times SU(2)_R$ and $SU(2)_{L+R}$ in the pion example. Not knowing $\mathcal{L}_{SB}$, we do not know $G$ or $H$, but the gauge invariance of the electroweak theory requires $\mathcal{L}_{SB}$ to be $SU(2)_L \times U(1)_Y$ invariant and the requirement of unbroken electromagnetic gauge invariance requires the ground state to be $U(1)_{\text{EM}}$ invariant. Therefore we know in particular that the global groups $G$ and $H$ are at least as big as $SU(2)_L$ and $U(1)_{\text{EM}}$ respectively, that is, $G \supset SU(2)_L$ and $H \supset U(1)_{\text{EM}}$. This is all we need to derive the low-energy theorems. By a method similar to the current algebra derivation (13) of Equation 55, it is shown in the first paper cited in (12) that the analogous low-energy theorem is

$$M(w^+w^- \rightarrow z\bar{z}) = \frac{1}{\rho} \frac{s}{v^2}.$$  \hspace{1cm} 56.

The corresponding domain of validity of Equation 56 is

$$s \ll \min \{M_{SB}^2, (4\pi v)^2\}.$$  \hspace{1cm} 57.

The presence of two parameters in Equation 56, $\rho$ and $v^2$, where only one, $F_\pi$, appears in Equation 55, is a reflection of our ignorance of the residual invariance group $H$. As remarked in Section 1, if $H$ includes a custodial $SU(2)$ under which $w$ transforms as a triplet, then $\rho = 1$ is protected against potentially large $O(\lambda_{SB})$ corrections (25). In particular
this explains the absence of $O(\lambda) = O(G_F m_h^2)$ quantum corrections to $\rho = 1$ at one loop in the minimal Higgs model (40). A limited converse to the observation of (25) emerges as a corollary to the current algebra derivation of Equation 56: if $\rho = 1$, then the low-energy interactions of the Goldstone boson triplet, $w$, are custodial SU(2) invariant (12). However, there is no assurance that the custodial SU(2) is an exact symmetry of $\mathcal{L}_{SB}$.

Finally we use the equivalence theorem to obtain the low-energy theorems for the longitudinally polarized gauge bosons. The complete list includes three independent amplitudes that can be chosen as

$$M(W_L^+ W^-_L \rightarrow Z_L Z_L) = \frac{1}{\rho} \frac{s}{v^2}$$  \hspace{1cm} 58.

$$M(W_L^+ W^-_L \rightarrow W_L^\pm W_L^-) = -\left(4 - \frac{3}{\rho}\right) \frac{u}{v^2}$$  \hspace{1cm} 59.

$$M(Z_L Z_L \rightarrow Z_L Z_L) = 0$$  \hspace{1cm} 60.

and four others that follow by crossing symmetry,

$$M(W_L^+ Z_L \rightarrow W_L^\pm Z_L) = \frac{1}{\rho} \frac{t}{v^2}$$  \hspace{1cm} 61.

$$M(W_L^+ W_L^+ \rightarrow W_L^\pm W_L^\mp) = M(W_L^- W_L^- \rightarrow W_L^\mp W_L^\pm) = -\left(4 - \frac{3}{\rho}\right) \frac{s}{v^2}.$$  \hspace{1cm} 62.

Equations 58–62 apply in the domain of validity common to both the equivalence theorem and the low-energy theorems,

$$M_w^2 \ll s \ll \min\{M_{SB}^2, (4\pi v)^2\}.$$  \hspace{1cm} 63.

If $M_{SB}$ is light, as for instance in a Higgs model with $m_h < 200$ GeV, then Equation 63 is the empty set. For the low-energy theorems to have an ample domain of validity we probably need $M_{SB} > O(1)$ TeV.

2.3 Unitarity and the Symmetry-Breaking Scale

The energy dependence of the low-energy amplitudes, Equations 58–62, cannot continue to arbitrarily high energy since they would eventually violate unitarity (11). There is a close analogy to the classical unitarity bounds (18) on Fermi’s effective theory of the weak interactions that, as discussed in Section 1.1, is both conceptual and numerical. Since $v^2 = (\sqrt{2}G_F)^{-1}$, the growth of the low-energy $W_L^\pm W_L^\mp$ amplitudes is proportional to $G_F E^2$ ($E$ is the WW center-of-mass energy), just like the fermion-fermion amplitudes of the Fermi theory. In the latter case we now understand the unitarity bounds (18) as applying to the mass scale of the electroweak
gauge sector, that is, $M_w$, since it is $W$ exchange that cuts off the growth of the fermion-fermion scattering amplitudes. In this context the Fermi constant is understood as the square of the ratio of the gauge coupling to the gauge sector mass scale,

$$G_F = \frac{g^2}{4\sqrt{2}M_w^2}. \quad 64.$$  

In $W_LW_L$ scattering, we expect the cutoff to be provided by exchange of quanta from $\mathcal{L}_{SB}$ (e.g. the Higgs boson in Higgs models) and we can understand the Fermi constant as proportional to the ratio of the coupling strength to the mass scale of the symmetry-breaking sector, $G_F \propto \lambda_{SB}/M_{SB}^2$. For instance, in the minimal Higgs model we have $\lambda = m_h^2/2\nu^2$ or

$$G_F = \frac{\sqrt{2}\lambda}{m_h^2}. \quad 65.$$  

As discussed below, the strength of the couplings—$g^2$ or $\lambda$—is correlated with whether the mass scales in the denominators of Equations 64 or 65 saturate their respective unitarity bounds. The jury is in for Equation 64—$M_w$ is small and $g$ is weak—but we are still waiting for a verdict in the case of Equation 65. The verdict for Equation 64 was predicted by electroweak unification that requires $g \approx O(e)$ to be weak and, therefore, $M_w$ to be small relative to the unitarity bound. Similarly, for Equation 65 supersymmetric models predict that $\lambda_{SB} \approx O(g^2)$ is weak and therefore that $M_{SB}$ is small, but dynamical models of symmetry breaking, like technicolor, suggest strong coupling and a large mass scale.

Decomposing the unitarity equation

$$\text{Im } M_{ij} = \sum_k M_{ik}(M_{kj})^\dagger \quad 66.$$  

into channels of definite angular momentum

$$a_J(s) = \frac{1}{32\pi} \int d(\cos \theta_{CM}) P_J(\cos \theta_{CM}) M(s, \theta_{CM}), \quad 67.$$  

we obtain the condition of partial wave unitarity

$$|a_J(s)| \leq 1. \quad 68.$$  

Setting $\rho = 1$ we extrapolate the low-energy theorem Equation 58 as a model of the absolute value of the $J = 0$ partial wave

$$|a_0(W_L^+W_L \rightarrow Z_LZ_L)| = \frac{s}{16\pi\nu^2}. \quad 69.$$
Unitarity therefore requires dynamics to cut off the growth of Equation 69 at a scale $\Lambda_{SB}$ bounded by

$$\Lambda_{SB} \lesssim \sqrt{16\pi v^2} = 1.7 \text{ TeV}.$$  

At the scale of the cutoff and above (but below the region of large inelasticity) we have

$$|a_j(s)|_{s \gtrsim \Lambda_{SB}^2} = O\left(\frac{\Lambda_{SB}^2}{16\pi v^2}\right).$$  

Equation 71 reveals the promised correlation between the strength of the interactions of $\mathcal{L}_{SB}$ and the mass scale at which they occur. If $\Lambda_{SB} \lesssim 0.5 \text{ TeV}$ then $|a_j| \lesssim 1/10$, which indicates weak scattering that can be analyzed perturbatively, whereas for $\Lambda_{SB} \gtrsim 1 \text{ TeV}$ we find $|a_j| \gtrsim 1/3$, close enough to saturation of unitarity to imply a strong interaction not amenable to perturbation theory. As in the example of the Fermi theory, where the analogous cutoff is provided by W exchange, the most plausible cutoff mechanism for Equation 69 is exchange of quanta from $\mathcal{L}_{SB}$, in which case the cutoff $\Lambda_{SB}$ can be identified with the mass of the quanta

$$\Lambda_{SB} = O(M_{SB}).$$  

It is instructive to see how this works in the minimal Higgs model. In contrast to the preceding discussion, we work in unitary gauge. To lowest order in $g^2$ we decompose the $W^+_L W^-_L \rightarrow Z_L Z_L$ amplitude into gauge sector and symmetry-breaking sector contributions,

$$M(W^+_L W^-_L \rightarrow Z_L Z_L) = M_{\text{gauge}} + M_{\text{SB}}.$$  

$M_{\text{gauge}}$ is due to t- and u-channel W exchange and to the four-point contact interaction contained in $\mathcal{L}_{\text{gauge}}$, Equation 26. Neglecting order $M_W^2/s$ it is

$$M_{\text{gauge}} = \frac{g^2}{4 \cos^2 \theta_W} \frac{M_W^2}{M_W^2} s$$  

or, using Equations 34 and 42,

$$M_{\text{gauge}} = \frac{1}{\rho} \frac{s}{v^2}. $$  

Equation 75 illustrates the "bad" high-energy behavior discussed in Section 1.1 that causes massive vector boson theories to be non-renormalizable, but the reader may have noticed that it is also identical to the low-energy theorem Equation 58! It is the task of $M_{\text{SB}}$ in Equation 73 to cancel the bad high-energy behavior and to ensure renormalizability,
but at the same time if $M_{SB}$ is negligible at low-energy compared to $M_{gauge}$ then we obtain an alternative proof of the low-energy theorems (12).

Both points are illustrated by evaluating $M_{SB}$, which in tree approximation in the minimal model is given by $s$-channel Higgs boson exchange,

$$M_{SB} = M_{Higgs} = \frac{-g^2}{4M_WM_Z} \frac{s^2}{s - m^2_h},$$

(Notice that we do not apply the equivalence theorem here since we must work consistently in unitary gauge, using for instance the $hW_LW_L$ coupling of Equation 52.) For $s \ll m^2_h$ we see that $|M_{SB}| \ll M_{gauge}$, verifying the low-energy theorem in tree approximation: the complete proof (12) requires establishing $|M_{SB}| \ll M_{gauge}$ to all orders in $\lambda_{SB}$ for $s \ll M^2_{SB}$. On the other hand, taking $s \gg m^2_h$ and using $\rho = 1$ as we are entitled to do in the minimal Higgs model, we find Equation 76 does indeed cancel the bad high-energy behavior of Equation 74. The sum for $s \gg m^2_h$ is then

$$M(W^+_LW^-_L \rightarrow Z_LZ_L)|_{s \gg m^2_h} = \frac{g^2 m^2_h}{4 M^2_W},$$

or, using Equations 33, 34, and 67,

$$|a_0(W^+_LW^-_L \rightarrow Z_LZ_L)|_{s \gg m^2_h} = \frac{m^2_h}{16\pi \rho^2} = \frac{\lambda}{8\pi}.$$  

Comparing Equation 78 with Equation 71 verifies Equation 72 for this example: that is, $m_h$, which in the minimal model is identified with $M_{SB}$, does indeed play the role of the cutoff $\Lambda_{SB}$ in Equation 71.

The previous illustration of $\Lambda_{SB} \approx M_{SB}$ was only to lowest order in $\lambda$ and is therefore not trustworthy if $\lambda$ (or $m_h$) is too large. It is difficult to give strong interaction examples since we have not yet learned to solve strong coupling theories. But we can illustrate the validity of Equation 72 for a strong interaction by drawing on our experimental knowledge of hadron physics. In analogy to Equation 69, the extrapolation of the pion low-energy theorem

$$|a_0(\pi^+\pi^- \rightarrow \pi^0\pi^0)| = \frac{s}{16\pi F^2_\pi}$$

would require a cutoff

$$\Lambda_{hadron} \leq 4\sqrt{\pi F_\pi}.$$  

Since hadronic interactions are strong, the inequality should actually be saturated and we expect
\( \Lambda_{\text{hadron}} \approx 4\sqrt{\pi} F_{\pi} \approx 700 \text{ MeV}, \)

so that

\( \Lambda_{\text{hadron}} \approx O(m_{\text{hadron}}), \)

once again verifying Equation 72.

Generically we expect two possibilities, illustrated in Figure 2. Figure 2a illustrates the behavior of the low-energy theorem amplitudes for weakly coupled (presumably Higgs) theories: the growth of the \( W_L W_L \) amplitudes is cut off by narrow resonances at a mass scale well below 1 TeV. For strongly coupled theories, Figure 2b, the amplitudes saturate unitarity and there are broad resonances in the TeV region where the strong interaction sets in.

This picture is the basis for a conservative model (11) of the experimental signal from a strongly coupled \( \mathcal{L}_{\text{SB}} \). In this model the partial wave amplitudes are extrapolated to the unitarity limit without including the resonances that are likely to occur and would considerably enhance the experimental signal. The model then represents a worst case scenario (intuitively unlikely) in which instead of Equation 72 we have \( M_{\text{SB}} \gg \Lambda_{\text{SB}} \). The experimental signals are then the strong interaction \( W_L^+ W_L^- \), \( W_L^+ Z_L \), and \( Z_L Z_L \) continuums, as discussed in Section 6.5.

### 2.4 Unitarity and Fermion Mass Generation

It is also possible to use unitarity to bound the scale of fermion mass generation (21). As already stated in Section 1.2, for a fermion \( f \) of mass \( m_f \) the bound on the corresponding scale of the mass-generating physics is

![Figure 2](image-url)

**Figure 2** Typical behavior of partial wave amplitudes. (a) Weak coupling, i.e. narrow resonance(s) much lighter than 1 TeV, with the amplitude well below the unitarity limit. (b) Strong coupling, i.e. broad resonances at the TeV scale and saturation at the order of the unitarity limit.
where $\xi = 1$ for leptons and $\sqrt{3}$ for quarks. The upper limit is inversely proportional to the fermion mass $m_\ell$ and for the known fermions corresponds to much larger values than the order TeV bound on the symmetry-breaking scale $\Lambda_{SB}$ given in Equation 70. For instance, for the electron the bound is $\sim 6 \times 10^6$ TeV whereas for the top quark with $m_t > 25$ GeV (41) the corresponding bound is $\Lambda_t < 70$ TeV. Only for a heavy quark of mass $m_Q \approx 1.8$ TeV does the upper bound on $\Lambda_Q$ approach the value of the bound on $\Lambda_{SB}$.

In Higgs boson models, the bound on $\Lambda_\ell$ corresponds to the mass of the Higgs boson(s) that makes the dominant contribution to the fermion mass. In dynamical models, like technicolor, the bound applies to the scale at which the corresponding mass-inducing vacuum condensate is generated (the technicolor scale in technicolor models) rather than the scale of the quanta that transmit the condensate to the ordinary fermion sector (the extended technicolor scale in technicolor models). For the three generations of fermions now recognized there is no known natural mechanism that would saturate the bound on $\Lambda_{m_\ell}$—unlike the bound on $\Lambda_{SB}$ that, as discussed in Section 2.3, is naturally saturated if $\mathcal{L}_{SB}$ is strongly interacting. However, a fourth generation with quark masses of order $\sim 2$ TeV would interact strongly with $W_L$ and $Z_L$ (33) and would naturally saturate the bound with $\Lambda_Q \approx \Lambda_{SB} \approx 2$ TeV in a dynamical model with strongly coupled $\mathcal{L}_{SB}$.

Equation 83 is derived by considering the chirality-violating scattering amplitude, $\bar{f}_+ f_+ \rightarrow W_L^+ W_L^-$, where the subscripts on $\bar{f}$ and $f$ denote helicity. The corresponding chirality-violating amplitude would vanish in the chiral limit, $m_\ell = 0$, but for $m_\ell \neq 0$ would exhibit bad high-energy behavior if the fermion mass is not generated by spontaneous symmetry breaking. Just as in Equation 73 we decompose the tree approximation amplitude

\[ M(\bar{f}_+ f_+ \rightarrow W_L^+ W_L^-) = M_{\text{gauge}} + M_{\text{SB}} \]

where $M_{\text{gauge}}$ denotes gauge sector exchange contributions. Here, as in the corresponding discussion of Equation 73 and Section 2.3, we choose the unitary gauge and do not use the equivalence theorem. In particular, $M_{\text{gauge}}$ is the sum of $s$-channel photon and $Z$ boson exchanges and $t$-channel exchange of the weak isodoublet partner $f'$ of the fermion $f$. Neglecting the masses of $f$, $f'$, $W$, and $Z$ relative to the center-of-mass energy $\sqrt{s}$, we obtain for the $J = 0$ partial wave amplitude

\[ a_0(\bar{f}_+ f_+ \rightarrow W_L^+ W_L^-)_{\text{gauge}} \approx \frac{\sqrt{2G_F m_\ell \sqrt{s\xi}}}{16\pi}. \]
For $\sqrt{s} > 16\pi/\sqrt{2G_F m_c \xi}$, $M_{\text{gauge}}$ would violate unitarity; therefore $M_{\text{SB}}$ must cancel the growth of $M_{\text{gauge}}$ at a scale $\Lambda_f$ bounded as in Equation 3.3. For instance, in the minimal Higgs boson model the cancellation is accomplished by the s-channel Higgs boson exchange contribution

$$a_0(\gamma^+ f \rightarrow \gamma \gamma)_{\text{Higgs}} \approx -\frac{\sqrt{2G_F m_t \xi \sqrt{s}}}{16\pi} \frac{s}{s - m_h^2}.$$ 86.

3. TECHNICOLOR

Technicolor (10) is based on the knowledge of strong interaction dynamics acquired from the study of QCD. In technicolor models the symmetry-breaking Lagrangian $\mathcal{L}_{\text{SB}}$ is presumed to be an unbroken, confining, non-Abelian gauge theory, like $\mathcal{L}_{\text{QCD}}$ but with an intrinsic mass scale of order $v/F \approx 246$ GeV/92 MeV $\approx 2700$ times larger than in QCD (see 42 for a review). Technicolor models do not suffer from the technical naturalness problem described in Section 1.4—instability against quantum fluctuations—because they do not have the quadratic divergences of theories with elementary scalar bosons but only the gentle logarithmic divergences of fermions interacting with massless gauge bosons. Viewed in isolation, the technicolor mass scale is just a parameter introduced to fit the weak scale $G_\gamma \sim 2^{-1/2}$, and as such no insight is gained into the deeper puzzle of the hierarchy between $v$ and $M_{\text{Planck}}$. However, a unified theory might some day relate both $v$ and $F_\pi$ to a higher scale, much as $F_\pi$ (or $\Lambda_{\text{QCD}}$) is related to $M_{\text{GUT}}$ in grand unified theories encompassing $SU(3)_{\text{Color}} \times SU(2)_L \times U(1)_Y$ (43; see, however, 44 for a generic difficulty).

Technicolor has a certain solidity because of its grounding in QCD, where the presumed symmetry-breaking mechanism (45) is known experimentally to function as hypothesized. A confining gauge theory is a deeply beautiful mathematical construction; having chosen this mathematics once, nature may well have used it again. On the other hand, we may be guilty of overgeneralizing from limited experience. Experiment will decide once we have constructed colliders that can probe the multi-TeV domain.

Technicolor provides an elegant mechanism to break the electroweak symmetry and generate W and Z masses, but it staggers when confronted with the problem of generating quark and lepton masses. An additional force is needed to transmit the technicolor condensate to the ordinary fermions [e.g. extended technicolor (46)]; it may at the same time induce flavor-changing neutral currents conflicting with experimental upper bounds. Several approaches to this problem are being actively explored (see Section 3.4), but as of this writing no clear solution has emerged.
3.1 The Basic Idea

Following Farhi & Susskind (42), it is amusing and instructive to consider the SU(3)_{Color} × SU(2)_L × U(1)_Y model in the absence of an additional electroweak symmetry-breaking sector,

\[ \mathcal{L}_{\text{SB}} = 0. \]  

Equation 87 is misleading since it suggests that SU(2)_L × U(1)_Y is unbroken, i.e. M_w = M_z = 0. This is wrong—in fact electroweak symmetry would be spontaneously broken by QCD and instead of Equation 87 we should write

\[ \mathcal{L}_{\text{SB}} = \mathcal{L}_{\text{QCD}}. \]  

To be precise consider QCD with two quark flavors, u and d, taken to be massless, m_u = m_d = 0. The theory then has an exact SU(2)_L × SU(2)_R global symmetry that breaks spontaneously to the SU(2)_{L+R} isospin subgroup with three massless Goldstone bosons, the pions. The symmetry-breaking vacuum condensate is the scalar quark bilinear (45)

\[ \langle \bar{u}_L u_R + \bar{d}_L d_R \rangle \neq 0 \]  

with nonvanishing SU(2)_L and U(1)_Y charges. For pure QCD this would be the end of the story, but in the presence of the gauge sector of the electroweak Lagrangian, Equation 26, the Higgs mechanism occurs. The pions disappear from the meson spectrum while the W and Z bosons acquire longitudinal modes and masses,

\[ M_w = \frac{g F_\pi}{2} = 29 \text{ MeV}, \]  

with M_z = M_w/\cos \theta_w assured by the ordinary isospin symmetry (see Section 2.2).

To derive Equation 90 consider the vacuum polarization tensor (with imaginary part proportional to theνν annihilation cross section),

\[ \Pi^{\mu\nu}(p) = - \int d^4x e^{-ipx} \langle T^* J_L^{\mu}(x) J_L^{-\mu}(0) \rangle_0 \]  

where J_L^{\mu} is the weak current

\[ J_L^{\mu}(x) = \bar{\psi}_L(x) \tau^i \gamma^\mu \psi_L(x) \]  

with ψ_L = (u, d)_L the weak quark doublet, τ^i the conventional Pauli
matrices, and $J^+ = (J^-)^* = J^1 + iJ^2$. The pion decay constant is defined by

$$\langle 0 | J^+_L^\mu | \pi^- (p) \rangle = \frac{F_\pi}{\sqrt{2}} p^\mu.$$  \hfill (93)

Gauge invariance implies conservation of the polarization tensor, $p_\mu \Pi^{\mu\nu}(p) = 0$, so that $\Pi^{\mu\nu}(p)$ is determined by a single Lorentz scalar $\Pi(p^2)$,

$$\Pi^{\mu\nu}(p) = i(p^2 g^{\mu\nu} - p^\mu p^\nu) \Pi(p^2).$$  \hfill (94)

The pion pole contribution is

$$\Pi^{\mu\nu}(p) = -\frac{i}{p^2} p^\mu p^\nu \frac{F_\pi^2}{2} + \cdots,$$  \hfill (95)

where the factor $1/p^2$ is the propagator of the massless pion. Consequently the scalar function $\Pi(p^2)$ acquires a singularity at $p^2 = 0$,

$$\Pi(p^2) = \frac{F_\pi^2}{2} \frac{1}{p^2} + \cdots.$$  \hfill (96)

The weak polarization tensor is of interest not because of any imminent proposal to build a $\bar{v}v$ collider but because it controls the quantum corrections to the W and Z propagators and therefore the W and Z masses. Choosing the Landau gauge, the lowest order W propagator is

$$D_{0\nu}^{\mu\nu}(p) = -i \left( g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} \right) \frac{1}{p^2}.$$  \hfill (97)

In higher orders we sum the geometric series

$$D^{\mu\nu} = D_0^{\mu\nu} + \frac{g^2}{2} D_0^{\alpha\beta} \Pi_{\alpha\beta} D_0^{\delta\gamma} + \frac{g^4}{4} D_0^{\mu\nu} \Pi_{\alpha\beta} D_0^{\delta\gamma} \Pi_{\rho\sigma} D^{\delta\gamma} + \cdots$$  \hfill (98)

The pole in $\Pi(p^2)$ then induces a singularity at nonvanishing $p^2$ in the W propagator,

$$\frac{1}{p^2 [1 - \frac{1}{2} g^2 \Pi(p^2)]} = \frac{1}{p^2 - \frac{1}{2} g^2 F_\pi^2},$$  \hfill (99)

which results in the W mass promised in Equation 90. (Contributions to $\Pi$ not singular at $p^2 = 0$ are absorbed in the wave function renormalization or induce finite higher order corrections to the W propagator.)
In fact this derivation of Equation 99 is a general derivation of the Higgs mechanism, more general than the one sketched in Section 1.3. It exhibits the essential features of the mechanism: a massless spin-zero particle coupled to the gauge current gives a mass to the associated gauge boson. In particular, it is not necessary that there be a physical Higgs particle $h$ with vacuum condensate $v$. Comparing Equations 34 and 99 we see that the role of the vacuum condensate $v$ in the Higgs boson model is played more generally by the coupling $F_\tau$ of the Goldstone boson to the gauge current, Equation 93. [Of course in the Higgs boson model the two are one and the same—a fact familiar to students of the sigma model (22).]

The QCD spectrum contains no scalar meson that is a strong candidate to identify with the physical Higgs boson in the world with $\mathcal{L}_{SB} = \mathcal{L}_{QCD}$. There must, however, be $J = 0$ states with the correct couplings to ensure good high-energy behavior of the $W_L W_L$ scattering amplitudes discussed in Sections 2.3 and 2.4, but they may be, and in this case probably are, predominantly broad resonances and/or multiparticle states. In QCD chiral symmetry breaking is induced by the condensate of the quark bilinear field, Equation 89, rather than a scalar boson condensate, and good high-energy behavior is assured by the hadron continuum that is dual (47) to the $I = J = 0$ quark-antiquark continuum.

To turn this example into a model that correctly reproduces the $W$ and $Z$ masses, we let $\mathcal{L}_{SB}$ be a confining gauge theory with a spontaneously broken $SU(2)_L \times SU(2)_R \to SU(2)_L + R$ symmetry and with Goldstone boson–gauge current coupling (defined as in Equation 93 with $\pi$ replaced by the Goldstone boson $W$) given by

$$ F_\pi^{TC} = v = 246 \text{ GeV}. $$

Our experience from QCD is most reliable if $\mathcal{L}_{SB}$ has $SU(N_{TC})$ gauge interactions. In that case we are most confident of the hypothesized global symmetry breaking, and in the large $N_{TC}$ approximation we can naively estimate the technimeson masses. Technimeson masses are to leading order independent of $N_{TC}$ (48) whereas $F_\pi^{TC}$ is proportional to $\sqrt{N_{TC}}$. [The $N_{TC}$ dependence of $F_\pi^{TC}$ (or $F_\pi^{QCD}$) is easily deduced for Equation 93, since the electroweak gauge current is the sum of $N_{TC}$ (or $N_{QCD} = 3$) color diagonal terms, while the color singlet Goldstone boson wave function is a sum of $N_{TC}$ color diagonal $\bar{q}_{TC} q_{TC}$ pairs normalized by a factor $1/\sqrt{N_{TC}}$ (or $1/\sqrt{3}$ for QCD).] Since $F_\pi^{TC}$ and $F_\pi^{QCD}$ are normalized to their experimental values, 246 GeV and 92 MeV respectively, the result is

$$ \frac{M_{\text{Technimeson}}}{M_{\text{Ordinary meson}}} \approx \sqrt{\frac{3}{N_{TC}}} \frac{F_\pi^{TC}}{F_\pi}. $$

For example, for $SU(4)$ technicolor the technirho mass is estimated at
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\[ m_{\rho T} = \sqrt{\frac{3}{4}} F_{\pi}^{TC} m_\rho = 1.8 \text{ TeV}. \]

Since meson widths scale like \( N_{TC}^{-1} \) for large \( N_{TC} \) (48), the corresponding width is

\[ \Gamma_{\rho T} = \frac{3}{4} \frac{m_{\rho T}}{m_\rho} \Gamma_\rho = 260 \text{ GeV}. \]

As discussed in Section 5, a \( \sqrt{s} = 40 \text{ TeV} \) proton-proton collider with luminosity \( L = 10^{33} \text{ cm}^{-2} \text{s}^{-1} \) is a minimal machine for observing the signal of the SU(4) technirho.

Before considering the formidable difficulties raised by fermion masses, I want to address a question that may trouble the reader: what is the role of the pion in symmetry breaking as it actually occurs? The answer is that both the QCD and \( \mathcal{L}_{SB} \) condensates contribute to \( v^2 = F_\pi^2 + F_{SB}^2 \). The pion and the Goldstone triplet \( \omega \) mix via the gauge current vacuum polarization, with mixing angle given by \( \sin \theta_{\pi-\omega} = F_\pi/v = 1/2700 \). Therefore \( W_L \) has a small component in its wave function from QCD and the physical pion has a small component from \( \mathcal{L}_{SB} \). It would be very interesting to test \( W_L-\pi \) mixing directly, but the effect seems too small to isolate.

3.2 Quark and Lepton Masses

The basic technicolor mechanism for electroweak symmetry breaking and gauge boson generation is simple and elegant enough to have a chance to be right, but the same claim cannot be made for any attempt until now to explain fermion mass generation in the technicolor context. It is easy to see that because of chiral symmetry, quarks and leptons do not acquire masses from the simple technicolor mechanism described in Section 3.1. By definition chiral transformations act differently on the left and right projections, Equation 20, of a fermion \( \psi \),

\[
\delta \psi_L \rightarrow e^{i e_L} \psi_L,
\delta \psi_R \rightarrow e^{i e_R} \psi_R,
\]

where \( e_L \neq e_R \) are constants. Since gauge interactions only couple \( \psi_L \) to \( \psi_L \) and \( \psi_R \) to \( \psi_R \),

\[
\bar{\psi} \gamma^\mu \psi = \bar{\psi}_L \gamma^\mu \psi_L + \bar{\psi}_R \gamma^\mu \psi_R,
\bar{\psi} \gamma^\mu \gamma_5 \psi = \bar{\psi}_L \gamma^\mu \gamma_5 \psi_L + \bar{\psi}_R \gamma^\mu \gamma_5 \psi_R,
\]

they are invariant under chiral transformations. Fermion mass terms couple left to right chirality, Equation 21, and therefore are not invariant.
under Equation 104. In the basic technicolor model, quarks and leptons only have $SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$ gauge interactions, so chirality is unbroken and the quarks and leptons are massless.

To generate quark and lepton masses, one must enlarge the basic model by adding an additional force, analogous to the Yukawa force, Equation 22, of Higgs boson models, that transmits the chirality-violating vacuum condensate to the quarks and leptons. The technicolor vacuum condensate is the chirality-violating techniquark bilinear,

$$\langle q_L^{TC} q_R^{TC} \rangle_0 \neq 0,$$

like the QCD condensate, Equation 89. The most economical extension is then to introduce a "sideways" force between the techniquarks and the ordinary quarks and leptons that can transmit the chirality-violating condensate of the former to the latter.

Without specifying the precise nature or origin of this sideways force we can represent it at low energy by a four-fermion interaction that, perhaps after Fierz rearrangement, includes terms of the structure

$$\mathcal{L}_{\text{sideways}} = \frac{1}{\Lambda_{\text{sideways}}^2} (q_L q_R) (q_L^{TC} q_R^{TC}) + \text{hermitian conjugate}. \quad 107.$$

With appropriate choices of the $G_{TC} \times SU(3)_{\text{color}} \times SU(2)_L \times U(1)_Y$ charges of the techniquarks, Equation 107 can be made consistent with the four gauge invariances. The factor $1/\Lambda_{\text{sideways}}^2$ is analogous to $G_F$ in the effective Fermi Lagrangian with $\Lambda_{\text{sideways}}$ the arbitrary (for now) mass scale of the sideways interaction. Equations 106 and 107 together imply a mass for an ordinary quark (or lepton) of

$$m_q = \frac{\langle q^{TC} q^{TC} \rangle_0}{\Lambda_{\text{sideways}}^2}. \quad 108.$$

Extended technicolor (ETC) is a particular realization of this idea in which the technicolor group is embedded in a larger gauge symmetry, $G_{TC} \subset G_{\text{ETC}}$ (46). Irreducible representations of $G_{\text{ETC}}$ are taken to include both techniquarks, $q^{TC}$, and ordinary quarks and leptons, denoted generically as $q$. Schematically, an irreducible representation is then $(q^{TC}, q^{TC}, \ldots, q, q', \ldots)$. The technicolor gauge bosons are the subset acting exclusively in the $q^{TC}$ sector, while the ETC gauge bosons that connect $q^{TC}$'s to $q$'s mediate the sideways force. By hypothesis $G_{\text{ETC}}$ breaks spontaneously to $G_{TC}$ and the sideways gauge bosons acquire a mass $M_{\text{ETC}}$. Then Equation 107 emerges as a low-energy approximation to sideways gauge boson exchange for $E \ll M_{\text{ETC}}$. Just as $G_F \approx g^2/M_W^2$, we relate $\Lambda_{\text{sideways}}$ in Equation 107 to the ETC coupling constant and mass,
The sideways scale can be estimated from Equation 108 by first making a crude estimate of the technicolor condensate based on the value of the QCD condensate. From current algebra (49) we have

\[
\frac{1}{\Lambda_{\text{sideways}}^2} \approx \frac{g_{\text{ETC}}^2}{M_{\text{ETC}}^2}.
\]

where \( m_{u,d} \) are the current quark masses, \( m_u + m_d \approx 12 \text{ MeV} \) (50). Using isospin symmetry, \( \langle ar{u}u \rangle_0 = \langle ar{d}d \rangle_0 \), and the experimental values for \( F_\pi \) and \( m_N \), we find \( \langle ar{u}u \rangle_0 \approx 17 F_\pi^3 \). Therefore an SU(3)_TC technicolor theory with one techniquark SU(2)_L doublet would have

\[
\langle q^{\text{TC}} q^{\text{TC}} \rangle_0 \approx 17 v^3.
\]

More generally for SU(\( N_{\text{TC}} \)) and \( N_{\text{doublet}} \) many weak doublets, we might expect

\[
\langle q^{\text{TC}} q^{\text{TC}} \rangle_0 \approx \frac{1}{N_{\text{doublet}} \sqrt{\frac{3}{N_{\text{TC}}}}} 17 v^3.
\]

For example, consider \( N_{\text{TC}} = 4 \) and \( N_{\text{doublet}} = 4 \), corresponding to three techniquark doublets (one per ordinary color) and one technilepton doublet. Then combining Equations 108 and 112 we have

\[
m_q = \frac{0.06 \text{ TeV}^3}{\Lambda_{\text{sideways}}^2}.
\]

For instance, for the charm quark, \( m_c \approx 1 \text{ GeV} \), we require \( \Lambda_{\text{sideways}} \approx 8 \text{ TeV} \). The electron requires a much larger scale, \( \sim 360 \text{ TeV} \), while the lower bound on the top quark mass (41), \( m_t > 25 \text{ GeV} \), implies a severe upper bound, <1.5 TeV, on the corresponding sideways scale.

### 3.3 Problems

The description of extended technicolor has been deliberately vague because there are many possible variants, most rather complicated, and none very attractive or successful. For a description of some of the possibilities and some simple, schematic examples, the reader can consult the review of Farhi & Susskind (42). The models are defined by specifying the irreducible ETC representations and the pattern of ETC symmetry breakdown. For instance, if an ETC representation contains quarks of different flavors and equal electric charge, then the extended technicolor interactions include horizontal (i.e. generation-changing) interactions and the theory has dangerous flavor-changing neutral currents. In such models
the scale of $\Lambda_{\text{sideways}}$ needed to generate the strange quark mass is more than an order of magnitude smaller than the upper bound required by the $K_L-K_S$ mass difference (51).

The large range of quark and charged-lepton masses, $m_t/m_e > 5 \times 10^4$, requires a variation in $\Lambda_{\text{sideways}}$ of two orders of magnitude, which demonstrates the inadequacy of a discussion based on a single scale. One possibility is to place all like-charge quarks (or leptons) in a single ETC representation and to rely on ETC symmetry breaking to generate a hierarchy of sideways gauge boson masses that would be inversely reflected in the hierarchy of fermion generations. To be viable this approach requires a (now missing) ingredient to suppress the flavor-changing neutral currents of the horizontal gauge bosons. Another approach is to assume that only the heaviest generation acquires mass directly by the ETC mechanism, with lighter generations acquiring mass in a cascade of radiative corrections originating from the heavy generation—a dynamical version of the scalar-based models of Fritzsch (52).

Even models without flavor-changing neutral currents in lowest order face the problem in the next order of perturbation theory, from an ETC loop amplitude. The analogous amplitude, with ETC bosons and techniquarks replaced by $W$ bosons and charm quarks, saturates the $K_L-K_S$ mass difference (53), leaving little room for additional contributions. The ETC contribution is proportional to $g_{\text{ETC}}^2/M_{\text{ETC}}^2 \approx g_{\text{ETC}}^2/\Lambda_{\text{sideways}}^2$, which results in a bound (51) $\Lambda_{\text{sideways}}/g_{\text{ETC}} > 80 \text{ TeV}$. For plausible values of $g_{\text{ETC}}$, say $g_{\text{ETC}} \gtrsim 1$ or $\alpha_{\text{ETC}} = g_{\text{ETC}}^2/4\pi > 0.1$, this bound exceeds the value of $\Lambda_{\text{sideways}}$ computed from Equation 113 by a factor of four for a strange quark mass of $m_s = 150$ MeV. Given the many uncertainties in the estimate, the discrepancy is worrisome but not definitive.

Comparable sensitivity may be possible in $B\rightarrow\bar{B}$ mixing. For instance, if the ETC loop contribution to $B_s\rightarrow\bar{B}_s$ mixing can be bounded from above by the value of the standard model loop contribution, then the result of (54) reexpressed in terms of $\Lambda_{\text{sideways}}$ is

$$\frac{\Lambda_{\text{sideways}}^2}{g_{\text{ETC}}^2} > \frac{v^2}{16\pi\alpha} \frac{1}{V_{ts}^2} \left(\frac{1.7 \text{ GeV}}{m_t}\right)^2.$$  \hspace{1cm} (114.)

Taking the KM matrix element $V_{ts}$ at the center of the allowed range (55), $|V_{ts}| \approx 0.044$, and assuming $g_{\text{ETC}} \gtrsim 1$, we obtain the bound

$$\Lambda_{\text{sideways}} > 9 \text{ TeV} \left(\frac{37 \text{ GeV}}{m_t}\right),$$  \hspace{1cm} (115.)

to be compared with $\Lambda_{\text{sideways}} \approx 4 \text{ TeV}$ for $m_b = 5 \text{ GeV}$ from Equation 113:.

Two other potentially serious problems are identified in (51). First, in
the absence of a principle to suppress the phases of the ETC couplings, $CP$ violation in the $K\bar{K}$ system from the ETC box graph would be four orders of magnitude larger than what is observed (for a discussion of $CP$ violation in ETC theories, see 56). Second, pseudo-Goldstone bosons, discussed below, can also mediate dangerous flavor-changing interactions in lowest order.

Because of their large flavor symmetries, ETC models typically contain many more than the three Goldstone bosons needed to give mass to the $W$ and $Z$. To avoid potentially embarrassing long-range forces, models are designed so that the corresponding global symmetries are explicitly broken by the ETC, $SU(3)\text{Color}$, and/or electroweak interactions (10, 46, 57–59). The would-be Goldstone bosons then acquire masses, becoming “pseudo-Goldstone bosons.” Masses acquired in this way are model dependent, but the lightest tend to be from a few GeV to some tens of GeV. Decay signatures are also model dependent. In a recent assessment of the experimental situation Eichten et al (60) concluded that existing data do not preclude the existence of pseudo-Goldstone bosons.

All these difficulties are daunting. In particular, the problem of flavor-changing neutral currents requires a qualitative innovation, like the GIM mechanism (61) that suppresses $SU(2)_L \times U(1)$ flavor-changing neutral currents.

### 3.4 Solutions?

Several new approaches are being explored in response to the problems sketched in the previous section. None are yet demonstrably successful but all are interesting and worthy of more work. One approach maintains the conventional technicolor and ETC framework but supposes special dynamics—slow running of the technicolor coupling constant (62, 63). This changes the relationship between the ETC condensate and the weak scale $G_F^{-1/2}$ to mitigate the flavor-changing neutral current and pseudo-Goldstone boson problems. Other approaches involve more substantial revision of the basic framework. In ultracolor theories (64), a light ($< 1$ TeV) Higgs boson emerges as a bound state of confining ultracolor dynamics. At the cost of tuning a parameter, the ultracolor mass scale can be much greater than the natural $\sim 2$ TeV scale of technicolor, also alleviating the flavor-changing neutral current and pseudo-Goldstone boson problems.

The most far reaching proposal is that ordinary quarks and leptons are bound states of (more?) elementary fermions known generically as preons (65), and that $SU(2)_L \times U(1)_Y$ is broken by preon condensates (66). In other words, preons are identified with techniquarks or ultraquarks. Quark and lepton masses are then the properties of a bound-state spectrum, and
the large global symmetries that spawned the pseudo-Goldstone bosons can be eliminated. However the compositeness scale cannot be much greater than the weak scale, $G_F^{1/2}$, so that flavor-changing neutral currents remain a concern.

These ideas are described briefly below. See (67) for a review with additional references.

3.4.1 STAGNANT MODELS

The difficulties of extended technicolor have their origin in the property—known for QCD and assumed for technicolor—that the energy scales of confinement and chiral symmetry breaking are not very different. This has the consequence that the scale of the symmetry-breaking condensate, $(\langle q\bar{q}\rangle_0)^{1/3}$ and the parameter $\Lambda$ at which the coupling becomes strong are of the same order. For instance, from Equation 110 we deduced $(\langle u\bar{u}\rangle_0)^{1/3} \approx 200$ MeV, which is well within the range for $\Lambda_{\text{QCD}}$. This led in turn to the estimate of Equation 112 and the attendant problems of flavor-changing neutral currents and pseudo-Goldstone bosons. In slowly running theories the chiral condensate may survive to energies much greater than the confinement scale, resulting in an enhanced value for the condensate.

Before explaining the preceding sentence, let us first explore its consequences. Suppose the right side of Equation 112 were enhanced by an additional factor $R \gg 1$:

$$\langle q^{TC}\bar{q}^{TC}\rangle_0 \approx R \frac{1}{N_{\text{doublet}}} \sqrt{\frac{3}{N_{TC}}} 17v^3.$$  \hspace{1cm} 116.

The scale $\Lambda_{\text{sideways}}$ required for fermion masses, Equation 108, would be increased by $\sqrt{R}$ and flavor-changing neutral currents suppressed by $1/R$. The masses of the color singlet pseudo-Goldstone bosons—the lightest and potentially most embarrassing—are proportional to (10)

$$M_{\text{color singlet PGB}}^2 \propto \frac{\langle q^{TC}\bar{q}^{TC}\rangle^2}{v^2\Lambda_{\text{sideways}}^2}$$  \hspace{1cm} 117.

so that the right side would be enhanced by one power of $R$ relative to its value in a nonstagnant technicolor theory.

Now we return to the relationship between the running of the coupling constant and the value of the chiral condensate. Naively the condensate is given in lowest order by the closed fermion loop [or, more carefully, from the gap equation (62, 63)],

$$\langle q^{TC}\bar{q}^{TC}\rangle \propto \int d^4p \frac{1}{p + \Sigma(p)} \propto \int dp p \Sigma(p);$$  \hspace{1cm} 118.

whereas from spectral function sum rules (68)
\[ v^2 \propto \int \frac{dp}{p} [\Sigma(p)]^2, \]

where \( \Sigma(p) \) is the technifermion self-energy. Therefore the techniquark condensate is more sensitive to the high-energy behavior of \( \Sigma(p) \) than is \( v = F_\pi^{TC} = (\sqrt{2}G_F)^{-1/2} \). The renormalization group implies that \( \Sigma(p) \) varies slowly (logarithmically) until it is damped like \( 1/p^2 \) at energies above the chiral symmetry-breaking scale (69). In stagnant theories the coupling constant runs more slowly so that the damping is deferred to higher energies, resulting in enhancement of \( \langle q^{TC}q^{TC} \rangle \) relative to \( v^2 \).

Though originally proposed in the context of nonasymptotically free theories with finite fixed points (62), the scenario has recently been applied to asymptotically free gauge theories with small one-loop \( \beta \) functions (63). Numerical studies in the latter case exhibit models in which the technifermion condensate can be enhanced by two orders of magnitude relative to naive estimates (see the last paper cited in 63).

3.4.2 COMPOSITE HIGGS BOSON: ULTRACOLOR Ultracolor theories (64) are confining gauge theories that break \( SU(2)_L \times U(1)_Y \) indirectly by spawning a pseudo-Goldstone boson sector that mimics the sub-TeV minimal Higgs boson model. This is accomplished at the cost of adding a second hypercharge interaction \( U(1)_A \) to \( SU(2)_L \times U(1)_Y \) with coupling constant \( g_A \), which in one version is tuned to equal

\[ g_A^2 = (1 - \varepsilon) \frac{3g^2 + g'A^2}{5}. \]

Here \( g \) and \( g' \) are the \( SU(2)_L \) and \( U(1)_Y \) coupling constants and \( \varepsilon \) is tuned to obey \( 0 < \varepsilon \ll 1 \). Then the \( W \) mass is small relative to the ultracolor condensate scale \( F_\pi^{UC} \),

\[ M_w = \frac{\varepsilon}{2} g F_\pi^{UC}, \]

while the mass of the new gauge boson is unsuppressed,

\[ M_{Z_A} = \frac{1}{2} g_A F_\pi^{UC}. \]

Consequently \( M_{Z_A} \gg M_w \) and since \( F_\pi^{UC}/v = \varepsilon^{-1} \ll 1 \) the scale of the ultracolor spectrum can be much greater than the \( O(2) \) TeV scale of technicolor.

As in technicolor models, ultracolor interactions have a global symmetry \( G \) that breaks to a subgroup \( H \) at the scale \( F_\pi^{UC} \). However, unlike technicolor models, \( SU(2)_L \times U(1)_Y \) is not broken by the ultracolor condensate but is a subgroup of the unbroken group \( H \). The strategy is to break
SU(2)_L \times U(1)_Y by weak quantum fluctuations but this cannot be accomplished by the SU(2)_L \times U(1)_Y interactions themselves since they alone would align the ground state to preserve the SU(2)_L \times U(1)_Y symmetry (57). Enlarging the electroweak gauge group to SU(2)_L \times U(1)_Y \times U(1)_A with g_A tuned as above causes the vacuum condensate to point slightly away from the SU(2)_L \times U(1)_Y preserving direction. It is possible to construct models containing a complex scalar SU(2)_L doublet of Goldstone bosons—precisely the minimal Higgs boson doublet described in Section 1.4—such that three become W^±_L and Z_L in the usual way while the fourth is made a massive pseudo-Goldstone boson by the U(1)_A interactions. The low-energy symmetries dictate that in the approximation \varepsilon \ll 1 this pseudo-Goldstone boson have the same low-energy interactions with itself and with the gauge bosons as the standard Higgs boson of the minimal model. Given a particular model, the mass is calculable.

In the limit \varepsilon \rightarrow 0 and F^U_C \rightarrow \infty with \varepsilon F^U_C held fixed, the model is indistinguishable from the minimal Higgs boson model. On the other hand, for 0 < 1 - \varepsilon \ll 1 it is indistinguishable from ordinary technicolor, since for 1 - \varepsilon \ll g_A arbitrarily small, the effects of the (now very light) Z_A boson would be undetectable and Equation 121 would require F^U_C = v.

Fermion masses can be incorporated by enlarging the ultracolor group to an extended ultracolor group, EUC (70). Then fermion masses are given by

\[ m_f \propto \left( \frac{F^U_C}{\Lambda_{\text{sideways}}} \right)^2 v. \]

Compared to the ETC estimate, proportional to \nu^2/\Lambda_{\text{sideways}}, the \Lambda_{\text{sideways}} scale is raised by \nu/\nu = \varepsilon^{-1} \gg 1, so that flavor-changing neutral currents are suppressed. Since pseudo-Goldstone boson masses receive contributions proportional to \nu F^U_C, they are elevated above the tens of GeV region predicted by ETC models (71).

Despite these phenomenological advantages, existing ultracolor models are implausible because of the required tuning. Perhaps the basic idea can eventually find a more natural setting.

3.4.3 COMPOSITE QUARKS AND LEPTONS The most profound possibility is that electroweak symmetry breaking is a consequence of forces that bind preons into quarks and leptons. The family structure and masses of quarks and leptons are then properties of a bound-state spectrum computable from the preonic theory (for reviews with additional references, see 67 and 72). Therefore the large global symmetries and attendant pseudo-Goldstone bosons of extended technicolor theories are eliminated. Elec-
ELECTROWEAK SYMMETRY BREAKING

Electroweak symmetry breaking is due to a condensate of preonic fields. If the preonic theory is itself a confining gauge theory, the symmetry-breaking condensate is generated just as in the basic technicolor models. Then the compositeness scale is of order $F/v$ times the 1 fermi $= 10^{-13}$ cm scale of QCD or about $5 \times 10^{-17}$ cm, at the edge of the scales probed by existing experiments. If this idea is correct, quark and lepton substructure will be discovered in the next decade if not sooner!

One manifestation of quark and lepton compositeness would be non-point-like form factors that can be probed in deep inelastic lepton-nucleon scattering or in electron-positron annihilation (73). The conventional quantities $\Lambda^\text{QED}$ used to parameterize deviations from quantum electrodynamics in Bhabha scattering can also be regarded as parameterizations of form factors effects. Thus the present experimental limit (74) $\Lambda^\text{QED} > 500$ GeV corresponds to a distance scale of $(500 \text{ GeV})^{-1} \approx 5 \times 10^{-17}$ cm. Just as nuclei interact by nucleon exchange and nucleons interact by quark exchange, quarks and leptons would interact by preon exchange, giving rise to anomalous four-fermion interactions such as (75)

$$L_4 = \frac{1}{\Lambda^2_{\text{composite}}} (\bar{q}q)(\bar{q}q), \quad 124.$$ 

where the Lorentz structure is suppressed. $\Lambda^{-1}_{\text{composite}}$ is proportional to the bound-state size. Preon interchange is inevitable for diagonal interactions (i.e. all four $q$’s identical) and will also occur in off-diagonal processes when the bound states have preon constituents in common. Limits from Bhabha scattering, $e^+e^- \to e^+e^-$, exclude anomalous interactions down to $\sim 2 \times 10^{-17}$ cm (74, 76), very near the predicted scale. At the Stanford Linear Collider (SLC) and Large Electron-Positron Collider (LEP), Bhabha scattering will be probed to distances smaller by a factor of $\sim 5$, and at the SSC quark-quark scattering can be probed to a scale $\sim 30$ times smaller (77). Definitive tests will be possible even allowing for an order-of-magnitude uncertainty in the estimated size of the effects.

Off-diagonal interactions of the form of Equation 124 could include flavor-changing neutral currents that would be experimentally unacceptable for $\Lambda_{\text{composite}}$ as small as the envisioned few-TeV scale. Chivukula and Georgi have exhibited a preonic GIM mechanism suppressing flavor-changing neutral currents to acceptable levels that follows if the preon Lagrangian has the same global symmetry, $[SU(3) \times U(1)]^5$, that is respected by the gauge interactions of the standard model (66). The same mechanism would also suppress flavor-changing neutral currents in ETC models. It implies deviations from standard model predictions for a variety of $K$ and $B$ meson weak interaction phenomena.
If quarks and leptons are bound states of size $1/\Lambda_{\text{composite}} < 10^{-16}$ cm, their Compton wavelengths are much larger than their radii, very different from the light hadrons, for which $m^{-1} \approx R$. The challenge to find and solve such a theory is enormous. It is, however, extremely encouraging that experiment will soon give a clear pronouncement on the viability of composite models at the technicolor scale.

4. SUPERSYMMETRY

In additional to the minimal model sketched in Section 1.3, many other Higgs boson models can be devised to give the W and Z bosons mass consistent with $\rho = 1$. If the Higgs potential has a custodial SU(2) symmetry, then $\rho = 1$ holds to all orders in the Higgs sector interactions. For example, this can be accomplished with arbitrarily many complex doublet Higgs fields. The two complex-doublet model can contain an additional “Pececi-Quinn” symmetry that would explain the absence of strong CP violation otherwise expected from nonperturbative effects in QCD (78). The model predicts a very weakly coupled pseudo-Goldstone boson, the axion (79), which is the subject of great experimental interest.

Larger representations than doublets are also possible. For instance, a model consisting of three real triplets can be constructed with a custodial SU(2) symmetry (80). However, a single complex triplet (or two real triplets) that could give neutrinos a Majorana mass (81) cannot be made custodial SU(2) symmetric. The triplet must have a small vacuum expectation value in order to avoid an unacceptable deviation from $\rho = 1$. In such models the dominant contributions to the W and Z masses could come, for example, from a Higgs doublet. But even if the triplet vacuum expectation value is small enough to make negligible contributions to $M_W$ and $M_Z$, there are still strong constraints from custodial SU(2) violating one-loop corrections. In particular, independent of the value of the triplet condensate the mass of the doubly charged member of the triplet must be less than 250 GeV to be consistent with the measured value of $\rho$ (82).

While Higgs boson models exist that are satisfactory as renormalizable field theoretic descriptions of the known experimental facts, they are unsatisfactory when viewed from a deeper perspective. With only one known exception, the discussion of Section 1.4 applies to all models with elementary Higgs bosons: in the absence of cancellation all have quadratic divergences that destabilize the electroweak scale, requiring finely tuned subtractions in each order of perturbation theory. The exception is provided by supersymmetric theories (9) in which the quadratic divergences cancel between bosonic and fermionic loops, with only gentle logarithmic diver-
gence remaining that do not require fine-tuning (83). The stabilizing mechanism is the combined consequence of chiral symmetry and supersymmetry. As discussed in Section 3.2, chiral symmetric interactions at high energy cannot communicate their large intrinsic scales to fermion masses. Supersymmetry links scalar and fermion masses so that scalar masses are also protected by the chiral symmetry of the fermionic superpartners.

Like the possibility of composite fermions discussed in Section 3, the discovery of supersymmetry would be an immense physics bonanza, with implications reaching far beyond the problem of electroweak symmetry breaking. Supersymmetry is a profound generalization of the notion of space-time symmetry, enlarging the Poincaré algebra with spinor transformations that connect fermions and bosons. Global supersymmetry at the weak interaction scale could be the low-energy remnant of local supersymmetry ("supergravity") or of superstrings at the Planck scale, \(10^{19}\) GeV. Both supergravity and superstrings would unify gravity with the other forces, and superstring theories offer in addition the intoxicating vision of a true Theory of Everything—a finite theory in which all properties of matter are determined from a single parameter, the Planck mass.

The hierarchy stability problem is the clue that supersymmetry may be discovered at the few hundred GeV scale of electroweak interactions. The quadratic divergences of the Higgs sector are cured by cancellation of loop contributions between the ordinary particles and their superpartners, e.g. between contributions to the Higgs self-energy from the W boson and its spin-1/2 partner the wino, \(\tilde{W}\). Since supersymmetry is not an exact symmetry, the cancellation is not exact but leaves a finite contribution, proportional to the difference of the masses squared of particles and superpartners. The cutoff introduced in Section 1.4 is then replaced by the scale of supersymmetry breaking. Calculations show that if the supersymmetry-breaking scale is much larger than 1 TeV then fine-tuning (to a few percent or less) again becomes necessary (84). Therefore if supersymmetry is to solve the hierarchy stability problem, superpartner masses cannot be much greater than 1 TeV.

From the perspective introduced in Section 2, supersymmetric theories are typically (though not necessarily) weakly coupled, even if the supersymmetry-breaking scale is as large as 1 TeV. As discussed below, the supersymmetric models under study typically require the Higgs bosons responsible for the W and Z masses to have Higgs sector coupling constants of the order of the electroweak gauge coupling strengths. Though such models can have heavy Higgs bosons, those that dominate the W and Z masses tend to be light and to resemble the light Higgs boson of the Weinberg-Salam model.
4.1 Minimal Supersymmetric Extension of the Standard Model

The minimal supersymmetric extension of the standard model (85) is useful as a concrete illustration of electroweak symmetry breaking in supersymmetric models. Unlike more general supersymmetric models (e.g. the simplest "string-inspired," nonminimal possibility discussed in Section 4.2) the minimal extension makes rather definite predictions about the mass spectrum of Higgs bosons. Most strikingly it predicts that the lightest Higgs boson $h$ cannot be heavier than the $Z$ boson, $m_h \leq M_Z$. This lack of ambiguity is a nice pedagogical feature of the minimal extension, but the reader should not be lulled into thinking that it is a general property of supersymmetric models.

Renormalizable models of the simplest and phenomenologically most viable form of supersymmetry, i.e. $N = 1$ supersymmetry, have two kinds of supermultiplets. Chiral supermultiplets contain scalar and fermionic partners. They include the matter fields (leptons with their slepton partners and quarks with their squark partners) and the scalar Higgs fields with their fermionic higgsino partners. In gauge theories there are also vector supermultiplets consisting of the gauge bosons and their fermionic gaugino partners. The reader can consult specialized reviews of the theoretical concepts (86) and phenomenological implications (87) of supersymmetry.

It is instructive to consider first the form of the scalar potential that causes $SU(2)_L \times U(1)_Y$ symmetry breaking. The constraints of supersymmetry and gauge invariance dictate the form (86)

$$V = F_i^* F_i + \frac{1}{2} (D_a D_a + D_Y D_Y).$$

The index $i$ is summed over all chiral supermultiplets $\Phi_i$ and $a = 1, 2, 3$ is summed over the three degrees of freedom of $SU(2)_L$, $D_Y$ being connected with $U(1)_Y$. The $F_i$ are related to the superpotential $W$ by

$$F_i = \partial W/\partial \phi_i$$

where $\phi_i$ is a scalar component of chiral supermultiplet $\Phi_i$. The superpotential $W$ is in turn the sum of terms cubic, quadratic, linear, and constant in the chiral superfields $\Phi_i$ with constraints from supersymmetry and gauge invariance as discussed below. The $D_a$ terms, summed over $a = 1, 2, 3$ for the three $SU(2)_L$ generators, are given by

$$D_a = g \phi_i^* T^a_{ij} \phi_j$$

where the $T^a_{ij}$ is the (reducible) $SU(2)_L$ matrix representation for all the $\phi_i$, and $g$ is the $SU(2)_L$ gauge coupling constant. $D_Y$ is the analogous $U(1)_Y$ term.
Two important properties of supersymmetric models follow from the form of the scalar potential, Equation 125. The first is completely general: at least two complex doublet Higgs fields are needed for a viable model. The second is a property of the minimal extension, which has just two complex Higgs doublets, and is not completely general: the Higgs sector of the minimal extension is weakly coupled. That is, the generic coupling strength $\lambda_{SB}$ introduced in Sections 1 and 2 is of electroweak strength, $g^2$ and $g'^2$, in the minimal extension. Though unpopular with model builders, one can imagine strong contributions to $\lambda_{SB}$ in nonminimal models. As a consequence of the arguments presented in Section 2, instead of $m_h \leq M_Z$ we could then have $m_h \gg M_Z$.

To understand these remarks it is necessary to know more about the superpotential $W$ from which the $F$ terms in the scalar potential are determined (86). Supersymmetry may be softly broken by terms in the Lagrangian with operator dimension three or less (e.g. $m\phi^3$ or $m^2\phi^2$), but to avoid quadratic divergences the operator dimension-four interactions (e.g. $\phi^4$) must be supersymmetric. This is assured for $F_i^*F_i$ if the superpotential $W$, Equation 126, itself has the supersymmetry transformation property of a chiral superfield. Since products of superfields of a given chirality are superfields of the same chirality, whereas products of chiral superfields of differing chirality are not, the cubic terms in $W$ must contain factors of the same chirality. This means that we cannot allow the supersymmetric generalization of Equation 43 from the Weinberg-Salam model, where the up quark mass is contributed by the Higgs field $\Phi$ and the down quark mass by the conjugate field $\Phi^c$, since $\Phi$ and $\Phi^c$ have opposite chirality. Rather we need at least two separate Higgs fields, for instance, one to provide mass to $T_{3L} = +\frac{1}{2}$ fermions and the other to $T_{3L} = -\frac{1}{2}$ fermions. [This construction also avoids dangerous flavor-changing neutral interactions from Higgs boson exchange (88).] For the first fermion generation, the cubic terms in $W$ for the minimal extension are

$$W_3 = \kappa_u H_2 Q U^c + \kappa_d H_1 Q D^c + \kappa_e H_1 L E^c$$

where $H_{1,2}$ are the two complex SU(2)$_L$ Higgs doublet superfields, $Q$ and $L$ are the quark and lepton SU(2)$_L$ doublets, and $U^c$, $D^c$, $E^c$ are SU(2)$_L$ singlets. Projecting the scalar fields from $H_{1,2}$ and the fermion fields from $Q$, $L$, $U^c$, $D^c$, and $E^c$, the reader will recognize the familiar Yukawa interactions that generate fermion masses when $H_1$ and $H_2$ acquire vacuum expectation values. In two-component notation for the fermions, the fermion mass-generating Yukawa interactions are given in terms of the superpotential by

$$D_Y = g'\phi_i^* Y_i\phi_j.$$
where h.c. denotes a hermitian conjugate and \( \phi_i \) is the scalar superpartner of \( \psi_i \).

The Higgs interactions that would induce SU(2)_L \( \times U(1)_Y \) breaking are forbidden in the superpotential \( W \) by the requirement that \( W \) be gauge invariant. For instance, a term \( H_2 H^*_2 H_2 \) in \( W \) would violate SU(2)_L and U(1)_Y. Therefore we must look to the \( D \) terms in Equation 125 for the necessary Higgs boson interactions. While the quartic Higgs interactions of the Weinberg-Salam model, Equation 32, have an arbitrary coupling strength \( \lambda \), the quartic interactions of the \( D \) terms have the electroweak coupling strengths \( g^2 \) and \( g'^2 \) as seen by substituting Equations 127 and 128 into Equation 125. This is because the \( D \) terms arise from the supersymmetric generalization of Pauli’s gauge-covariant derivative, Equation 28. They are determined by the combined requirements of supersymmetry and gauge invariance.

The superpotential \( W \) also contains “soft” terms, of operator dimension two or less, that generate the quadratic and constant terms in the scalar potential, which, together with the quartic terms discussed above, induce the asymmetric ground state. These “soft” terms in \( W \) may break supersymmetry but must be SU(2)_L \( \times U(1)_Y \) gauge invariant. The reader can consult the literature for the details of the analysis (89–92). Here I only summarize the principal results.

The minimal supersymmetric extension contains two complex Higgs boson doublets or four charged and four neutral fields. Two charged and one neutral field are consumed in the Higgs mechanism, leaving five physical Higgs particles. They are a charged pair \( H^\pm \), two scalars \( H \) and \( H' \), and a pseudoscalar \( P \). Their masses are determined by two parameters. The first is

\[
\tan \beta = \frac{v_2}{v_1},
\]

where \( v_1 \) and \( v_2 \) are the vacuum expectation values of the Higgs fields \( H_1 \) and \( H_2 \) that mix to form the eigenstates \( H \) and \( H' \). To assure the correct \( W \) and \( Z \) masses they must satisfy \( v_1^2 + v_2^2 = v^2 = (\sqrt{2}G_f)^{-1} \). The second parameter, \( m_{12}^2 \), is the coefficient of a soft supersymmetry-breaking term in the scalar potential that mixes \( H_1 \) and \( H_2 \). To avoid fine-tuning problems \( m_{12} \) cannot be much larger than 1 TeV.

In terms of these two parameters, the mass of the pseudoscalar is

\[
m_P^2 = 2m_{12}^2 \sin 2\beta.
\]
The remaining four masses can be written in terms of \( m_p \) and the W and Z boson masses,

\[
m^2_{H^\pm} = m_p^2 + M_W^2
\]

\[
m^2_{H,H'} = \frac{1}{2} [m_p^2 + M_Z^2 \pm \sqrt{(m_p^2 + M_Z^2)^2 - 4m_p^2M_Z^2\cos^22\beta}].
\]

Equation 133 implies that \( H^\pm \) is heavier than the W boson. Equation 134 implies that one scalar, say \( H \), is lighter than the Z boson while the second, \( H' \), is heavier. Two extreme limits are of interest. If \( v_1 = v_2 \) then \( \beta = \pi/4 \) in which case \( m_p \) and \( m_{H^\pm} \) are minimized for given \( m_Z \), \( H \) is massless, and \( m_{H'} = m_p^2 + M_Z^2 \). The other extreme is \( v_2 \ll v_1 \) and \( \beta \ll 1 \). Then \( P, H^\pm, \) and \( H' \) become very heavy, \( \gg m_p \), while \( H \) becomes degenerate with \( Z \).

The physical Higgs scalar eigenstates \( H \) and \( H' \) are determined in terms of the original scalars \( H_1 \) and \( H_2 \) by mixing angle \( \alpha \), with \( H = H_2 \cos \alpha - H_1 \sin \alpha \) and \( H' = H_1 \cos \alpha + H_2 \sin \alpha \). The angle \( \alpha \) satisfies the relationship

\[
\tan 2\alpha = \frac{m_h^2 + m_Z^2}{m_p^2 - m_Z^2} \tan 2\beta.
\]

The couplings of \( H \) and \( H' \) to quarks, leptons, and gauge bosons are easily evaluated in terms of \( \alpha \) and \( \beta \). Relative to the coupling of the Weinberg-Salam model Higgs boson, the corresponding scale factors for the Yukawa couplings of the lighter scalar are \( \cos \alpha/\sin \beta \) for \( \bar{H}_u \) and \( \sin \alpha/\cos \beta \) for \( \bar{H}_d \). The scale factor for the gauge boson coupling of the heavier scalar, \( H'(WW + ZZ) \), is \( \cos(\beta - \alpha) \).

The couplings of \( H \) and \( H' \) can be studied as a function of their masses (93). For given values of \( m_{H_1} \) and \( m_{H_1} \), Equation 134 determines \( m_p \) and \( \beta \) and Equation 135 then determines \( \alpha \). The result for \( \cos^2(\alpha - \beta) \), which is the suppression factor for \( \Gamma(H' \rightarrow WW + ZZ) \) relative to the Weinberg-Salam model, is at first sight surprising. For \( m_{H_1} > 2M_Z \) the suppression is typically two or more orders of magnitude, much smaller than a typical value of \( \cos^2(\alpha - \beta) \) in the uncorrelated \( \alpha, \beta \) plane. However, the result can be understood intuitively by using the equivalence theorem, Section 2.1. Above \( ZZ \) threshold the Weinberg-Salam model Higgs boson decays predominantly to longitudinally polarized W and Z pairs. As shown in Section 2.1 the relevant cubic coupling from the scalar potential is \( \lambda v \) with \( \lambda = m_h^2/2\pi^2 \) becoming a strong coupling as \( m_h \) increases. But in the supersymmetric model \( \lambda \) is fixed at \( (g^2 + g'^2)/8 \) independent of \( m_{H_1} \) so that the coupling of \( H' \) to the Goldstone bosons, \( H'(ww + zz) \), is never strong. The suppression of the \( H'(WW + ZZ) \) coupling makes observation of the \( H' \) very difficult.
One can similarly study the H and H′ quark and lepton couplings in the $(m_H, m_{H'})$ plane (93). The result is that for $m_H \ll M_Z$ the lighter scalar H closely mimics the Weinberg-Salam model Higgs boson. As $m_H$ increases toward its upper limit of $M_Z$, its coupling to up (or down) quarks is suppressed and its coupling to down (or up) quarks is enhanced, by large factors at the upper end of the range. The experimental search for H is similar to the search for the Weinberg-Salam model Higgs boson.

Global supersymmetry at the electroweak scale by itself adds nothing to our understanding of the quark and lepton masses. Just as in the Weinberg-Salam model, fermion masses are introduced arbitrarily by choosing the Yukawa coupling constants in the super potential, Equation 129. As discussed in Section 4.2, the possible connection of low-energy supersymmetry to physics at the Planck scale offers hope that this arbitrariness may eventually be removed from the theory.

### 4.2 Inspiration from on High ($10^{19}$ GeV)

Global supersymmetry stabilizes the scale of spontaneous symmetry breaking by elementary Higgs scalars but gives no insight as to its physical origin. In the simplest models the scale is introduced by hand, by means of the parameters of the soft supersymmetry-breaking terms that drive the Higgs potential to its $SU(2)_L \times U(1)_Y$ asymmetric minimum. It is natural to speculate that global supersymmetry is a consequence of local (space-time-dependent) supersymmetry, just as electric charge conservation follows from the local gauge symmetry of electrodynamics (94). This is a tremendously exciting possibility. It implies a quantum theory of gravity, which is why theories with local supersymmetry are referred to as supergravity theories (see 95 for a thorough review). And it suggests possible explanations of the origin of the electroweak scale in terms of higher energy scales, including the $10^{19}$-GeV Planck scale of gravity.

It is easy to understand why local supersymmetry necessarily implies a quantum theory of gravity. The charge, or generator, of the simplest ($N = 1$) supersymmetry transformation is a spin-1/2 fermionic operator $Q$ that transforms the bosons and fermions of supermultiplets into one another. It is intimately connected with the Poincaré invariance of spacetime because of the anticommutation relation

$$\{Q, Q^\dagger\} = 2\gamma^\mu P_\mu,$$

where $\gamma_\mu$ are Dirac matrices and $P_\mu$ is the four-momentum operator, the generator of space-time displacements. Therefore the product of two space-time-dependent supersymmetry transformations is a space-time-dependent transformation of space-time—which is to say, a general coordinate transformation. Since invariance under general coordinate transformations is
the gauge invariance of general relativity, any locally supersymmetric theory is certain to encompass gravity.

Extending the Pauli minimal substitution trick, Equation 28, the covariant derivative of local supersymmetry requires a gauge field that transforms like the space-time derivative $\partial/\partial x_\mu$ of the $J = 1/2$ spinor transformation parameter $\epsilon(x)$, hence a spinor gauge field with a vector index, $\psi^\mu$. This “gauge fermion” is known as the gravitino because the supersymmetry generator $Q$ connects it to the graviton, the $J = 2$ gauge boson of general coordinate invariance. While $\psi^\mu$ might have had spin 1/2 or 3/2, supersymmetry requires $J = 3/2$ so that it can form a supermultiplet with the graviton. Since unbroken local supersymmetry requires massless gauge particles, the simplest supermultiplet contains just the massless gravitino and the massless graviton. Being massless, each has just two spin states, satisfying the requirement that a supermultiplet must have an equal number of bosonic and fermionic degrees of freedom.

In nature local supersymmetry must be broken since unbroken local supersymmetry would imply exact global supersymmetry and degenerate supermultiplets. Spontaneous breaking of local supersymmetry by the super-Higgs mechanism (96) is an elegant means of breaking supersymmetry in the spectrum while leaving general coordinate invariance intact. Global supersymmetry is broken when a scalar “auxiliary” field (a so-called $F$ or $D$ term like, for instance, the $F$ and $D$ terms in Equations 126–128) acquires a scalar condensate. The supersymmetric generalization of Goldstone’s theorem then implies that a spin-1/2 fermionic superpartner of this scalar is massless: it is the Goldstone fermion or goldstino. If supersymmetry is gauged, that is, if it is local, then the supersymmetric generalization of the Higgs mechanism occurs. The massless gauge fermion, i.e. the gravitino with two spin states, consumes the two spin states of the goldstino, becoming a massive gravitino with four spin states. The $J = 2$ tensor graviton remains massless.

We can now bring the discussion back to the subject of this review. It is possible that local supersymmetry breaking determines the soft supersymmetry-breaking terms that were introduced arbitrarily to break $SU(2)_L \times U(1)_Y$ in the minimal supersymmetric extension of the standard model, as discussed in Section 4.1. In the most attractive models the soft supersymmetry-breaking terms are generated by purely (super)gravitational interactions. This implies interesting constraints on the low-energy theory, even though the details of the Planck or GUT scale dynamics is very uncertain. In particular, the gravitational ansatz implies a universality of the soft terms with consequences discussed below. The reader should consult (97) for a review of a variety of supergravity models and their implications for $SU(2)_L \times U(1)_Y$ symmetry breaking.
A common feature of these models is the introduction of a “hidden sector” of particles that are gauge singlets under all the interactions of the observable sector, including SU(3)$_{\text{Color}} \times$ SU(2)$_{L} \times$ U(1)$_{Y}$ as well as any other nongravitational interactions of ordinary matter. Local supersymmetry is assumed to be spontaneously broken by an auxiliary field vacuum condensate in the hidden sector. The mechanism that forms this condensate need not be specified. It could result from strong gravitational interactions at the Planck scale or from nongravitational interactions at a lower scale, perhaps the GUT scale or an intermediate scale. In any case the effect of the hidden sector condensate is communicated to the observable sector by (super)gravitational interactions (98) with a universal coupling strength.

In the simplest realization of this idea the low-energy observable sector consists precisely of the minimal supersymmetric extension of the standard model. At the scale $M_{X}$ the soft supersymmetry-breaking terms have the form (99)

$$L_{\text{soft}} = AmW_{3} + Bm\mu H_{1}H_{2} + m^{2}\sum_{a} |\phi_{a}|^{2} + M \sum_{a} (\lambda_{a})^{2} \lambda_{a},$$

where $W_{3}$ is the cubic superpotential defined in Equation 129, the index $a$ sums over all scalars including $H_{1}$ and $H_{2}$, and $\lambda_{a}$ are the gauginos. The scalars and gauginos therefore acquire universal masses, $m$ and $M$ respectively, both typically of the order of the gravitino mass (except perhaps for “no-scale” models as discussed below) induced by the super-Higgs mechanism. $A$ and $B$ are unknown dimensionless numbers, presumed of order 1. The mass parameter $\mu$ is the coefficient of a quadratic term, $\mu H_{1}H_{2}$, that can in general appear in the superpotential $W$. Then at some large scale, typically assumed to be $M_{\text{GUT}}$, the potential for the scalars $H_{1}$ and $H_{2}$ is

$$V(H_{1}, H_{2})|_{M_{\text{GUT}}} = \frac{g^{2} + g'^{2}}{8} (|H_{1}|^{2} - |H_{2}|^{2})^{2} + (m^{2} + \mu^{2})$$
$$\times (|H_{1}|^{2} + |H_{2}|^{2}) + B\mu m H_{1}H_{2}. $$

This appears to be a disaster: $m^{2} + \mu^{2}$ is positive so there is no value of $B$ that allows the formation of a finite nonvanishing Higgs condensate. For $B \gtrsim -2(m^{2} + \mu^{2})/\mu m$ the potential is minimized at $H_{1} = H_{2} = 0$ and SU(2)$_{L} \times$ U(1)$_{Y}$ is unbroken, while for smaller values of $B$ the potential is unstable against large values of $H_{1} = H_{2} \neq 0$.

The “disaster” is good news in disguise because it shows that the universality of the supergravitational couplings has imposed nontrivial constraints on the minimal model. It is not really a disaster because Equation
138 describes the physics at the scale $M_{\text{GUT}}$, not at the electroweak scale $M_w$ that is relevant for $SU(2)_L \times U(1)_Y$ breaking. The potential at the scale $M_w$ must be obtained from Equation 138 by using the renormalization group to evolve the parameters down from the scale $M_{\text{GUT}}$ (100, 101). The crucial point is that flavor symmetry breaking causes the coefficients of $H_1^2$ and $H_2^2$ to evolve differently, breaking the deadlock between vanishing condensates and instability. In the absence of a fourth generation of quarks and leptons, the dominant effect is due to the Yukawa coupling of the top quark to $H_2$ (see Equation 129). In order for this mechanism to work, the top quark cannot be too light. The details depend on the top quark mass and on the unknown parameters $A$ and $B$, with two possible scenarios as described below.

After applying the renormalization group, the Higgs potential can be written in the form

$$V(H_1, H_2)|_{M_w} = \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 + m_1^2 |H_1|^2$$

$$+ m_2^2 |H_2|^2 + m_{12}^2 (H_1 H_2 + H_1^* H_2^*),$$

which is precisely the potential used in the analysis of the minimal supersymmetric extension described in Section 4.1. In order to form a non-vanishing condensate, the determinant of the $H_1, H_2$ mass matrix must be negative

$$m_1^2 m_2^2 - m_{12}^4 < 0.$$  \hspace{1cm} 139.

Here $m_1^2 > 0$ is little changed from its value, $m^2 + \mu^2$, at $M_{\text{GUT}}$ while $m_2^2$ has evolved downward. Spontaneous symmetry breaking could be triggered either if $m_2^2$ becomes negative or if $m_2^2$ is positive but $m_{12}^2$ is large enough to satisfy Inequality 140. The first case (100) imposes constraints on the top quark mass, $m_t \lesssim 55$ GeV, while the second (101) depends largely on the particular value of the parameter $B$. The reader should consult the review by Hall (97) for a more complete discussion.

The electroweak scale is determined in principle in supergravity models, but its relationship to the local supersymmetry-breaking scale $M_X$ and to the Planck scale is very model dependent. In the earliest models (98) the mass scale of the soft supersymmetry-breaking terms, Equation 137, is fixed by the gravitino mass, $M_\chi$, which in turn is related to the scales $M_X$ and $M_{\text{Planck}}$ by the tree approximation relation

$$M_\chi \simeq \frac{M_X^2}{M_{\text{Planck}}^4}.$$  \hspace{1cm} 141.

Identifying $M_\chi$ with the electroweak scale, say $M_\chi \simeq M_w$, we then find
$M_X \cong 10^{11}$ GeV as the scale of the hidden sector condensate. In “no-scale” models (102) the identification of the gravitino mass with the electroweak scale is not necessary. The soft supersymmetry-breaking terms, Equation 137, are not determined in tree approximation by local supersymmetry breaking at the scale $M_X$ (103), which is therefore freed of the constraint imposed by $M_W \cong M_X^2/M_{\text{Planck}}$. Rather the electroweak scale is determined by radiative corrections with dimensional transmutation (104), as occurs in the Coleman-Weinberg model (105) of electroweak symmetry breaking (see Section 4.3). That is, the tree approximation effective potential at the Planck scale is flat (in particular, mass terms in the Higgs potential vanish) and the minimum is determined by radiative corrections that have a logarithmic dependence on the energy scale. The result (104) is a relationship of the form

$$M_W \cong M_{\text{Planck}} e^{-4\pi\kappa^2}$$

where $\kappa$ is the dominant Yukawa coupling constant, perhaps of the top quark.

### 4.3 Superstrings

Supergravity models are neither finite nor renormalizable and must be viewed as effective “low”-energy theories that are cut-off at the Planck mass. Here “low” means low compared to $M_{\text{Planck}}$ and might, for instance, be larger than $M_{\text{GUT}}$. Superstring models (106) hold out the promise of being truly finite theories, encompassing gravity and everything else, giving rise to supergravity at low energy. String theories generalize the notion of a local quantum field, which is a function of space-time, $\psi = \psi(x)$, to quantum fields with arguments that are extended 1+1 dimensional entities or strings, $\psi = \psi(\sigma)$, with the string $\sigma = \sigma(x)$ embedded in space-time. Though it was originally believed that only a few finite string theories could be constructed, all requiring enlarging space-time to include additional compactified spatial dimensions, a large class of four-dimensional theories has subsequently been found (107).

Superstring theory is a booming industry for obvious reasons: it has potentially unmatched promise and is very poorly understood. However, the jump to the Planck scale could prove to be a leap over too many orders of magnitude of unknown physics. The mathematics of quantization is not yet well developed, and understanding of dynamics is even farther away. This is neither the place nor the author to review these developments. I therefore restrict myself to a little-finger-nail sketch of the original string scenario as it bears on the issue of electroweak symmetry breaking. This is useful as a cautionary tale since the string scenario forces us to go beyond the minimal supersymmetric model discussed in Section 4.1.
Of the original $D > 4$ dimensional finite string theories, the most promising is the heterotic string (108) requiring $D = 10$, i.e. six extra spatial dimensions. The natural scale of the dynamics and the spectrum is $M_{\text{Planck}} = 10^{19}$ GeV, but the zero modes give rise to a sub-Planck scale spectrum. In fact, the effective theory obtained by neglecting the Planck scale spectrum is just a $D = 10$ supergravity theory with calculable (in principle) corrections of order $M_{\text{Planck}}^{-2}$. Dynamics that we are far from understanding must induce compactification of the extra six spatial dimensions. The result is a four-dimensional supergravity theory with a non-Abelian gauge symmetry that is a subgroup of $E_6$ in the case of the originally proposed compactification mechanism (for a review, see 109). From here the story proceeds as in the preceding discussion. The structure of the ten-dimensional theory implies the existence of a hidden sector in the $D = 4$ supergravity theory, so that the local supersymmetry can be broken spontaneously in the hidden sector with the breaking transmitted to the observable sector by (super)gravitational interactions. The result may then be a softly broken supersymmetric theory at or below the TeV scale.

The particles of the low-energy effective theory in this scenario lie in the $27$ representation of $E_6$, though $E_6$ is not a symmetry of the four-dimensional world. One $27$ representation is expected per generation, and the number of generations is in principle determined. The $27$ chiral supermultiplets include the familiar $5 + \bar{10}$ of SU(5) (i.e. up and down quarks in two chiralities and three colors, the electron in two chiralities, and the left-handed neutrino); a charge conjugate neutrino $\nu^c$ that could combine with $\nu$ to give a Majorana mass; a charge-$1/3$ color triplet Majorana fermions; two complex SU(2)$_L$ Higgs doublets; and a neutral Higgs singlet.

The crucial deviation from the minimal supersymmetric extension of the standard model is the existence of the singlet Higgs field $N$. Where $n_g$ is the number of generations, there are $2n_g$ complex doublets and $n_g$ singlets, but the relevant Higgs potential can be written in terms of just the three linear combinations that get vacuum expectation values. As in Section 4.1, the real neutral components of the combinations that get vacuum expectation values are labeled $H_1$ and $H_2$ for the doublets while the singlet is $N$. Now in addition to the super Yukawa interactions displayed in Equation 129, the superpotential acquires a term that is cubic in Higgs fields,

$$\delta W_3 = \frac{1}{2} \sqrt{\lambda} NH_1 H_2.$$  

From Equations 143, 126, and 125 we then find that the Higgs potential of the minimal model, Equation 139, is augmented by several new terms (e.g. 96) including a new quartic interaction in $H_{1,2}$:
Whereas in the minimal model the quartic interactions come only from $D$ terms and are therefore fixed at electroweak strength, the coupling $\lambda$ is on the same footing as the Yukawa couplings that also appear in the $F$ terms. In principle $\lambda$ is determined in string models, but in practice it is unknown. We cannot say with certainty that it is big or small, so we can also not be sure that the symmetry-breaking sector is weakly coupled and that there must be a light Higgs boson. If we assume $\langle N \rangle = 0$, Equation 134 is modified, becoming

$$m_{H,H'}^2 = \frac{1}{2} [m_H^2 + M_Z^2 \pm \sqrt{(m_H^2 + M_Z^2)^2 - 4m_H^2M_Z^2\cos^2 2\beta - A}]$$

where

$$A = 2\lambda v^2 \sin^2 (2\beta)(m_H^2 + \sin^2 \theta_W M_Z^2).$$

For $A \neq 0$ the lightest Higgs boson $H$ need not be lighter than $M_Z$. For large $\lambda$ it could be appreciably heavier.

In particular superstring-inspired models, constraints on $\lambda$ prevent $m_H$ from being much heavier than $M_Z$ (110). Or if perturbative unification is assumed—that all couplings including $\lambda$ are small enough that perturbation theory is reliable between $M_W$ and $M_{\text{GUT}}$—then $m_H$ cannot be heavier than about 170 GeV (111). However, in general the necessity of a light Higgs boson is not guaranteed.

5. HIGGS SECTOR DYNAMICS

Except for the caveats expressed in Section 4.2, supersymmetric models tend to have weakly interacting Higgs sectors that can be analyzed perturbatively and are therefore not expected to exhibit interesting nonperturbative dynamics. Higgs models without supersymmetry are unnatural in the sense that they are unstable against quantum fluctuations from the highest energy scales in the theory and require implausible fine-tuning, as discussed in Section 1.4. Theories containing only scalar quanta are thought to have an even more serious affliction known as “triviality” that may also afflict theories of scalars with gauge bosons and/or fermions (112). Trivial theories do not exist at all as interacting field theories in the limit that the cutoff $\Lambda$ is taken to infinity. They are only consistently defined if the renormalized coupling constant vanishes as $\Lambda \to \infty$, i.e. as trivial free field theories. Scalar $\phi^4$ models, where $\phi$ can have any number of components, are known to be trivial for space-time dimension $D > 4$. 

and are widely believed to be trivial at $D = 4$ (113 and references cited there).

Despite or perhaps because of these difficulties, there is much interest in the nonperturbative dynamics of strongly coupled scalar field theories and strongly coupled Higgs sectors in particular. One promising line of research is to regularize the theory with a space-time lattice so that it can be studied by numerical methods, including renormalization group Monte Carlo methods, as in the study of quantum chromodynamics in the strong coupling phase. (See 114 for reviews of lattice Higgs studies.) Though not yet definitive, these investigations tend to confirm the triviality of the Weinberg-Salam model but leave open the possibility of interpreting it as an effective low-energy theory. Numerical studies correlate the mass of the Higgs boson $m_h$ with the finite cutoff $\Lambda$ that represents the mass scale of new physics. The results imply an upper limit on $m_h$ of order 800 GeV. This approach is reviewed in Section 5.3.

Another approach to the strongly coupled Higgs sector rests on the conjectured extension to four dimensions of dynamics known to occur in certain two-dimensional field theories. These two-dimensional scalar models possess a "hidden" gauge symmetry (115), which seems only a technical artifact in lowest order but is found in higher orders to be a true gauge theory with dynamically generated propagating gauge bosons (116). This approach has been applied to QCD (117, 118) and more recently to the Weinberg-Salam model (119, 120) as described in Section 5.2. In the Weinberg-Salam model it predicts the existence of vector mesons that might correspond to the technirho states of technicolor models or might be considerably lighter.

In Section 5.1 I briefly describe some amusing dynamics that can occur in weakly coupled Higgs models. In gauge theories with vanishing leading order Higgs mass, the gauge symmetry is broken by radiative corrections if the gauge and Higgs coupling constants are small (105). The Higgs boson mass is determined in terms of the gauge boson mass. Dimensional transmutation occurs: a dimensionless parameter in the initial Lagrangian is traded for a dimensionful parameter in the solution. The result is tantalizing, though no principle is known to explain the assumed vanishing of the bare Higgs mass, which is technically unnatural in the sense of Section 1.4. More generally, minimization of the one-loop effective potential in the one Higgs doublet model implies a lower bound on the Higgs boson mass and upper bounds on fermion masses, also discussed in Section 5.1.

5.1 Symmetry Breaking by Radiative Corrections

The starting point for this discussion (105) is the Weinberg-Salam model with the potential given by $V = \lambda(\Phi^+\Phi)^2$ so that there is no Higgs mass
term. The classical minimum is then at $\langle \Phi \rangle = 0$, $\text{SU}(2)_L \times \text{U}(1)_Y$ is unbroken, and the $W$ and $Z$ bosons remain massless. S. Coleman and E. Weinberg (105) and S. Weinberg (121) developed a formalism to incorporate the effects of quantum fluctuations on the Higgs potential. They compute a quantity, called the effective potential in Reference 105, using an expansion in $1/h$ or, equivalently, in the number of loops. The effective potential is a sensible quantum theoretical generalization of the classical potential since its minima are the ground states of the theory. The criteria for convergence of the loop expansion are discussed below.

Following Coleman & Weinberg (105) it is instructive to begin with a simple $\text{U}(1)$ gauge theory, i.e. massless scalar quantum electrodynamics. The Lagrangian is

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (D_\mu \phi) \bar{\phi} D^\mu \phi - V(\phi),$$

where $\phi = \phi_1 + i \phi_2$ is complex, $D^\mu = \partial^\mu - ieA^\mu$ is the covariant derivative, and $V(\phi)$ is the scalar potential.

$$V = \frac{\lambda}{4} (\phi^* \phi)^2.$$ 148.

Classically the minimum is at $\phi = 0$ and the photon is massless.

To one-loop order the effective potential is the sum of all one-loop diagrams containing internal photon and scalar boson lines and arbitrarily many external scalar boson fields. The effective potential is a function of $\phi_c^2 = \phi_{1c}^2 + \phi_{2c}^2$ where $\phi_c$ denotes the expectation value $\langle \phi \rangle_c$ in an unspecified state also characterized by the subscript $c$. Varying $\phi_c$ corresponds to surveying the candidate ground states in which the expectation value of $\phi$ is taken. We sum over diagrams with arbitrarily many external scalars because the condition $\langle \phi \rangle_c \neq 0$ means that the scalar boson can propagate into the candidate vacuum state $c$. The result for the one-loop potential is

$$V(\phi_c) = \frac{\lambda}{4} \phi_c^4 + \frac{5\lambda^2}{32\pi^2} + \frac{3\lambda^4}{64\pi^2} \phi_c^4 \left( \ln \frac{\phi_c^2}{M^2} - \frac{25}{6} \right),$$

where $M$ is an arbitrary scale chosen to define the renormalized coupling constant $\lambda$.

If $\phi_c/M$ is either very big or very small, the quantum correction in Equation 149 would overpower the first, classical term and the expansion would not be reliable. Since $M$ is arbitrary and since the goal is to find $\langle \phi \rangle$, the vacuum expectation value in the true minimum, Coleman & Weinberg (105) make the adroit choice $M = \langle \phi \rangle$.

Minimization of $V$ requires an interplay of the classical term, $O(\lambda)$, and the one-loop terms, $O(e^4)$ and $O(\lambda^2)$, so we assume that $\lambda$ is of order $e^4$...
and drop the $O(\lambda^2)$ contribution (actually an unnecessary restriction, as discussed below). Then the minimization condition

$$0 = \frac{dV(\phi_c)}{d\phi_c}_{\phi_c=\langle \phi \rangle} = \left( \lambda - \frac{11e^4}{16\pi^2} \right) \langle \phi \rangle^3$$

implies a relationship between the coupling constants

$$\lambda = \frac{11e^4}{16\pi^2}$$

if the symmetry is broken, $\langle \phi \rangle \neq 0$. In that case the potential can be rewritten

$$V(\langle \phi \rangle) = \frac{3e^4}{64\pi^2} \phi_c^4 \left( \ln \left( \frac{\phi_c^2}{\langle \phi \rangle^2} \right) - \frac{1}{2} \right),$$

the scalar mass is

$$m_\phi^2 = \frac{d^2V(\phi_c)}{d\phi_c^2}_{\phi_c=\langle \phi \rangle} = \frac{3e^4}{8\pi^2} \langle \phi \rangle^2,$$

and the photon mass is

$$m_r^2 = e^2 \langle \phi \rangle^2 = \frac{3e^2}{8\pi^2} m_\phi^2.$$

In this example one-loop fluctuations have spontaneously broken the U(1) gauge symmetry. Comparing Equation 152 with the starting point defined by Equations 147 and 148, we see a dimensional transmutation: the two dimensionless parameters $\epsilon$ and $\lambda$ have been replaced by $\epsilon$ and the dimensionful parameter $\langle \phi \rangle$. A similar phenomenon occurs in QCD (122): asymptotic freedom implies that the effective coupling $\alpha_s(Q^2)$ vanishes as $Q^2 \rightarrow \infty$ like $1/\ln Q^2$ with a known coefficient $C$. Therefore $\alpha_s$ can be defined either by specifying its dimensionless value at some renormalization scale $Q_0^2$ or by specifying the value of a dimensionful parameter $\Lambda_{QCD}$ that sets the scale of the logarithm: $\alpha_s(Q^2) = C \ln (Q^2/\Lambda_{QCD})$. To obtain the transmuted result, Equation 152, we followed a similar procedure, choosing $\langle \phi \rangle$ as the renormalization point for the coupling $\lambda$ and using Equation 152 as the counterpart of the boundary condition on $\alpha_s(Q^2)$ at infinity.

For the Abelian example, Coleman & Weinberg (105) show by a renormalization group analysis that the assumption $\lambda = O(e^4)$ is unnecessary and that it is sufficient for $\lambda$ and $e^2$ to be small compared to 1. However, no renormalization group analysis has been carried out for the SU(2)$_L \times$ U(1)$_Y$ model.
The analysis for the SU(2)_L × U(1)_Y model with a single Higgs doublet is similar (105). The potential is given by Equation 31 with v = 0. Then the theory is specified by three dimensionless parameters: the SU(2)_L and U(1)_Y gauge coupling constants g and g' and the quartic coupling constant λ. Neglecting fermionic contributions (only important for fermion masses of order M_w), we obtain the minimization condition

\[ m_h^2 = \frac{3}{32\pi^2} [2g^2M_w^2 + (g^2 + g'^2)M_Z^2]. \]

Using the lowest order expressions for G_F, M_w, M_Z, and sin^2 θ_w, we can rewrite this as

\[ m_h^2 = \frac{3\alpha^2}{8\sqrt{2}G_F} \left( \frac{2 + \sec^4 \theta_w}{\sin^4 \theta_w} \right) = (9.75 \text{ GeV})^2 \]

where α = 1/129 at the scale M_w. A Higgs boson at this mass is not ruled out experimentally. Since the presumed 3P_0(0++) state is tantalizing nearby, at 9.860 ± 0.001 GeV (55), there could be appreciable mixing, depending on the precise value of m_h (123).

Coleman & Weinberg (105) are careful to observe that the expansion fails for large \( \ln (\phi_c / \langle \phi \rangle) \), so that there could be deeper minima outside the range of the analysis. Suppose, for instance, that the expansion parameter is \( (g^2/4\pi^2) \ln (\phi_c / \langle \phi \rangle)^2 \) and also suppose, conservatively, that the expansion fails when the parameter equals 1/3. This occurs for \( (g^2/4\pi^2) \ln (\phi_c / \langle \phi \rangle)^2 \approx 10^7 \). If the true minimum were in this domain, then the minimum found in this analysis would be reinterpreted as a very shallow local minimum at the scale of \( 10^{-7} \times 10^5 \text{ GeV} \cong 10 \text{ keV} \). On the other hand, a two-loop computation of the effective potential suggests that the above estimate is unduly pessimistic and that the loop expansion may be valid for values of \( \phi_c \) as large as the Planck mass (124).

The minimization condition \( \lambda \propto g^4/\pi^2 \) is reminiscent of \( \lambda \propto g^2 \) in the minimal supersymmetric model (Section 4.1). The extra factor \( g^2/\pi^2 \) means that the Higgs boson is lighter than in the SUSY model, where the typical scale for the lightest neutral Higgs boson is \( O(M_w) \). No rationale has been given to explain the vanishing of the Higgs boson mass in lowest order, which is technically unnatural in the sense of Section 1.4.

In the SU(2)_L × U(1)_Y model with one Higgs doublet, the requirement that the one-loop effective potential have an SU(2)_L × U(1)_Y breaking minimum implies a lower bound on the Higgs boson mass and an upper bound on fermion masses. Including a bare Higgs mass and neglecting fermion masses, we find that a local minimum exists with \( \langle \phi \rangle \neq 0 \) for any value of \( m_h \) but if \( m_h \) is too small there is a deeper minimum at the origin,
\( \langle \phi \rangle = 0 \), which leaves \( SU(2)_L \times U(1)_Y \) unbroken. The condition for the minimum at nonvanishing \( \langle \phi \rangle \) to be deeper implies the lower bound (125)

\[
m_h^2 \gtrsim \frac{3\alpha^2}{16\sqrt{2}G_F} \left( \frac{2+\sec^4 \theta_W}{\sin^4 \theta_W} \right) = (6.9 \text{ GeV})^2
\]

157.

i.e. \( m_h \) must be at least \( 1/\sqrt{2} \) of the Coleman-Weinberg value, Equation 156.

If for \( m_h \lesssim 6.9 \text{ GeV} \) the universe somehow reached the metastable symmetry-breaking vacuum at an early time, then a calculation of the tunneling amplitude shows that the lifetime of the metastable vacuum would be longer than the \( \sim 10^{10} \) year lifetime of the universe for \( m_h \gtrsim 260 \text{ MeV} \), which considerably weakens the bound (126). However, this is not a likely scenario in the inflationary cosmological model, for which the bound is more probably raised to the Coleman-Weinberg value (127) or perhaps just below it (128). As the universe cools from very high temperature \( T \), the effective potential has a large mass-like term, \( \sim g^2 T^2 h^2 \), which chooses the symmetric vacuum at \( h = 0 \). For \( m_h \) smaller than the Coleman-Weinberg value, Equation 156, the universe supercools before arriving at the symmetry-breaking vacuum, \( h = v \); this results in a baryon-to-photon ratio far below what is observed (127). At or just below the Coleman-Weinberg value, the QCD chiral symmetry phase transition can trigger an earlier transition to the usual electroweak symmetry-breaking vacuum, thus decreasing the duration of supercooling and providing the possibility of an acceptable baryon-to-photon ratio (128).

It is instructive to consider the full one-loop effective potential including both the contribution of the Higgs boson with arbitrary mass and the top quark (129)

\[
V(\phi_c) = \frac{m_h^2}{8} \left[ 2D\phi_c^4 \ln \left( \frac{\phi_c^2}{v^2} \right) - (3D-1)\phi_c^4 + (4D-2)v^2\phi_c^2 \right],
\]

158.

where the minimum is at \( \phi_c = v = 246 \text{ GeV} \). The constant \( D \) is given by

\[
D = \frac{6M_w^4 + 3M_z^4 + M_h^4 - 12m_t^4}{16\pi^2 v^2 m_h^2}.
\]

159.

Here \( m_h^2 = V''(\phi_c = v) \) is the Higgs boson mass and \( M_k^2 = [3\lambda - (\mu^2/\phi_c^2)]v^2 \), with \( \lambda \) the quartic coupling constant and \( -\mu^2/2 \) the coefficient of \( \phi_c^2 \) in the tree level potential. The factor \( m_t^4 \) is really a mnemonic for a sum over all fermions including possible additional generations,
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\[ m_t^4 \rightarrow \frac{1}{3} \sum \xi_t m_t^4, \]

where \( \xi_t \) is a color factor, three for quarks and one for leptons.

The condition for stability of the broken symmetry vacuum is \( D < 1 \) or

\[ m_h^2 \geq \frac{3}{16\pi^2} (2M_W^4 + M_Z^4 - 4m_t^4). \]

If we neglect \( m_t \), Inequality 161 is equivalent to Inequality 157. As \( m_t \) approaches \([(2M_W^4 + M_Z^4)/4]^{1/2} = 80 \text{ GeV} \) from below, the bound on \( m_h \) is weakened and eventually disappears.

For \( D < 0 \) the potential, Equation 158, is unbounded from below, which suggests an upper limit on the mass of the top quark (130–132) that would be 80 GeV if the Higgs boson mass were much smaller. For heavy Higgs bosons the quantity \( M_h^4 \) (defined below Equation 159) tends to cancel the top quark contribution in \( D \), weakening the upper bound on \( m_t \). For \( m_h \geq 1 \text{ TeV} \) the perturbation expansion (and the loop expansion) begins to fail (31), though at \( m_h = 1 \text{ TeV} \) explicit calculations of higher order effects still give quite small corrections in some examples (133). For smaller masses, say \( m_h \approx 0.5 \text{ TeV} \), the perturbation expansion is probably reliable and much larger values of \( m_t \) are allowed. The upper bound on \( m_t \) is also weakened by two-loop effects and by allowing for the possibility of a long-lived metastable vacuum (129). Including these effects, Flores & Sher (129) find that values of \( m_t \) as large as 200 GeV are allowed for \( m_h < 200 \text{ GeV} \).

5.2  Hidden Gauge Symmetry

The low-energy interactions of Goldstone bosons are determined by symmetry considerations, as discussed in Section 2.2. This is elegantly represented by nonlinear effective Lagrangians (134, 135), which are nonrenormalizable and are only intended to represent the physics in the domain of validity of the low-energy theorems. For instance, the linear sigma model (22) has the scalar potential

\[ V = \frac{\lambda}{4} (\sigma^2 + \pi^2 - F_\pi^2)^2 \]

(like the one-doublet Higgs model, Equation 31), with an \( SU(2)_L \times SU(2)_R \) global symmetry. Assigning the minimum of the potential to \( \langle \sigma \rangle = F_\pi \) and redefining \( \sigma \rightarrow \sigma + F_\pi \) we can exhibit the linear model in the broken phase, as in the Equation 32. The symmetry breaks to the diagonal vector subgroup \( SU(2)_{L+R} \) (i.e. isospin), the \( \sigma \) acquire a mass \( 2\lambda F_\pi^2 \), and the pions are massless, being the Goldstone bosons associated with the three broken
(axial) SU(2)_{L-R} generators. A simple way to obtain a nonlinear representation is to consider the limit \( \lambda \to \infty \) with \( F_\pi \) held constant so that \( m_\sigma^2 = 2\lambda F_\pi^2 \to \infty \). Then the potential \( V \) is frozen at \( V = 0 \) and \( \sigma \) is no longer a propagating field. The theory is now specified by the kinetic energy terms

\[
\mathcal{L}_{KE} = \frac{1}{2}(\partial \sigma)^2 + \frac{1}{2}(\partial \pi)^2
\]

and the nonlinear boundary condition that freezes the potential at \( V = 0 \)

\[
\sigma^2 = F_\pi^2 - \pi^2.\]

The interactions are recovered from the \( \sigma \) kinetic energy term by substituting Equation 164 into Equation 163 and expanding \( \sqrt{F_\pi^2 - \pi^2} \) in powers of \( \pi^2/F_\pi^2 \). The result is an infinite series of coordinate space interactions including terms of the form \( F_\pi^{-2} \pi \partial \pi \partial \sigma \) that imply low-energy theorems like Equation 55. The Lagrangian is relevant for energies small compared to the masses of massive particles and to \( 4\pi F_\pi \), the scale of loop corrections.

Nonlinear representations are in fact infinitely nonunique. Though the example given above is easy to understand, other representations reveal the underlying symmetries more clearly. The elegant nonlinear representations of Reference (135) are best suited to the discussion of hidden gauge symmetry, but a description of these representations would not be appropriate for this review. I limit myself here to a brief description of the essential points of the hidden gauge symmetry hypothesis. To see the details, the reader can, for example, consult References (120) and (136).

We consider a nonlinear Lagrangian with global symmetry group \( G \) that breaks spontaneously to a global subgroup \( H \). The Goldstone bosons are in one-to-one correspondence with those generators of \( G \) that are not generators of \( H \), i.e. the generators spanning the coset space \( G/H \). (\( G/H \) can be thought of as group transformations of \( G \) defined "modulo" \( H \), i.e. transformations regarded as identical if they are equal up to a transformation in \( H \).) The nonlinear Lagrangian can be linearized by introducing a set of auxiliary vector fields \( V_\alpha \) one for each generator of \( G \). The result is a linear representation of the global symmetry group \( G \) that admits in addition a local gauge invariance of the subgroup \( H \) with the vector fields \( V_\alpha \) that correspond to the generators of \( H \) transforming like gauge bosons. The gauge symmetry is at this point only a mathematical device for linearizing the theory. The Lagrangian has no kinetic energy terms for any of the \( V_i \) so they do not propagate. They can be eliminated by the equations of motion, returning the theory to its original nonlinear form.

An exciting development in certain solvable 1+1 dimensional field theories is that the hidden gauge symmetry found using the construction sketched above (115) actually becomes a dynamical symmetry (116) when
quantum corrections are included. The quantum corrections induce kinetic energy terms for the hitherto auxiliary gauge fields $V_{\alpha}$, which therefore become true dynamical degrees of freedom. The conjecture that a similar mechanism may operate in 3+1 dimensions is the basis of phenomenological models that have been proposed for QCD and for electroweak symmetry breaking.

In the application to QCD the $\rho$ meson is interpreted as the gauge boson of the hidden symmetry, and relationships are deduced among masses and coupling constants that were previously obtained using current algebra, PCAC, and vector meson dominance (117, 118, 136). In the application to electroweak symmetry breaking (119, 120) the masses of the new gauge bosons are not fixed. It is claimed that no experimental evidence excludes the possibility that they might be as light as 200 GeV. Because they mix with the W and Z they would induce deviations from the predictions of the standard $SU(2)_L \times U(1)_Y$ model in a wide range of observable processes (144).

The strategy of this approach is to use the known structure of the Goldstone boson sector to try to deduce properties of the spectrum and dynamics of the massive states in the symmetry-breaking sector. In a hadron physics analogy, it is like using the sigma model to predict the physics of the vector mesons rather than merely as a low-energy approximation to the chiral dynamics of QCD. In this sense it is like the Skyrme model (138), which also seeks to extrapolate out of the low-energy Goldstone boson sector to encompass the baryons. There are two critical issues: first, whether the gauge bosons become dynamical as conjectured and, second, whether the theory so constructed oversteps its own foundation by reaching out of the low-energy domain. Of course, as emphasized by Casalbuoni et al (119, 120, 137), one could regard the model as an experimentally testable hypothesis independent of the steps that motivated its construction.

5.3 Triviality and Lattice Regularization

The probable triviality of $\phi^4$ theories in four dimensions can be understood intuitively from the renormalization group equation for the coupling constant $\lambda$. Define $t = \ln Q/A$, where $A$ is the cutoff imposed on Feynman integrals to make the theory finite and $Q$ is a low-energy renormalization scale, chosen at the scale of the phenomena to be studied (e.g. $m_h$ or $M_W$ in electroweak theory). Then for an interaction $\mathcal{L} = (\lambda/4)\phi^4$ the renormalization group equation for the renormalized coupling constant $\lambda_R(t)$ is

$$\frac{d\lambda_R(t)}{dt} = \beta(\lambda_R, t) = \frac{3\lambda_R^2}{2\pi^2} + \cdots,$$

165.
where the one-loop contribution to the beta function is shown on the right side. For example, assuming the one-loop contribution is the exact answer we can easily integrate Equation 165 from a finite value $t = \ln \frac{Q}{\Lambda} < 0$ up to the cutoff $Q = \Lambda$ or $t = 0$. The result is

$$\lambda_R(t) = \frac{\lambda_0}{1 - \frac{3}{2\pi^2} \lambda_0 t},$$

166.

where $\lambda_0 \equiv \lambda_R(t = 0)$ is the coupling constant evaluated at the cutoff, i.e. the “bare” coupling. As we attempt to remove the cutoff by taking $\Lambda \to \infty$ we find that $\lambda_R(Q) \to 0$ for any value of $\lambda_0$ at any fixed finite $Q$.

The conclusion does not depend on the functional form we assumed for beta function but only on its positivity in the physical domain, $\lambda > 0$ and $t < 0$. In that case Equation 165 has only the infrared fixed point at the origin. Starting from the cutoff $Q = \Lambda$ with $\lambda_R(Q) = \lambda_R(\Lambda) \equiv \lambda_0 > 0$, a decrease in $Q$, $dQ < 0$, implies $dt = \frac{dQ}{Q} < 0$ and $d\lambda_R = \beta dt < 0$. Continuing to decrease $Q$, $\lambda_R$ will continue to decrease until it reaches the fixed point at $\lambda_R = 0$. For any finite value of $Q$ we therefore find that $\lambda_R(Q) \to 0$ as we take $\Lambda \to \infty$ while $\lambda_0$ is held fixed at any finite value. Triviality is then a consequence of the positivity of the beta function.

To determine whether $\phi^4$ theories in $3 + 1$ dimensions are trivial we need to know if the theory has other infrared fixed points besides the one at $\lambda_R = 0$. This requires going beyond perturbation theory to study the theory in the strong coupling domain. The original study used an approximate renormalization group analysis (112). Later studies used strong coupling expansions with Padé approximants to extrapolate into the strong coupling domain (see 139 for a review). Triviality has been proved for $\phi^4$ theories in $D > 4$ dimensions (140) but not for $D = 4$.

With the growth of computing power and the development of lattice regularization to study strong coupling QCD, it is natural to use the same tools to study scalar field theories in the strong coupling domain. Numerical simulations of $\phi^4$ theories on a lattice in four dimensions tend to confirm triviality (141). The most recent studies, performed in the symmetric phase where $\langle \phi \rangle = 0$, leave little room for doubt (142). A study of the broken phase also indicates triviality (143) but has not yet been verified with a large scale numerical simulation comparable to the study of the symmetric phase. The broken phase is of course the relevant one for Higgs models.

Since perturbative gauge interactions are unlikely to change the qualitative nature of the scalar interaction in the strong coupling domain, it is probable that Higgs models of the electroweak interactions are also trivial.
This conclusion is supported by numerical studies of Higgs models on a lattice (144), though definitive studies remain to be done. Additional references can be found in the reviews of Jeršák and Montvay (114).

While Higgs models are probably trivial, they may still be useful as effective low-energy theories below a cutoff $\Lambda$ where new physics emerges. As expected from the qualitative argument given above, the numerical studies show that the renormalized coupling at a finite energy scale $\lambda_R(Q)$ is a decreasing function of $\Lambda/Q$ for fixed bare coupling $\lambda_0 = \lambda_R(\Lambda)$. Dashen & Neuberger (145) observed that this behavior implies an upper limit on the Higgs boson mass, since $m_H^2$ is proportional to $\lambda_R(v)$ but $m_h$ cannot be greater than $\Lambda$. Increasing $\lambda_R(v)$ in order to increase $m_h$ we decrease $\Lambda$ until it eventually crosses $m_h$ and the Higgs model ceases to be a sensible effective low-energy theory.

A very crude estimate of the upper bound can be obtained from the one-loop renormalized coupling constant (145). Taking $\lambda_0 \to \infty$ in Equation 166 we find

$$[\lambda_R(v)]_{\text{MAX}} = \frac{2\pi^2}{3 \ln \left( \frac{\Lambda}{v} \right)}.$$  \hspace{1cm} 167.

From Equations 33 and 34 we have

$$\frac{m_h^2}{M_W^2} = \frac{8\lambda_R(v)}{g^2},$$  \hspace{1cm} 168.

where $g$ is the SU(2)$_L$ coupling constant. The upper bound is then

$$m_h \leq \frac{4\pi}{\sqrt{3g}} \frac{M_W}{\left( \ln \left( \frac{\Lambda}{v} \right) \right)^{1/2}}$$  \hspace{1cm} 169.

or $O(1 \text{ TeV})$ for $\ln (\Lambda/v) \approx O(1)$.

Subsequent numerical studies of Higgs models on a lattice agree with this crude estimate. With lattice regularization the cutoff is $\Lambda = a^{-1}$ where $a$ is the distance between adjacent points on the lattice. Requiring $m_h < a^{-1}$ or $m_h < \frac{1}{2}a^{-1}$, several groups (146) obtain the upper bound $m_h \leq 10M_W$, in order-of-magnitude agreement with Equation 169. Perturbative estimates of Hasenfratz & Hasenfratz (146) also indicate that $\Lambda/m_h$ increases exponentially as $m_h$ decreases from the upper bound. Therefore we might say that the light Higgs boson is "deeply trivial" since the physics underlying it is many orders of magnitude beyond the electroweak scale. The very heavy Higgs boson is "superficially trivial" since its triviality will be
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readily exposed by discovery of the underlying physics at a scale not far beyond its own mass.

In the next section we leave theoretical value judgements and aspersions behind, to consider instead the prospects for acquiring real experimental evidence of the symmetry-breaking sector.

6. EXPERIMENTAL STATUS AND PROSPECTS

The experimental search for the mechanism of electroweak symmetry breaking is a difficult undertaking. It is difficult even if the answer is waiting at relatively modest energies because of the typically miniscule couplings of Higgs bosons to ordinary matter. For instance, from Equation 46 the coupling of the standard model Higgs boson to the electron is \( \kappa_e = \frac{g_m_e}{2M_w} = 2 \times 10^{-6} \) so that \( \kappa_e^2/4\pi = 3 \times 10^{-13} \), ten orders of magnitude smaller than the fine-structure constant \( \alpha = 1/137 \). If the answer is new strong interactions above 1 TeV—technicolor or one of its variants or something completely unexpected—then their discovery requires new high-energy facilities. The good news is that even in this case we know from unitarity (cf Section 2) that the search is not open-ended and that signs of the new physics must emerge in the WW channel at or below \( (8\sqrt{2}/G_F) \approx 1.7 \) TeV. As discussed below, a pp collider with \( \sqrt{s} = 40 \) TeV and \( L = 10^{33} \) cm\(^{-2}\) s\(^{-1}\) or an e\(^+\)e\(^-\) collider with \( \sqrt{s} = 2-3 \) TeV and \( L = 1-2.5 \times 10^{33} \) cm\(^{-2}\) s\(^{-1}\) are minimal facilities for this strong interaction WW physics. The former corresponds to the design parameters of the SSC (Superconducting Super Collider), which can be built with existing technology (147) but poses great challenges in experimental design (148). The latter offers a more benign experimental environment (149) but the necessary linear collider technology remains to be demonstrated and developed (150).

Given the tiny interaction with ordinary matter it is not surprising that existing searches for light Higgs bosons are far from comprehensive. The situation will improve dramatically with the operation of new facilities. A comprehensive search for the standard model Higgs boson below \( \sim 40 \) GeV may be possible when the Z factories—the Stanford Linear Collider (SLC) at SLAC and the Large Electron-Positron Collider (LEP) at CERN—have produced Z's of order \( 5 \times 10^6 \). LEP 200, the second stage of LEP scheduled for the early 1990s, will probably run at \( \sqrt{s} = 180 \) to 190 GeV, extending the search to perhaps 70 GeV in the Higgs boson mass. In the mid 1990s the Large Hadron Collider (LHC), a 16-TeV pp collider under study at CERN could carry the search to 600 GeV. In the same time frame the SSC at \( \sqrt{s} = 40 \) TeV could search to the 1-TeV limit.
suggested by the renormalization group studies (cf Section 5.3) and on into the strong interaction domain beyond 1 TeV.

There is one potential gap in this scenario of new facilities. If the standard model Higgs boson is too heavy to detect at LEP 200 ($m_h \geq 70$ GeV) and lies between the $\bar{t}t$ and $ZZ$ thresholds ($2m_t < m_h < 2M_Z$), then it might not be detectable at the LHC or SSC where the $h \to \bar{t}t$ signal may be lost in the $\bar{t}t$ QCD background. Though alternative strategies for the pp colliders are still under study, a high-energy $e^+e^-$ collider with $\sqrt{s} \gtrsim 320$ GeV and $L \gtrsim 10^{32}$ cm$^{-2}$ s$^{-1}$ might be needed. The TLC (TeV Linear Collider under study at SLAC) with $\sqrt{s} = \frac{1}{2}$ to 1 TeV and $L = 10^{33}$ cm$^{-2}$ s$^{-1}$ or CLIC (CERN Linear Collider at CERN) with $\sqrt{s} = 1$ to 2 TeV and $L = (1-4) \times 10^{33}$ cm$^{-2}$ s$^{-1}$ could fill this potential gap. The SSC is also important in this scenario, since the absence of strong WW interactions above 1 TeV would be an unambiguous signal to press the search below 1 TeV.

In the remainder of this section I review previous searches and future prospects. In discussing Higgs bosons I focus on the standard model for simplicity, with some comments on multi-Higgs doublet models as required by supersymmetry. Experimental bounds for the standard model case can always be translated into constraints on the parameters $\alpha$ and $\beta$ of the two-doublet model introduced in Section 4.

Section 6.1 is a review of Higgs boson decay modes as a function of the Higgs boson mass. Section 6.2 is a description of existing experimental constraints, while Sections 6.3 and 6.4 are concerned with future prospects for Higgs searches below and above the ZZ threshold. Signals of dynamical symmetry breaking above 1 TeV are discussed in Section 6.5.

6.1 Higgs Boson Decays

6.1.1 DECAYS TO GAUGE BOSONS, HEAVY QUARKS, AND LEPTONS. For Higgs bosons above charm threshold, $m_h \gtrsim 3.7$ GeV, the predominant decay modes are reliably computed from the Yukawa coupling $\kappa_t = gm_t/2M_w$, Equation 46, and from the $hWW$ and $hZZ$ couplings, $gM_w$ and $gM_Z/cos \theta_w$ respectively. Above WW and ZZ threshold, the WW and ZZ decays predominate for any value of $m_t$ if there are only three generations. Except very near threshold the W’s and Z’s are predominantly longitudinally polarized. The WW partial width is

$$\Gamma(h \to WW) = \frac{G_F m_h^3}{8\sqrt{2}\pi} \beta_w \left( \beta_w + \frac{12M_w^4}{m_h^4} \right),$$

where $\beta_w$ is the W velocity in the h rest frame. $\Gamma(h \to ZZ)$ is given by one half of the same expression with $W \to Z$. At $m_h = 1$ TeV, $\Gamma(h \to
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WW + ZZ) ≃ 500 GeV, and at 1.4 TeV the width and mass are equal. Like the renormalization group (Section 5.3), lowest order perturbation theory seems to be saying that nature is not likely to present us with a Higgs boson heavier than 1 TeV. Surprisingly, although \( m_h = 1 \) TeV marks the onset of strong coupling (31), the one-loop correction to Equation 170 for \( m_h = 1 \) TeV is only about 5% (133).

Below WW threshold but above charm threshold, the Higgs boson decays predominantly to the heaviest accessible quark or lepton pair. The decay width is

\[
\Gamma(h \rightarrow \ell\ell) = \frac{G_F m_h^2 b \beta^2}{4 \sqrt{2} \pi} \tag{171}
\]

where \( \beta = 3 \) for quarks and 1 for leptons. Therefore for \( 4 < m_h < 10 \) GeV we expect charm and \( \tau \tau \) decays to dominate with relative rates of \( \bar{c}c : \tau\tau \approx 3m_c^2 : m_\tau^2 \approx 2:1 \). Above \( \bar{b}b \) threshold, the \( \bar{c}c + \tau\tau \) fraction decreases to \( \sim 5\% \). Since \( m_t > 25 \) GeV the \( \bar{b}b \) partial width would be \( \lesssim 4\% \) of the total width above \( \tilde{t}t \) threshold.

6.1.2 DECAYS TO LIGHT HADRONs Below charm threshold the decays to light hadrons (made of u, d, and s quarks) are not so readily understood, essentially because the masses of the light hadrons are not dominated by the "bare" or "current" quark masses that determine the Yukawa coupling constants, Equation 46. The latter at \( m_u \approx 4 \) MeV, \( m_d \approx 7 \) MeV, and \( m_s \approx 150 \) MeV (50, 52) are substantially smaller than the "constituent" quark masses of the nonrelativistic quark model, \( m_u \approx m_d \approx 300 \) MeV and \( m_s \approx 500 \) MeV. The nucleon owes its mass predominantly to the gluons that dress the bare quarks (151) and perhaps also to a significant \( s\bar{s} \) component in the \( \bar{q}q \) sea (152).

The coupling of a light Higgs boson, \( m_h \lesssim 1 \) GeV, to the light hadrons can be estimated by observing (26) that it couples like a dilaton, the Goldstone boson that would have existed if scale invariance were spontaneously broken. In fact, if all masses are set equal to zero, scale invariance is not broken spontaneously but rather by gauge interactions. This is seen most directly in the trace anomaly (153, 154), which is to scale symmetry what the chiral anomaly is to chiral symmetry. Using the trace anomaly it is possible to estimate the nucleon-Higgs coupling (152),

\[
\kappa_N = \frac{g}{2M_w} (70 \text{ MeV}) n_Q \tag{172}
\]

where \( n_Q \) is the number of heavy quarks; \( n_Q = 3 \) for c, b, t in the absence of a fourth generation.

Equation 172 is much larger than what we would find from Equation
using current quark masses, but it is also a factor $\sim 5$ smaller than what we would obtain by treating the nucleon as an elementary fermion in Equation 46. The physical intuition behind Equation 172 is that the nucleon mass is dominated by the gluonic component of its wave function, which couples to the stress-energy-momentum tensor by virtue of the trace anomaly. Though it probably suggests the right order of magnitude, Equation 172 is not quantitatively reliable because it is based on a leading order evaluation of the anomaly. Unlike the chiral anomaly, the trace anomaly is not well represented at low energy by the leading order but requires knowledge of the exact QCD beta function (154). Equation 172 may also be modified if there is an important $\bar{s}s$ component in the nucleon wave function (152).

The coupling of a light Higgs boson to pions can be estimated by a similar method, using the lowest order trace anomaly and chiral symmetry (155). The result is

$$\kappa_\pi = -\frac{g}{2N_c} \frac{2}{9} \left( \frac{m_h^2 + \frac{11}{2} m_\pi^2}{m_h^2} \right) \tag{173}$$

where $n_Q = 3$ heavy quarks are assumed. Equations 173 and 171 together imply for $2m_\pi < m_h < 1$ GeV that the dominant $\pi\pi$ and $\mu\mu$ decays are in the ratio

$$\frac{\Gamma(h \to \pi\pi)}{\Gamma(h \to \mu\mu)} = \frac{1}{27} \frac{m_h^2}{m_\mu^2} \left( 1 + \frac{11}{2} \frac{m_\pi^2}{m_h^2} \right) \frac{p_\pi}{p_\mu^2}. \tag{174}$$

The $\pi\pi$ decay dominates over most of that range, by 4:1 at $m_h = 1$ GeV. Between $m_h = 1$ and 2 GeV, $\bar{K}K$ and $\eta\eta$ final states are expected to occur by the same mechanism and with the same strength (155). As for Equation 172, Equations 173 and 174 are only believable as order-of-magnitude estimates.

6.1.3 DECAY TO TWO PHOTONS Rare decay modes are also of interest, for instance, in searching below the ZZ threshold at the LHC and SSC for Higgs bosons that may be unobservable in their dominant $t\bar{t}$ or $b\bar{b}$ decay modes. The two-photon decay mode might be useful if $h \to b\bar{b}$ dominates (156). It is given by a one-loop amplitude (26, 157),

$$\Gamma(h \to \gamma\gamma) = \frac{\alpha^2 G_F m_h^3}{64\sqrt{2}\pi^3} \left| \sum_i F_i Q_i^2 \right|^2, \tag{175}$$

where $Q_i$ are the charges of the quanta in the loop. For $\tau = 4m_t^2/m_h^2$ the coefficients $F_i$ are given for quanta of spin 0, $\frac{1}{2}$, 1 respectively by $\tau(1-\tau)I^2, -2\tau[1+(1-\tau)I^2]$, and $2+3\tau+3\tau(2-\tau)I^2$, where (157)
For very light Higgs bosons, \( m_h \ll 1 \text{ GeV} \), the width is decreased by a negative contribution from the QCD trace anomaly that reduces the fermionic contribution to the amplitude by about half but is negligible for heavier Higgs bosons (158).

6.2 Present Status

Even for very light Higgs bosons, direct experimental constraints are few and far between. They suffer from uncertainties in the couplings to light hadrons that were evident in Section 6.1 and from additional theoretical uncertainties described below.

6.2.1 Atomic and Nuclear Physics Very light Higgs bosons might create observable effects in the spectra of muonic atoms. X-ray anomalies in barium and lead were at one time interpreted as possible evidence for a scalar below 22 MeV (159) but the anomalies were not confirmed in subsequent studies (160).

Searches for forbidden nuclear transitions provide plausible bounds on the Higgs boson in the MeV range. A negative search (161) for the forbidden transition \( ^{16}\text{O}^* \rightarrow ^{16}\text{O}(\text{ground state}) + e^+ e^- \) easily has sufficient sensitivity to exclude a Higgs mass in the interval \( 1.03 < m_h < 5.54 \text{ MeV} \) when Equation 172 is used for the nucleon-Higgs coupling with \( n_Q = 2 \). A reexamination of data from (161) on a similar forbidden transition \( ^{4}\text{He}^* \rightarrow ^{4}\text{He}(20.1 \text{ MeV}, 0^+) \) failed to confirm the claim that the bound could be extended to 18 MeV (162). A more recent measurement of the helium transition is able to exclude \( 2.8 < m_h < 11.5 \text{ MeV} \) assuming Equation 172 with \( n_Q = 4 \) (163). (The value \( n_Q = 4 \) is a reasonable choice since for this range of Higgs boson mass the strange quark is indeed heavy.)

6.2.2 Radiative Decay of Heavy Quarkonium Returning to high-energy physics, a light Higgs boson could be produced in radiative decays of heavy quarkonium, \( J/\psi \) and \( \Upsilon \). In a nonrelativistic approximation for the quarkonium states and to leading order in QCD, the (corrected) partial width is (164)

\[
\frac{\Gamma(V \rightarrow h\gamma)}{\Gamma(V \rightarrow \mu^+\mu^-)} = \frac{G_F m_V^3}{4\sqrt{2}\pi\alpha} \left( 1 - \frac{m_V^2}{m_h^2} \right). \tag{177}
\]
However the $O(\alpha_s)$ QCD correction is large, decreasing Equation 177 by almost a factor of 2 in the case of $\Upsilon(9.46)$ (165). Since the leading correction is so big we may wonder whether the next order is also large and therefore whether the perturbation expansion is valid. Using the $O(\alpha_s)$ QCD result the CUSB collaboration reports that they are able to exclude $m_h < 3.9$ GeV by combining data from $\Upsilon(9.46)$ and $\Upsilon''(10.36)$ (166). However further doubt is raised by conflicting calculations of the relativistic corrections to Equation 177, which claim to decrease the prediction by another factor of two (167) (thereby evading the CUSB data) or to increase it by a large factor (168). The interpretation of the data will be in question until the theoretical uncertainties as to both the QCD and relativistic corrections are resolved.

6.2.3 HEAVY QUARK DECAY Heavy quark decay by Higgs boson emission, $Q \rightarrow qh$, has been advocated as an effective search method (169, 170) and has been used as evidence against a Higgs boson interpretation of the narrow state $\xi(2220)$ seen in $J/\psi$ radiative decay (171). Here, too, the conclusions are uncertain because of an unsettled theoretical problem concerning the exclusive decay $K \rightarrow h\pi$ and the inclusive decays $D \rightarrow hX$ and $B \rightarrow hX$. All are computed in the spectator model using the quark level amplitude $Q \rightarrow hq$, where $Q = s, c, \text{or } b$ and $q = d, u, \text{or } s$ respectively. Explicit calculation indicates (169) that the dominant contribution is given by a sum of one-loop graphs containing a heavy quark $Q'$. The explicit result for the decay of the pseudoscalar $P$ containing heavy quark $Q$ as constituent and heavy quark $Q'$ in the loop is (169)

$$\frac{\Gamma(P \rightarrow hX)}{\Gamma(P \rightarrow e\bar{\nu}X)} = \frac{|V_{QQ}V_{\bar{Q'}q}|^2}{|V_{Qq}|} \frac{27}{64\pi^2} \left(\frac{m_Q}{m_Q'}\right)^4 \text{LIPS}$$

where $V_{ij}$ are the KM matrix elements and LIPS denotes Lorentz-invariant phase space factors. The important feature is the factor $(m_Q/m_Q')^4$, which for $B$ decay becomes $(m_t/m_b)^4 \approx 5^4$ using the present lower bound (41) on $m_t$. Equation 178 implies that the inclusive branching ratio is $BR(B \rightarrow hX) \approx 10^{-3} \times (m_t/25 \text{ GeV})^4(1 - m_Q^2/m_b^2)$, which is at the percent level if $m_t \lesssim 50$ GeV.

This explicit result is in conflict with the general observation that the Higgs boson couples to matter like a dilaton (26). Provided gluonic interactions are not important so that the trace anomaly can be neglected, the relevant amplitude is related to the trace of the stress tensor

$$\langle q, h|Q \rangle = \frac{i g}{2 M_w} \langle q|\theta_\mu^{\mu}|Q \rangle.$$

179.
A kinematical analysis then shows that for $m_q \ll m_Q$ the right side of Equation 170 is proportional to a factor $m_0^2$ that is not reflected in Equation 178. Consequently the general analysis implies an extra suppression factor of $(m_Q/M_W)^4$, which for $B$ decays is $10^{-3}$. If this conclusion is correct, these heavy quark decays would be much too rare to observe. It is therefore important either to find a flaw in the general analysis or to show that the explicit calculation fails to respect the proper constraints of scale invariance.

6.2.4 CHARGED HIGGS BOSONS Our discussion of existing searches has up to this point been restricted to the standard model, though in each case a bound on the parameter space of the two-doublet model is implied (cf Section 4). While nonminimal models typically have several free parameters and are therefore harder to constrain experimentally, they unambiguously predict the existence of charged Higgs bosons that would be produced in $e^+e^-$ annihilation with point-like cross sections. Far above threshold but well below the $Z$, the cross section $\sigma$ is given by $\sigma/\sigma_{\text{point}} \equiv R = Q^2/4$, where $Q$ is the Higgs boson charge and $\sigma_{\text{point}}$ is the QED cross section for $e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-$. Because of the p-wave phase space, $\propto Q^2$, the cross section grows slowly from threshold. A recently reported search for $e^+e^- \rightarrow H^+H^-$ at $\sqrt{s} = 46.8$ GeV was performed by looking for $H^\pm$ decays to $\tau\nu$, $c\bar{s}$, and $c\bar{c}$ (172). A lower bound $m_{H^\pm} > 19$ GeV (95% CL) was obtained, independent of the relative branching ratios of the three assumed decays.

Charged Higgs bosons can also give rise to large flavor-changing neutral currents in one-loop order. Analogous effects do not occur in the minimal one-doublet model because quarks get their masses from the vacuum expectation value $\langle h \rangle$, Equation 43, so that when the mass matrix is diagonalized the resulting quark eigenstates also have diagonal (flavor-conserving) interactions with $h$. In nonminimal models neutral currents due to the neutral Higgs bosons are similarly avoided by the prescription that all quarks of a given charge interact with only a single Higgs boson (88). This prescription is followed in the two-doublet supersymmetric models discussed in Section 4. But no such principle operates to provide GIM suppression of one-loop neutral flavor-changing interactions due to charged Higgs boson exchanges, which as a result can include large effects. For instance, the $K_S-K_L$ mass difference gets contributions proportional to $1/m_{H^\pm}^2$ to be compared with the usual GIM-suppressed contributions, $\propto m_t^2/M_W^4$ (173). These contributions are used by Abbott et al (173) to restrict the parameter space of the two-doublet models. Similar though stronger bounds are obtained from $CP$-violating contributions to the $K$ mass matrix (174) and from $B\bar{B}$ mixing (175), both effects amplified by large terms proportional to powers of the $t$ quark mass.
6.2.5  \(\xi(2220)\) as a Higgs Boson Candidate

The \(\xi(2220)\), a narrow state seen in \(\psi \rightarrow \gamma\xi\), \(\xi \rightarrow K_S K_S + K^+ K^-\), with \(\Gamma = 18 \pm 25\) MeV (176), has been considered as a Higgs boson candidate. Since the rate \(\Gamma(\psi \rightarrow \gamma\xi)\) is at least an order of magnitude larger than what is expected from Equation 177, it might be interpreted (177) as the lighter neutral Higgs scalar in a two-doublet model with enhanced couplings to \(T_3 = \pm 1/2\) quarks, as allowed by the parameters \(a\) and \(\beta\) discussed in Section 4.1. At this moment the Higgs boson interpretation seems unlikely because a state of the same mass and width has been observed in \(Kp\) scattering, decaying to \(K^+ K^-\) and \(K_S K_S\) (178). The cross section is many orders of magnitude larger than that of a Higgs boson, which would be unobservably small, and the angular momentum is \(J \geq 2\). (No spin measurement has been made in \(\psi \rightarrow \gamma\xi\).) These results suggest that the \(\xi\) is a hadron, perhaps a \(2^{++}\) state (179) or a \(4^{++}\) \(qg\) meikton (180, 181).

6.3  Prospects for Higgs Boson Detection Below ZZ Threshold

This and the next two sections are intended to give a bird’s-eye view of the future prospects for finding the symmetry-breaking sector. Since the experimental effort is either imminent, as in the case of SLC and LEP, or in the active planning stage, as for LEP 200, LHC, and the SSC, many detailed studies and reviews are available, several of which are cited below. In this section we consider the standard model Higgs boson with mass below the ZZ threshold. The discussion is readily extended to the lighter neutral Higgs scalar expected in supersymmetric models (cf Section 4).

6.3.1  \(Z \rightarrow \nu Z^*\)  

\(Z\) decay to the Higgs boson plus a lepton pair, proceeding by the mechanism \(Z \rightarrow hZ^*\), \(Z^* \rightarrow \ell\ell\), is a powerful method to search for Higgs bosons lighter than 50 GeV (182). Prospects for detecting the signal over backgrounds are good when the lepton \(\ell\) is an electron, a muon, or even a neutrino. Since it originates in a virtual \(Z\) boson, denoted by \(Z^*\) above, the \(\ell\ell\) pair tends to peak at the largest kinematically allowed invariant mass, a feature that helps distinguish the signal from backgrounds. The branching ratio for \(Z \rightarrow h\mu^+\mu^-\) relative to \(Z \rightarrow \mu^+\mu^-\) is shown in Figure 3, taken from (183). The branching ratio for \(Z \rightarrow h\nu\overline{\nu}\) assuming three neutrino species is six times larger.

The total \(Z\) production cross section at the resonance peak is

\[
\sigma(Z) = \frac{9}{\alpha^3} \frac{BR(Z \rightarrow e^+e^-)}{M_Z^3} \frac{87}{M_Z^2}
\]

where \(\alpha = 1/137\) and \(\sigma\) is expressed in nanobarns for \(M_Z\) given in GeV. Then for \(M_Z = 92\) GeV and \(BR(Z \rightarrow e^+e^-) = 0.033\) we have \(\sigma(Z) = 57\)
Figure 3  Comparison of the decay rates for $Z \rightarrow h\gamma$ and $Z \rightarrow h\mu^+\mu^-$. The curves show the ratios $\Gamma(Z \rightarrow h\gamma)/\Gamma(Z \rightarrow \mu^+\mu^-)$ (solid line) and $\Gamma(Z \rightarrow h\mu^+\mu^-)/\Gamma(Z \rightarrow \mu^+\mu^-)$ (dashed line) as a function of the mass ratio $M_h/M_Z$. The curves are computed for $\sin^2\theta_W = 0.2$ so that $M_Z = 94$ GeV (from 183).

For a luminosity $L = 10^{31} \text{ cm}^{-2} \text{s}^{-1}$ there are $6 \times 10^6$ $Z$'s produced in an experimental year of $10^7$ s. For $m_h = 40$ GeV we then find $\sim 55$ Higgs bosons produced in the process $Z \rightarrow hZ^*$, $Z^* \rightarrow e^+e^- + \mu^+\mu^-$, while for $m_h = 10$ GeV the yield is $\sim 800$. In these events the Higgs boson is detected inclusively as a peak in the spectrum of the invariant mass recoiling against the $e^+e^-$ or $\mu^+\mu^-$ pair. A Monte Carlo study (184) indicates that the principal background, from $Z \rightarrow t\bar{t}$ with both $t$ and $\bar{t}$ decaying semi-leptonically, can be cleanly separated from the signal for $m_h < 40$ GeV and perhaps even out to $m_h = 50$ GeV where the signal is three times smaller.

An earlier study (185) of the same reaction identified a possible difficulty for Higgs masses below 20 GeV. Even assuming optimistically that the $e^\pm$ energy resolution is $\Delta E/E = 0.1/\sqrt{E}$ (for $E$ expressed in GeV), the resolution of the recoil mass is seriously degraded, reaching $\Delta m_h/m_h = 1$ at
$m_h = 10$ GeV. The problem is more severe for muons with the assumed energy resolution $\Delta E/E = 0.15$. The problem is not discussed in the more recent study (184).

The decay $Z \rightarrow h\bar{\nu}$ is, of course, less clean but it compensates with a rate three times larger than the $e^+e^- + \mu^+\mu^-$ channel. The principal background is from events with neutrons and $K_L$'s that escape the detector. A Monte Carlo study for $m_h = 20$ and 30 GeV shows clear signals above background in events with two well-separated jets ($h \rightarrow b\bar{b}$ is assumed) and missing mass greater than 40 GeV (taking advantage of the propensity of the $\bar{\nu}\nu$ pair for large invariant mass) (184).

6.3.2 $Z^* \rightarrow hZ$ The reaction $e^+e^- \rightarrow Z^* \rightarrow hZ$ is an excellent discovery channel (26) for the Higgs boson above the range that can be found in $Z$ decay, say $40$ GeV < $m_h < 2M_Z$. This is like the process discussed in Section 6.3.1 with real and virtual $Z$'s, $Z$ and $Z^*$, interchanged. For a given Higgs boson mass the ratio $R = \sigma/\sigma_{\text{point}}$, where $\sigma_{\text{point}} \equiv \sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)$, is maximized at $s = \sqrt{2m_h M_z}$, where it is given by (185)

$$R_{\text{max}}(Zh) = \frac{3}{64} \left( \frac{M_Z}{38 \text{ GeV}} \right)^4 \frac{M_Z}{2\sqrt{2m_h}} [1 + (1 - 4 \sin^2 \theta_w)^2].$$

For $M_Z = 92$ GeV and $\sin^2 \theta_w = 0.23$ this is $R_{\text{max}}(Zh) = 53$ GeV/$m_h$. Then for Higgs bosons just below ZZ threshold, $m_h = 2M_Z$, we have $R_{\text{max}}(Zh) = 0.29$ with $s = (2\sqrt{2+1})M_Z = 350$ GeV so that $\sigma(Zh) = 0.26$ pb. A 350-GeV collider with luminosity $L = 2 \times 10^{32}$ cm$^{-2}$ s$^{-1}$ would then produce 400 events in this channel, 26 with $Z \rightarrow e^+e^- + \mu^+\mu^-$ and 78 with $Z \rightarrow \bar{\nu}\nu$. The maximum cross section for lighter Higgs bosons increases relative to this case as $(2m_h/m_Z) \times [(2\sqrt{2+1})M_Z/(\sqrt{2m_h + M_Z})]^2$.

This process was studied with a Monte Carlo simulation for an example taken in the center of the LEP 200 range, $\sqrt{s} = 160$ GeV and $L = 10^{32}$ cm$^{-2}$ s$^{-1}$ (184). For $m_h = 50$ GeV there are then 116 events in $e^+e^- \rightarrow hZ$, $Z \rightarrow e^+e^- + \mu^+\mu^-$ for a run of $T = 10^7$ s. With a modest sphericity cut $\geq 0.05$ the signal stands out clearly against the background from $e^+e^- \rightarrow \bar{b}b$ or $t\bar{t}$ followed by semileptonic decays of both heavy quarks. Barbiellini et al (185) again indicate a serious degradation of the recoil mass resolution below 20 GeV. Because of the natural $Z$ width, the resolution is not improved by constrained fitting of the lepton pair mass.

6.3.3 $Z \rightarrow h\gamma$ The radiative $Z$ decay $Z \rightarrow h\gamma$ is of interest because it tests the theory at the one-loop level and because it "counts" heavy fermions (183, 186, 187). It does not occur in leading order but by virtue of W boson and fermion loop diagrams. For $m_t \ll M_W$ the fermion contributions to the amplitude are damped relative to the W contribution by $m_t^2/M_W^2 \ll 1$, resulting in a large and unambiguous signal at high $p_T$.
while ultraheavy fermions with $m_r \gg M_w$ make a contribution of order one that is independent of $m_r$. For instance, a complete standard model generation of ultraheavy fermions (with massless or light neutrino) would contribute $-1/7$ of the W loop contribution to the amplitude, which implies a decrease of $\sim 30\%$ in the decay rate. Retaining only the W loop contribution, the decay width is (183)

$$\frac{\Gamma(Z \to h\gamma)}{\Gamma(Z \to \mu^+ \mu^-)} \approx 7.8 \times 10^{-5} \left(1 - \frac{m_h^2}{M_Z^2}\right)^3 \left(1 + 0.17 \frac{m_h^2}{M_Z^2}\right),$$

which is plotted in Figure 3 (see also 186, 187). For $m_h \ll M_Z$ this corresponds to $\sim 15$ events for $6 \times 10^6$ produced Z's (a $10^7$-second run at $L = 10^{31}$ cm$^{-2}$ s$^{-1}$) using $\text{BR}(Z \to \mu^+ \mu^-) = 0.033$. At $m_h = 40$ GeV the signal is reduced to $\sim 9$ events, about $1/3$ the signal for $Z \to hZ^*, Z^* \to e^+ e^-$ at the same Higgs boson mass.

For the Higgs bosons mass range $10 < m_h < 40$ GeV, the dominant decay mode is $h \to \bar{b}b$, with only a few percent for $h \to \tau^+ \tau^-$. If heavy quarks could be tagged with a high-efficiency vertex detector, the dominant background would come from $Z \to \bar{b}b\gamma$ or $\bar{c}c\gamma$. No feasibility study has been reported that determines the prospects of observing the signal above this background. Barbiellini et al (185) studied the background in a hypothetical case, assuming the Higgs boson decays predominantly to a 10-GeV heavy lepton, $h \to L^+ L^-$. They showed that a simple cut—requiring the $L^+ L^-$ pair to be in the hemisphere opposite to the photon—reduced the $Z \to L^+ L^- \gamma$ background far below the signal with negligible loss of signal for $m_h \lesssim 35$ GeV. The same cut would be less effective against the $\bar{b}b\gamma$ and $\bar{c}c\gamma$ backgrounds.

6.3.4 WW Fusion The WW fusion mechanism (188, 189) has been much studied in recent years as the preferred mechanism in pp and e$^+ e^-$ colliders to produce heavy Higgs bosons ($m_h \gtrsim 300$ GeV) and the quanta of dynamical symmetry breaking above 1 TeV—subjects discussed in Sections 6.4 and 6.5 respectively. It can also be used in e$^+ e^-$ colliders of sufficiently high energy to produce Higgs bosons in the mass range below ZZ threshold (149, 190). The process is like photon-photon scattering with the photons replaced by off-mass-shell $W$'s, $e^+ e^- \to e^+ e^- W^* W^*$, $W^* W^* \to h$. For sufficiently large $W^* W^*$ invariant mass, the cross section (but not the $p_t$ distribution) can even be evaluated by an effective $W$ approximation (191) closely analogous to the Weizsacker-Williams approximation familiar in photon-photon scattering (192).

Unlike the much more difficult situation in hadron colliders, discussed below, the signal for $e^+ e^- \to e^+ e^- h$, $h \to \bar{b}b$ or $\bar{t}t$, is observable over backgrounds without requiring $t$ and $b$ identification (186). (In pp colliders...
the analogous signal is not observable even assuming t and b identification.) The photon-photon scattering backgrounds, $e^+e^- \rightarrow e^+e^-\gamma\gamma$, $\gamma\gamma \rightarrow$ hadrons, are distinguished from the signal by the characteristic transverse momentum of the Higgs boson produced in WW fusion $p_T(h) \approx O(M_W)$. For $m_h$ between 100 and 200 GeV, the cross section after cuts dominates the background and varies from $10^{-1}$ to $10^{-2}$ pb at a $\sqrt{s} = 1$ TeV collider (190); this provides 1000 to 100 events for a 10$^7$-second experiment at $L = 10^{33}$ cm$^{-2}$ s$^{-1}$. The signal at a $\sqrt{s} = 2$ TeV collider is about twice as large: like photon-photon scattering at fixed $\gamma\gamma$ invariant mass, the cross section increases with increasing beam energy.

6.3.5 Toponium Depending on the mass of the $J^{PC} = 1^{--}$ toponium ground state $\theta$, the radiative decay $\theta \rightarrow h\gamma$ can provide a very effective Higgs boson search for $m_h \lesssim 0.7m_\theta$. As $m_\theta$ approaches $M_Z$, Equation 177 is no longer valid since it assumes $\theta \rightarrow \gamma^* \rightarrow \mu^+\mu^-$ and neglects the effect of the Z pole $\theta \rightarrow Z^* \rightarrow \mu^+\mu^-$. For $m_\theta$ within a few GeV of $M_Z$, the total width of $\theta$ is greatly increased so that $BR(\theta \rightarrow h\gamma)$ becomes hopelessly small, far below the percent level branching ratio expected for $m_\theta \lesssim 80$ GeV (184, 193, 194). Above the Z pole, the $\theta$ width is also enhanced by t-channel W exchange, $\theta \rightarrow \bar{b}b$, and eventually by the semiweak decay $\theta \rightarrow \bar{t}W^- + bW^+$, which again renders $\theta \rightarrow h\gamma$ unobservably small.

Though the possible relativistic corrections discussed in Section 6.2.2 should be much less important for toponium, the QCD one-loop correction is still a great source of uncertainty (165). The correction to $\Gamma(\theta \rightarrow h\gamma)$ is a multiplicative factor,

$$\frac{\Gamma(\text{one loop})}{\Gamma(\text{leading order})} = 1 - \frac{4\alpha_s}{3\pi}a(x),$$

where $x \equiv m_h^2/m_\theta^2$ and $a(x) \approx 10$ for $x \lesssim 0.8$ and increases sharply for $x \gtrsim 0.8$. Thus for $\alpha_s \approx 1/10$ the correction is large, which casts doubt on the reliability of the perturbation expansion. As discussed for $\psi$ and $\Upsilon$ in Section 6.2, it will therefore be difficult to give a precise interpretation to negative searches.

Nevertheless, with the plausible assumption that Equation 183 correctly describes the order of magnitude, it is clear that $\theta \rightarrow h\gamma$ is an excellent channel if $\theta$ is not too close to the Z or to the WW threshold. Decreasing the estimates of (193) to include the effect of Equation 183, for $m_\theta = 70$ GeV an experiment of 100 pb$^{-1}$ and the anticipated LEP beam spread could expect from $\sim 300 \theta \rightarrow h\gamma$ events for $m_h = 10$ GeV to $\sim 80$ events if $m_h = 60$ GeV. In addition to the uncertainties from Equation 183, these estimates are uncertain by about a factor of two because of the unknown value of the $\theta$ wave function at the origin (193). A Monte Carlo study...
for this case (193) indicates that the signal stands out clearly over the backgrounds for \( m_h \lesssim 55 \text{ GeV} \).

Exchange of a sufficiently light Higgs boson adds an appreciable short-range attractive force to the \( t\bar{t} \) potential, affecting the toponium spectrum and wave function. The 1S ground state is more deeply bound so that the 2S-1S mass splitting is increased (195). The wave function at the origin for the 1S ground state can be significantly increased, which would increase the annihilation decay widths, and a possible qualitative signal is inversion of the 1P-2S splitting (175). However, as observed by Athanasiu et al (175), because of the uncertainties in the QCD contribution to the binding potential it is difficult to make reliable quantitative predictions.

### 6.3.6 Hadronic Production

The Higgs boson with \( m_h < 2M_Z \) is copiously produced in multi-TeV proton-proton collisions by the gluon-gluon fusion mechanism (196). For instance in a \( 10^7 \)-second run at the SSC about \( 10^6 \) Higgs bosons would be produced with mass \( 100 \text{ GeV} < m_h < 2M_w \).

However if \( m_h > 2m_t \), so that \( h \rightarrow t\bar{t} \) is the dominant decay, that large signal is overwhelmed by an even larger QCD \( t\bar{t} \) background, which is about 100 times larger than the signal if we optimistically assume a 5% resolution for the invariant \( t\bar{t} \) mass (197) (and also assume perfect \( t \) quark tagging!). Much effort has been focused on this problem, with no encouraging results if \( h \rightarrow t\bar{t} \) dominates (197–200). If the Higgs boson is below the \( t\bar{t} \) threshold so that \( h \rightarrow \bar{b}b \) dominates, the prospects for detection are more promising though still far from established in the \( \bar{b}b \) decay channel (201) or in the rare \( \tau^+\tau^- \) (200) and \( \gamma\gamma \) channels (156).

### 6.3.7 Radiative Corrections

In principle the Higgs boson could betray its presence indirectly by inducing observable radiative corrections. The largest effects might be expected from the heavy Higgs boson, discussed in the next section. For instance, for \( m_h = 1 \text{ TeV} \), the quartic coupling constant is \( \lambda = m_h^2/2v^2 = 8 \) so that according to the equivalence theorem one-loop corrections to processes involving longitudinal \( W \)'s and \( Z \)'s could be of order \( \lambda/\pi^2 = O(1) \). Corrections of this order to the \( W \) and \( Z \) masses should then show up as a large correction to the tree level relationship \( \rho = 1 \), Equation 42. In fact, in the standard model all potentially large corrections that can be probed in low-energy processes are “screened” (40), in the case of the rho parameter because of the standard model custodial \( SU(2) \) symmetry discussed in Section 1.3.

The standard model Higgs sector corrections to \( \rho \) have been computed to two-loop order (202). Instead of being proportional to \( \lambda \) for large \( \lambda \), the one-loop correction is proportional to \( \alpha \ln \lambda \),

\[
\delta \rho_1 = -\frac{3\alpha}{16\pi \cos^2 \theta_W} \ln \frac{m_h^2}{M_W^2}.
\]

184.
Even for $m_h = 1$ TeV the correction is only $\delta \rho_1 = -2.8 \times 10^{-3}$, four times smaller than estimates of the current experimental uncertainty (24). The two-loop correction has the opposite sign,

$$\delta \rho_2 = 9.49 \times 10^{-4} \left( \frac{2\alpha}{\sin 2\theta_W} \right)^2 \frac{m_h^2}{M_W^2},$$

and is two orders of magnitude smaller than $\delta \rho_1$ for $m_h \leq 1$ TeV.

A complete one-loop calculation has been done for $e^+e^- \rightarrow W^+W^-$ in the standard model, in part to search for sensitivity to the value of the Higgs boson mass (203). As for the one-loop correction to $\rho$, the leading effect at large $m_h$ is logarithmic. For $\sqrt{s}$ from 180 to 1000 GeV the differential cross section varies by $\sim 5\%$ when $m_h$ is varied from 10 to 1000 GeV. Unfortunately the sensitivity to $m_h$ is greatest where the cross section is smallest.

The sensitivity of one-loop corrections to the Higgs sector of two-doublet models has been studied by Hollik (204) for processes that can be studied at LEP, SLC, and HERA (see also 205 for a review).

### 6.4 Prospects for Higgs Boson Detection Above ZZ Threshold

Unless the top quark mass nearly saturates the upper bound obtained from the rho parameter (33, 206), currently $m_t \lesssim 200$ GeV (24), WW fusion (188, 189) is the principal mechanism for production of the 1-TeV Higgs boson at pp or $e^+e^-$ colliders. (For $e^+e^-$ collisions WW fusion dominates regardless of the value of $m_t$.) In $e^+e^-$ scattering, $e^+e^- \rightarrow e^+e^-WW$, $WW \rightarrow h$ or in quark collisions, $qq \rightarrow qqWW$, $WW \rightarrow h$, the scattering fermions persevere into the final state. As in the more familiar photon-photon scattering process at $e^+e^-$ colliders, the kinematics favors collisions in which the final-state fermions retain a large fraction of the total energy and the cross section grows logarithmically with energy at fixed $m_h$. Therefore beam energy is at a premium. Even more than for other physics signals, not only the signal but also the signal-to-background ratio is rapidly enhanced by increasing the available energy.

To extend the Higgs boson search up to $m_h = 1$ TeV with colliders of luminosity $L = 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$ requires at least a $\sqrt{s} \simeq 40$ TeV proton-proton collider or a $\sqrt{s} \simeq 2$ TeV electron-positron collider, as discussed in Sections 6.4.2 and 6.4.3 below. The first corresponds to the design parameters of the SSC (147) while the second is a goal of design studies for CLIC (207).

A preliminary study has been performed to determine whether the Higgs
boson reach of a $\sqrt{s} = 16$ TeV pp collider [i.e. the LHC with 10-Tesla magnets (208)] could be extended to $m_h = 1$ TeV by increasing the luminosity to $L = 5 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ (209). The study concludes that it may be possible provided both electrons and muons can be detected and identified. However, as the authors are careful to observe, rate-associated problems such as radiation hardening and data acquisition already stretch the imagination at luminosities of $10^{33}$ cm$^{-2}$ s$^{-1}$. It is by no means clear that the associated problems at $L = 5 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ could be solved by the mid 1990s. If they are indeed solvable, a high-energy collider like the SSC would be well placed to reap the benefits because of the favorable signal-to-background ratios. That is, with larger signal-to-background ratio, the reach grows more steeply with increasing statistics. Signals that were barely visible at 1 TeV could be analyzed with the benefit of higher statistics, and the reach above 1 TeV could be extended.

In Section 6.4.1 we consider production mechanisms for heavy Higgs bosons at pp and $e^+e^-$ colliders, with a review of computational methods including the effective W approximation and the equivalence theorem. Sections 6.4.2 and 6.4.3 briefly describe what is known about the signals and backgrounds at multi-TeV pp and $e^+e^-$ colliders respectively. Excellent sources with additional references are the 1984 and 1986 Snowmass studies (210, 211) and the 1987 La Thuile study (212).

### 6.4.1 PRODUCTION MECHANISMS

At the multi-TeV pp colliders the two principal production mechanisms are gluon-gluon fusion (196) and WW fusion (188, 189), while only the latter is relevant at the TeV $e^+e^-$ colliders. (For pp colliders WW fusion is shorthand for the sum of WW and ZZ fusion.) In pp colliders the magnitude of the gluon-gluon fusion contribution scales quadratically with the mass of the heaviest quark, since the $ggh$ coupling is mediated by a quark loop. Like the $Z \rightarrow h\gamma$ loop amplitude discussed in Section 6.3.3, the amplitude is proportional to $m_q$ for $m_q$ less than the largest external scale, $m_h$ in this case, and is independent of $m_q$ for heavier quarks, which it therefore "counts." The gg fusion contribution is significant because of the very large gluon luminosity at the relevant energy scale, much larger than quark-quark luminosities at the same subprocess energy (60). For three generations with $m_t \approx 40$ GeV and $\sqrt{s} = 40$ TeV, $gg \rightarrow h$ dominates over $WW \rightarrow h$ for $m_h \lesssim 300$ GeV but quickly becomes negligible for larger $m_h$ (213). For $m_t = 200$ GeV $gg \rightarrow h$ dominates all the way up to $m_h = 1$ TeV, where it is almost twice the WW fusion contribution (213). For this discussion a 40-GeV top quark mass is usually assumed.

At TeV $e^+e^-$ colliders WW fusion is far more important than the $e^+e^- \rightarrow Zh$ discussed in Section 6.2. For $\sqrt{s} \gg m_h$ the former grows logarithmically (214)
\[ \sigma(e^+e^- \rightarrow \nu \bar{\nu} h) \approx \frac{\alpha^3 \ln \left( \frac{s}{m_h^2} \right)}{16 M_w^2 \sin^2 \theta_w} = 0.13 \ln \left( \frac{s}{m_h^2} \right) \text{ pb}, \]

while the latter falls like \( 1/s \), \( \sigma \approx (0.01/s) \) in picobarns for \( s \) in TeV\(^2\). In the regime of interest with \( m_h \) and \( \sqrt{s} \) of the same order of magnitude, WW fusion is still dominant. For instance, for \( m_h = 400 \text{ GeV} \) it dominates over \( e^+e^- \rightarrow Zh \) for \( \sqrt{s} \geq 500 \text{ GeV} \) (214).

The WW fusion cross sections for pp and \( e^+e^- \) colliders of various energies are shown in Figure 4 (taken from Reference 215). For \( m_h = 1 \text{ TeV} \) the pp cross section at \( \sqrt{s} = 40 \text{ TeV} \) is a factor of ten larger than at \( \sqrt{s} = 15 \text{ TeV} \) and \( \sim 40 \) times larger than the \( e^+e^- \) cross section at \( \sqrt{s} = 2 \text{ TeV} \). However, it will become clear in what follows that the advantage in rate of the 40-TeV pp collider over the \( e^+e^- \) collider is offset by the ability to utilize more of the \( e^+e^- \) signal. As discussed in Section 6.5, it is not known if a \( \sqrt{s} = 2 \text{ TeV} \) \( e^+e^- \) collider can also match the sensitivity of the SSC for the WW continuum signals of symmetry-breaking physics above.

![Figure 4](https://www.annualreviews.org/aronline)

**Figure 4** Higgs boson production cross sections for various pp and \( e^+e^- \) collider energies (from 215).
1 TeV. It certainly cannot match the reach of the SSC for resonances at \( \sim 2 \) TeV in the WW fusion channel.

The first calculations of WW fusion were done by numerical methods (188, 189). Subsequently, analytical expressions were derived for the double differential cross section in the final-state fermion energies (216) (useful for tagging) and for the differential cross section with respect to the Higgs boson three-momentum (215). In addition, a computationally simple approximation was derived, the effective W approximation (191), analogous to the more familiar Weizsacker-Williams approximation for photon-photon scattering (192). The effective W approximation is a small angle approximation that provides no information on the sometimes important transverse momentum distributions of the fermions and Higgs bosons. It is, however, generally adequate (see below) for calculating yields of heavy Higgs boson.

The result is an effective luminosity function for the probability to find colliding “beams” of longitudinally polarized gauge bosons \( V_1 \) and \( V_2 \) in incident fermions \( f_1 \) and \( f_2 \) (11, 191),

\[
\frac{\partial L}{\partial z} = \left( \alpha^2 \chi_1 \chi_2 \right) \frac{1}{\pi^2 \sin^4 \theta_W} \frac{1}{z} \left[ (1 + z) \ln \left( \frac{1}{z} \right) - 2 + 2z \right],
\]

where \( z \equiv s_{VV}/s_{ff} \) and \( \chi_1 \) are the \( f_1 \)-charged couplings, e.g. \( \chi_W = 1/4 \) for all fermions, \( \chi_{Z,\phi} = \left[ 1 + (1 - \frac{3}{8} \sin^2 \theta_w)^2 \right]/16 \cos^2 \theta_W \), etc. Equation 187 must be convoluted with the desired \( V_1 V_2 \) subprocess cross section, e.g. \( \sigma(V_1 V_2 \to h) \) if a narrow width approximation is appropriate for \( h \), and also with the quark distribution functions in the case of pp collisions.

The effective W approximation has been compared with analytical (215) and numerical (216, 217) evaluations of Higgs boson production. The most definitive results are probably the analytical calculations of Altarelli et al (215). They show good agreement for \( WW \to h \) for \( m_h \geq 500 \) GeV, with errors \( \lesssim O(10\%) \) and decreasing with \( m_h \) and \( \sqrt{s} \), while for the relatively less important process \( ZZ \to h \) the errors are roughly twice as large.

Care must also be taken in evaluating the subprocess cross section, sometimes characterized by \( \sigma(VV \to h) \) in narrow width approximation (e.g. 217a). At large values of \( m_h \), \( \Gamma_h \) is very large and \( \sigma(VV \to h) \) must be replaced by a calculation of what is actually measured, \( VV \to VV \) (i.e. \( W^+ W^- \to W^+ W^- , W^+ W^- \to ZZ, \) etc). There are then two possible ways of proceeding. Using the equivalence theorem (see Section 2) one can evaluate the scattering amplitudes for longitudinally polarized gauge bosons in an \( R \) gauge from the corresponding Goldstone boson amplitudes, including Higgs boson s-, t-, and u-channel exchange contributions (11). This procedure is computationally simple and automatically assures
the correct high-energy behavior. It provides a good approximation for $m_h$ and $\sqrt{s_{VW}}$ above 800 GeV, as shown for instance in Figure 5 (taken from Reference 17). The alternative is to use the unitary gauge, in which good high-energy behavior requires the cancellation of many diagrams involving gauge sector and Higgs sector exchanges (e.g. 215, 217, 218).

A potentially dangerous approximation found not uncommonly in the literature (216, 219) is to keep just the s-channel Higgs boson pole in the unitary gauge. By itself this amplitude has bad high-energy behavior and can lead to significant overestimates of the contribution from the high-energy tail of the resonance (220). The approximation is acceptable if the Higgs boson is not too heavy or too broad, say $m_h \leq 600$ GeV [as is the case for the applications considered by Cahn et al (219)].

The narrow width approximation, also not infrequent in the literature (15, 217a, 221), may misrepresent the heavy Higgs boson signal since it neglects the penalty of rapidly falling luminosity suffered on the high side of the resonance. It compensates by underestimating the signal on the low

![Figure 5](image_url)  
**Figure 5** Cross sections for the scattering of longitudinal gauge bosons ($WW \rightarrow WW$ and $WW \rightarrow ZZ$) in the standard model with $m_h = 1$ TeV. *Solid curves:* full perturbative calculation. *Dashed curves:* calculation based on the equivalence theorem (from 17).
side; however, as discussed below, the low side of the 1-TeV Higgs boson signal at a pp collider is typically lost in the $\bar{q}q$ annihilation background, in which case only the high side is observable.

It has been conjectured that the effective $W$ approximation may not provide correct high-energy behavior since the necessary $U$ gauge cancellations might be unhinged by the off-shellness of the initial-state bosons (222). However, viewed from the perspective of the $R$ gauge and the equivalence theorem (11) this is not a concern: correct high-energy behavior is guaranteed and off-shell corrections are $O(M^2_W/m_h^2)$. This conclusion is verified by a recent analysis in axial gauge, which concludes that the effective $W$ approximation is indeed valid to leading order in $g^2$ with corrections of order $O(M^2_W/m_h^2)$ (223).

All the above approximations must be approached with caution to be sure that they are applied within their domains of validity. They can then be very useful, providing needed checks on more exact computations. In particular the equivalence theorem is useful computationally at high energy as a check on the cancellations of the much more complicated $U$ gauge calculations and as a source of physical intuition (see Section 2).

6.4.2 SEARCHES AT pp COLLIDERS Methods of searching for heavy Higgs bosons at the SSC and LHC have been extensively studied during the last few years, though much more remains to be done. Recent results are summarized in (214, 220). Leptonic decay modes, $h \rightarrow ZZ \rightarrow \ell^+\ell^- \ell'^\pm \ell'^\mp$ and $h \rightarrow ZZ \rightarrow \ell^+\ell^- \tilde{\nu} \nu$ with $\ell = e$ or $\mu$, are the most straightforward, though the latter has at least one potential background that is not yet fully analyzed. Larger rates and, unfortunately, much larger backgrounds occur in mixed hadron-lepton decay modes such as $h \rightarrow WW \rightarrow \ell^+\ell^-\nu_\ell\bar{\nu}_\ell$. The purely hadronic decays are hopelessly overwhelmed by the four-jet QCD background.

The cleanest and rarest channel is $h \rightarrow ZZ \rightarrow \ell^+\ell^- \ell'^\pm \ell'^\mp$ with $\ell = e$ or $\mu$. With $BR(Z \rightarrow e^+e^-) = 0.033$, the branching ratio is $(4/3)(0.033)^2 = 1.5 \times 10^{-3}$. The background is from $q\bar{q} \rightarrow ZZ$ and $gg \rightarrow ZZ$, the latter (224) proceeding by a quark loop. Recent Monte Carlo simulations suggest that the Higgs boson can be detected in this channel for $m_h \leq 300$ GeV at the LHC (225) and for $m_h \leq 600$ GeV at the SSC (220). (Here and elsewhere unless otherwise stated both SSC and LHC are assumed to operate at $L = 10^{33}$ cm$^{-2}$ s$^{-1}$.) At these values of $m_h$ the Higgs boson appears as a recognizable peak above the continuum background. This decay mode is believed to offer the best chance for the LHC high-luminosity option to carry the search to $m_h \sim 1$ TeV (209).

The 1-TeV Higgs boson cross section $d\sigma/dM_{ZZ}$ is shown in Figure 6 for pp colliders of 10, 20, 30, and 40 TeV. It is shown again over the $\bar{q}q \rightarrow$
ZZ background in Figure 7 for $\sqrt{s} = 40$ TeV. At $m_h = 1$ TeV the width is $\Gamma_h = 0.5$ TeV and the Higgs boson appears as a broad enhancement over the background. One strategy at this mass is to impose cuts $|y_\ell| < 1.5$ and $m_{ZZ} > 1$ TeV to optimize the signal-to-background ratio while retaining a large fraction of the signal. With these cuts, a calculation using the effective W approximation and the equivalence theorem yields four signal events over a background (augmented by 50% for $gg \rightarrow ZZ$) of $1.5$ events for an integrated luminosity of $10^4$ pb$^{-1}$ at the SSC (14). (Here and below unless otherwise stated yields are quoted per $10^4$ pb$^{-1}$ corresponding to $L = 10^{33}$ cm$^{-2}$ s$^{-1}$ for $10^7$ s.) At the LHC the corresponding signal is a factor of ten times smaller while the background is about four times smaller. Such a signal at the SSC would not be statistically significant, but augmented by additional years of running and/or results from several experiments it would become significant. It would also be a valuable confirmation of larger signals detected in other channels.

Figure 6  Yields as a function of ZZ invariant mass for $h \rightarrow ZZ$ with $m_h = 1$ TeV. Rapidity $|y_\ell| < 1.5$ is required and $10^4$ pb$^{-1}$ is assumed for pp colliders of 10, 20, 30, or 40 TeV (from 11).
Figure 7  Yields as in Figure 6 for a 40-TeV pp collider. The short-dash line is the $qq \to ZZ$ background while the long-dash and solid lines are the sum of signal and background for the 1-TeV Higgs boson and the strong WW scattering model (Section 6.5) respectively (from 11).

Of course, to detect such a structureless signal it is necessary to know the magnitude of the background, and this requires a variety of calibrations in situ at the SSC to confirm knowledge of the relevant distribution functions and couplings (226). After such calibration studies are completed, the ZZ background should be known to within 30% uncertainty (220), sufficient accuracy given the expected 3:1 signal-to-background ratio.

Ascending the ladder of rates while descending on the scale of "purity" of signal, we come next to the decay $h \to ZZ \to E_T \nu \nu$ (11, 227). The branching ratio is six times larger than the previous decay, $1/3 \times 2 \times 0.066 \times 0.195 = 0.0086$, nearly 1%. Monte Carlo simulations have suggested a reach in this channel to $m_h \leq 600$ GeV at the LHC (225) and to at least 800 GeV at the SSC (220) (a comparable study for $m_h > 800$ GeV at the SSC has not yet been done). The two studies are not directly comparable because, while both considered the $qq \to ZZ$ and $gg \to ZZ$
backgrounds, only the SSC study considered the background from \( Z + \text{jet} \) where the jet generates large missing energy, imitating a \( Z \rightarrow \bar{\nu} \nu \) decay. This background is sharply reduced by cutting on visible hadronic transverse energy on the side opposite to the observed \( Z \rightarrow \ell^+ \ell^- \). A very hermetic detector was found to be critical. The analysis may be improved in the future by adding cuts involving event topology and the rapidity distribution of the visible hadronic clusters. The same background may be more dangerous at the LHC because of the less favorable signal-to-background ratio.

A calculation reported in (14), correcting a previously published result (227), indicates that the 1-TeV Higgs boson should be observable at the SSC in this mode, in agreement with the conclusion reached in (214). Requiring the observed \( Z \rightarrow \ell^+ \ell^- \) to satisfy \( p_T > 0.45 \) TeV and \( |y| < 1.5 \), we estimate the yield to be a signal of 27 events over a \((\bar{q}q \text{ or } gg) \rightarrow ZZ\) background of \( \sim 12 \) events. The \( Z + \text{jet} \) background has not yet been examined but is probably less dangerous than for \( m_h = 800 \) GeV since the background falls more steeply with \( p_T \) than the signal decreases with \( m_h \).

The mixed decay mode, \( h \rightarrow WW \rightarrow \ell \nu \bar{q}q \) with \( \ell = e \) or \( \mu \) and \( q\bar{q} = u\bar{d} \) or \( c\bar{s} \), has a large branching ratio, \( 2 \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{18} \). However, the QCD background from \( Wjj \) is two orders of magnitude bigger than the signal if we optimistically assume a 5% measurement of the dijet mass (228). Two approaches have been taken to attempt to winnow the signal from this enormous background. One method is to cut on the \( p_T \) of the jets, using the tendency of the longitudinally polarized W's from the signal to decay into jets transverse to the W line of flight, unlike the QCD dijets and the transversely polarized W's that both tend to produce jets along the line of flight. Applying this approach to the SSC for \( m_h = 800 \) GeV a parton level calculation results in a signal of 500 events over an equal background (229). Smaller but still encouraging yields have been reported based on Monte Carlo studies (230). The prospects of following this strategy in the real world are more difficult to assess than for the purely leptonic decays, being more sensitive to detailed aspects of jet physics and detector performance.

The second approach to the mixed modes borrows a trick from photon-photon scattering experiments at \( e^+e^- \) colliders, where detecting an \( e^\pm \) near the forward direction is a powerful way to isolate a clean sample of two-photon events. The analogous idea (219) is to tag the forward jets that occur in WW fusion, \( q\bar{q} \rightarrow q\bar{q}WW \), with transverse momentum of order \( M_W \). Of course, tagging suffers its own QCD background because of processes with a real \( W \rightarrow \ell \nu \) plus a dijet to mimic the second \( W \rightarrow \ell \nu \) and one or two jets near the forward direction that mimic the tagged quark or quarks. The necessary background calculation has not yet been performed.
A recent calculation assuming 100% efficient tagging for a 700-GeV Higgs boson resulted in 20 events for an LHC year and 160 events for an SSC year (221) (a “year” is always $10^7$ seconds). The QCD backgrounds after tagging were estimated at 12 and 140 events respectively; however, the authors remark that their background estimates will probably prove to be small by a factor of two. Highly segmented forward calorimeters would be essential.

An additional serious background would occur if $m_t > M_W$ so that the top quark decays by $t \rightarrow Wb$ (231). For $100 \leq m_t \leq 200$ GeV, the contribution to the $W^+W^-$ continuum from $\bar{t}t \rightarrow W^+W^-b\bar{b}$ would be two orders of magnitude larger than $q\bar{q} \rightarrow W^+W^-$, which probably eliminates any hope of detecting $h \rightarrow W^+W^-$. The leptonic decay signals from $h \rightarrow ZZ$ would not be affected.

Prospects for higher luminosity are most readily imagined for an experiment dedicated to muon detection, hence $h \rightarrow ZZ \rightarrow \mu^+\mu^-\mu^+\mu^-$ (232). The branching ratio for this mode is four times smaller than for $h \rightarrow ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-$ with $\ell = e$ or $\mu$, so a correspondingly larger increase in luminosity is needed. The 4-event signal over a 1½-event background cited above for the 1-TeV Higgs boson in the $e$ or $\mu$ mode at the SSC would become a 50-event signal over a 20-event background in the all-muon mode at $L = 5 \times 10^{34}$ cm$^{-2}$ s$^{-1}$. A detector that could detect electrons and muons at this luminosity, as contemplated in (209), would be a much more formidable undertaking. According to the conclusions of (209), the high-luminosity option is unlikely to confer any advantage for the other decay modes considered above. The mixed modes are overwhelmed by the much larger jet backgrounds due to event pile-up. Though the report does not explain why the $h \rightarrow ZZ \rightarrow \ell\ell\bar{\nu}\bar{\nu}$ mode cannot be utilized, the explanation may be degradation of the missing energy signal also due to event pile-up.

6.4.3 SEARCH AT $e^+e^-$ COLLIDERS Heavy Higgs boson searches at TeV $e^+e^-$ colliders do not have to contend with the fierce backgrounds found in pp colliders. As a result, detection of the purely hadronic decays $h \rightarrow WW \rightarrow q\bar{q}q\bar{q}$ is feasible at an $e^+e^-$ collider though unimaginable at a pp collider. The $e^+e^-$ colliders can therefore afford to give up an appreciable factor in raw cross section, as Figure 4 shows they indeed do for the collider energies under consideration.

The most complete study to date is contained in the proceedings of the La Thuile 1987 workshop (212), nicely summarized by Altarelli (214). A set of cuts against the dominant backgrounds is given and signals are estimated by Richard (233). A second recent study (234) uses the exact leading order matrix element (only heuristic in the strong coupling domain,
The WW fusion production mechanism, \( e^+e^- \rightarrow \bar{\nu}\nu WW \), \( WW \rightarrow h \) is larger than ZZ fusion, \( e^+e^- \rightarrow e^+e^- ZZ \), \( ZZ \rightarrow h \), by one order of magnitude. The largest signal occurs in \( h \rightarrow (WW \text{ or } ZZ) \rightarrow q\bar{q}q\bar{q} \) and results in four-jet events if top quarks are excluded, which could give rise to additional jets. Richard's analysis (233) assumes that \( m_t > M_W \) so that \( \sim 50\% \) of the \( h \) decays result in four-jet events. The signal is easily distinguished from \( e^+e^- \rightarrow W^+W^- \) and from the four-jet QCD background, \( e^+e^- \rightarrow 4 \) jets, by the sizeable missing energy carried off by the \( \bar{\nu}\nu \) pair in the \( e^+e^- \rightarrow \bar{\nu}\nu h \) signal. The \( e^+e^- \rightarrow WW\gamma \) background (235) is eliminated by requiring large missing mass recoiling against the four jets (17).

The most important backgrounds are from photon-photon scattering, \( e^+e^- \rightarrow e^+e^- \gamma\gamma \), \( \gamma\gamma \rightarrow WW \), and photon-W scattering, \( e^+e^- \rightarrow e\bar{\nu}W \), \( \gamma W \rightarrow WZ \). The \( \gamma\gamma \rightarrow WW \) background (237, 238) is distinguished from the signal by the \( p_T(\gamma\gamma) \) distribution—peaked at \( p_T = 0 \) for the background and at \( p_T = O(M_W) \) with a long high-side tail for the signal (see Section 6.4.1)—and by the angle between the \( \gamma\gamma \) line of flight (\( V = W \) or \( Z \)) and the beam axis, which is sharply peaked in the forward direction for the background and isotropic for the Higgs boson signal. The \( \gamma W \rightarrow WZ \) background (239) has a \( p_T \) distribution similar to the signal but differs from the signal by a sharply forward-peaked scattering angle (like \( \gamma\gamma \rightarrow WW \)).

The strategy followed in (233) uses the above characteristics along with an electron veto assumed (on assurances from the CLIC accelerator working group) to be implementable with high efficiency to within 50 mrad of the beam. Then a modest cut on the \( p_T \) of the four-jet system, \( p_T > 20 \) GeV, eliminates most of the \( \gamma\gamma \rightarrow WW \) events that pass the electron veto while leaving the signal intact. The electron veto is not as effective against the \( \gamma W \rightarrow ZW \) background that passes the \( p_T > 20 \) GeV cut because of the recoil of the initially radiated \( W \), so that the \( p_T(ZW) \) cut does not force the final-state electron away from the beam direction. The \( \gamma W \rightarrow WZ \) background is reduced principally by the angular cut, \( |\cos \theta| < 0.8 \), where \( \theta \) is the \( \gamma W \rightarrow WZ \) (or \( \gamma\gamma \rightarrow WW \)) scattering angle in the \( WZ \) center of mass.

Gunion & Tofighi-Niaki (234) used a much harder \( p_T(WW) \) cut, \( p_T > 150 \) GeV, to eliminate the \( \gamma\gamma \rightarrow WW \) background. However, this cut is ineffective against the \( \gamma W \rightarrow WZ \) background, which was not considered, and is therefore not optimal in the context of the more complete analysis.

Richard (233) presents signals and backgrounds before and after cuts for a range of center-of-mass energies and Higgs boson masses, including a rough simulation of some detector characteristics. We describe the results
for the 1-TeV Higgs boson produced at a $\sqrt{s} = 2$ TeV collider with $10^4$ pb$^{-1}$. The signal after acceptance and jet reconstruction but before cuts is 110 events, while the corresponding $\gamma\gamma \rightarrow WW$ and $\gamma W \rightarrow WZ$ backgrounds are 1200 and 600. After the cuts described above the signal is only reduced to 90 events while the backgrounds are 50 and 90 respectively. The reader can consult Figure 4 to get a rough idea of the comparative yields for other values of $m_h$ and $\sqrt{s}$.

6.5 Signals of Dynamical Symmetry Breaking Above 1 TeV

Even if the theoretical prejudice and/or wisdom that the Higgs boson cannot be heavier than 1 TeV is correct, it is no guarantee that the Higgs boson or whatever else may break the electroweak symmetry does in fact exist below 1 TeV. Even within the framework of an SU$(2)_L \times U(1)_Y$ gauge theory broken spontaneously by the Higgs mechanism, there is no guarantee that the Higgs boson exists at all, and there are models in which it does not. Many of these models predict other quanta below 1 TeV but their properties are uncertain and they may be difficult to find, and to recognize and interpret if they are found. Furthermore, at least one model exists without such quanta—technicolor with two flavors. It may not be phenomenologically viable but neither are the multiflavored technicolor models that predict light quanta (i.e. pseudo-Goldstone bosons). Since there are no satisfactory dynamical models, it is dangerous to rely heavily on the existing models for guidance.

The analysis reviewed in Section 2 uses only unitarity and symmetry properties valid in any spontaneously broken SU$(2)_L \times U(1)_Y$ gauge theory. Two alternatives emerge (Section 2.3 and Figure 2), classified according to the strength of the interaction of the symmetry-breaking sector [the indisputable fifth force (240)]. If the symmetry-breaking force is weak, then new weakly coupled quanta exist in the Higgs boson channel below 1 TeV—they are almost certainly Higgs bosons. (A weakly coupled continuum may be a logical alternative but is very far-fetched.)

The second alternative is that the symmetry-breaking force is strong, in which case the new quanta are to be found above 1 TeV and, probably, not far beyond 2 TeV. Whether the quanta exist below 2 TeV or not, the presence of the strong force will be manifested by strong scattering of longitudinally polarized W and Z bosons at WW center-of-mass energies between 1 and 2 TeV. If, as we have learned from the study of hadron physics, resonances form when strong scattering occurs, then there will be resonances in at least some of the WW, WZ, and ZZ channels not far above 2 TeV. In any case the presence or absence of the continuum signals is a general test of whether the symmetry-breaking sector is strongly coupled, regardless of the detailed nature of the spectrum. In this sub-
section we consider the ZZ, WZ, and WW continuum signals of strong interactions and one example of a resonance near 2 TeV.

The estimate of the continuum signal is based on the low-energy theorems, Equations 58–62, valid to all orders in the interactions of the symmetry-breaking sector (11, 12). As the analogous ππ low-energy theorems (13), e.g. Equation 55, are probably applicable for \( \sqrt{s} \approx 300 \) MeV, we might expect Equations 58–62 to apply below \( \sim 300 \) MeV \( \times v/F_\pi \approx 1 \) TeV. A crude model (11) of the strong continuum consists of extrapolating above 1 TeV, using Equations 58–62 as a model for the absolute value of the relevant partial wave amplitudes, which are

\[
a_{II} = a_{00}, a_{11}, a_{02}
\]

where \( I \) denotes the effective low-energy custodial SU(2). Above the energy at which partial wave unitarity is saturated, the amplitudes are set equal to one. For \( a_{00} \), saturation occurs at \( \sqrt{16\pi v^2} = 1.7 \) TeV, Equation 70, so in that case the model is (11)

\[
|a_{00}(s)| = \frac{s}{16\pi v^2} \theta(16\pi v^2 - s) + 1 \cdot \theta(s - 16\pi v^2).
\]

The detailed form is not critical, the essential point being to extrapolate smoothly from the known behavior at low energy into the domain above 1 TeV where the amplitude becomes strong, of order 1, without assuming enhanced resonant behavior. Continuum cross-section signals are then computed from WW, WZ, and ZZ fusion. The ZZ signals for 10, 20, 30, and 40 TeV colliders are shown in Figure 8. The signal at 40 TeV is seen with the \( \bar{q}q \rightarrow ZZ \) background in Figure 7.

The model is conservative in that it assumes no resonant behavior. For instance, the ZZ continuum signal for \( M_{ZZ} > 1 \) TeV is only half the corresponding signal from the 1-TeV Higgs boson. Comparatively little is gained from the high-energy region where \( |a_I| = 1 \) because of the rapidly falling luminosities at the relevant collider energies. The model also neglects higher partial waves, which would probably contribute at the energies where the extrapolation of the lower waves saturates unitarity. For the \( a_{00} \) and \( a_{11} \) waves, the model provides a good qualitative description of \( \pi\pi \) scattering data, which extrapolate smoothly from the domain of the low-energy theorem to the resonance region (241).

The experimental yields for this model have been estimated for pp colliders (11, 14) and e\(^+\)e\(^-\) colliders (17, 234). [Below unitarity saturation energies the \( m_h \rightarrow \infty \) limit considered in (234) is equivalent to the model described here.] Since the continuum signal is shifted to larger values of \( M_{ZZ} \) relative to the 1-TeV Higgs boson, it puts even greater stress on the beam energy. The most promising leptonic signal for the ZZ channel is

\[
ZZ \rightarrow \ell^+ \ell^- \bar{\nu}\nu
\]

with \( \ell = e \) or \( \mu \) (11). Requiring the observed Z to satisfy
Z Z: low energy theorem

Figure 8  Yields as a function of ZZ invariant mass for the strong WW continuum scattering model. Rapidity |y_2| < 1.5 is required and 10^5 pb^{-1} is assumed for pp colliders of 10, 20, 30, or 40 TeV.

|y| < 1.5 and p_T > 0.45 TeV, we obtain a yield at \( \sqrt{s} = 40 \) TeV of 15 events over a background of \( \sim 12 \) (14), including gg \( \rightarrow ZZ \). The corresponding signal at \( \sqrt{s} = 16 \) TeV is 1 event over a background of 4 or 5. The possible background from Z+jet has not yet been examined but is less pernicious than for the 800-GeV Higgs boson considered in (220).

Unlike the standard Higgs boson signal, strong interaction continuum signals also occur in the W^±Z and like-charge WW channels. WZ can be detected in clean leptonic decays WZ \( \rightarrow \ell \nu \bar{\ell} \bar{\nu} \), \( \ell = e \) or \( \mu \), with about 1% branching ratio, augmented to \( \sim 1.5\% \) if W \( \rightarrow \tau \nu \) is also feasible. Assuming only e's and \( \mu \)'s, we obtain an expected signal of \( \sim 7\frac{1}{2} \) events over a q\bar{q} \( \rightarrow WZ \) background of \( \sim 3 \) after cuts of |y_{wz}| < 1.5 and \( M_{WZ} > 1 \) TeV.

If either of the methods discussed in Section 6.4.2 to detect the mixed decay modes WW \( \rightarrow \ell \bar{\nu} q\bar{q} \) prove practicable, they can also be applied to the WW continuum signal. Including W^+W^-, W^+W^-, and W^-W^-, the net WW continuum signal with |y_{ww}| < 1.5 and \( M_{WW} > 1 \) TeV is \( \sim 2/3 \) of the h \( \rightarrow W^+W^- \) signal for \( m_h = 1 \) TeV (11).
The like-charge signal, \( W^+W^+ + W^-W^- \), not accessible to \( e^+e^- \) colliders, is of special interest because it is free of the \( \bar{q}q \) and/or \( gg \) annihilation backgrounds that occur in the \( W^+W^- \), \( W^\pm Z \), and \( ZZ \) channels at \( pp \) colliders. This means that the signal for lower mass gauge boson pairs may be observable, with the benefit of increased rate from higher quark effective luminosities. It also means that more of the signal comes from the domain of validity of the low-energy theorems. Of course, the charge can only be measured in leptonic decays so that \( M_{WW} \) cannot be determined. Taking only \( \ell = e, \mu \) the branching ratio for \( WW \rightarrow \ell\bar{\nu}\nu\ell\bar{\nu} \) is 0.028.

The true physics background, \( pp \rightarrow (W^+W^+ \text{ or } W^-W^-)X \), occurs in leading order by gluon exchange. It was recently computed and found to be negligible, smaller than the signal in the central region by two orders of magnitude (242). Other backgrounds, primarily from heavy quark decays, can be rejected by requiring the leptons to be isolated and by cutting against events with high \( p_T \) hadronic jets. Except for Drell-Yan production of \( t\bar{t} \) pairs, backgrounds will populate \( ++ \) and \( -- \) equally, in contrast to the signal for which \( ++ \) dominates by 3:1.

Because the equivalence theorem imposes a lower limit on the domain of validity of the low-energy theorems, \( M_{WW}^2 \gg M_W^2 \), Equation 63, the signal is computed with \( M_{WW} > 500 \text{ GeV} \) imposed. In fact, lower values of \( M_{WW} \) will contribute and increase the signal considerably, which is defined operationally by cuts on the observed leptons of \( |y_{\ell\bar{\nu}}| < 3 \) and \( p_T(\ell) > 50 \text{ GeV} \). The corresponding yield is 60 events at \( \sqrt{s} = 40 \text{ TeV} \) and \( \sim 4 \) at \( \sqrt{s} = 15 \text{ TeV} \) (242). For a light Higgs boson, \( m_h < 200 \text{ GeV} \), the signal would be entirely negligible, while for \( m_h = 1 \text{ TeV} \) it is \( \sim 2/3 \) of the value for the continuum model. A model based on the \( I = 2 \) pion-pion scattering data (243) gives a yield comparable to the value for the 1-TeV Higgs boson. The required \( e \) and \( \mu \) charge determinations are well within the range of experimental feasibility (244).

The \( W^+W^- \), \( ZZ \), and \( WZ \) continuum signals can also be observed in \( e^+e^- \) collisions. One study of signals and backgrounds estimated 11 events after stringent cuts in \( e^+e^- \rightarrow \nu\bar{\nu}W^+W^- \), \( W^+W^- \rightarrow ZZ \rightarrow \ell\nu\bar{\nu}q\bar{q} \), for \( 10^4 \text{ pb}^{-1} \) at \( \sqrt{s} = 3 \text{ TeV} \) (17). Another study obtained a total of 40 continuum model events in \( e^+e^- \rightarrow \nu\bar{\nu}WW \), \( WW \rightarrow q\bar{q}q\bar{q} \) with \( 0.5 < M_{WW} < 1.5 \text{ TeV} \) and \( p_T(WW) > 150 \text{ GeV} \) for \( \sqrt{s} = 2 \text{ TeV} \) and \( 10^{40} \text{ pb}^{-1} \) (234). Neither study considered the big \( \gamma W \rightarrow WZ \) background discussed in (233, 239). Comparing the raw signals for the continuum model and the 1-TeV Higgs boson, it seems that a signal of several tens of events over a background a few times larger might be extracted at \( \sqrt{s} = 2 \text{ TeV} \) with \( 40 \text{ pb}^{-1} \) using the strategy of (233) discussed in Section 6.4.3. A complete analysis is needed to determine whether the continuum model signal can be seen at a 2-TeV collider.
Though we have pessimistically concentrated here on continuum signals of strong WW scattering, where there are strong interactions there are probably also resonances. As a concrete example consider the technirho meson of $N_c = 4$ technicolor, with a mass of 1.8 TeV and width of 260 GeV—see Equations 102 and 103 in Section 3. In pp colliders both $\bar{q}q \rightarrow \rho_T$ (217a) and $WW \rightarrow \rho_T$ (11) contribute comparably to the production cross section. In analogy to the hadronic $\rho$ we expect the charged $\rho_T$ to decay predominantly to $WZ$, $\rho_T^+ \rightarrow W^+Z$, which can be detected in the leptonic decay mode $WZ \rightarrow \ell^+\ell^-\ell^\pm\ell^\mp$ that occurs with a 1% branching ratio for $\ell = e$ or $\mu$. Then at the SSC with $10^4$ pb$^{-1}$ the signal in the central region, $|y_w|$ and $|y_z| < 1.5$, is 12 events over a $\bar{q}q \rightarrow WZ$ background of only 1 event, while the LHC signal is 1 event over a somewhat smaller background. It is unlikely that this signal could be observed at a $\sqrt{s} = 2$ TeV $e^+e^-$ collider.

In this and the preceding section on heavy Higgs boson detection I have not made much of the fact that the signals are dominated by longitudinally polarized $W$'s and $Z$'s while the backgrounds are predominantly transverse. The decay angular distributions do not differ sharply enough and the statistics are typically too low to use polarization as an effective discriminant against the background. However, it will be important to analyze the $W$ and $Z$ polarizations of any potential signal to confirm the expected predominance of longitudinal modes.

7. CONCLUSION

Knowledge of the mechanism of electroweak symmetry breaking would complete the bosonic sector of the standard SU(3)$_C \times$ SU(2)$_L \times U(1)_Y$ model. It would provide an essential stepping stone toward an understanding of the quark and lepton spectrum and beyond that to deeper unification, perhaps eventually connecting the present standard model with the Planck scale of gravitational interactions. There is no way to know how far off these deeper goals are, but the first step is one we are ready to take now. The experimental program defined by the existing $p\bar{p}$ colliders, the soon to be commissioned $Z$ factories, and the multi-TeV pp and TeV $e^+e^-$ colliders, is guaranteed to find the symmetry-breaking mechanism. The guarantee is based on general considerations of symmetry and unitarity that place an upper limit on the energy scale at which the mechanism operates and allow us to estimate the magnitude of the experimental signals—much like the unitarity bounds (18) on the older Fermi theory of weak interactions.

Though they cannot directly probe the Higgs boson channel, the existing $p\bar{p}$ colliders at CERN and Fermilab can discover signals that would point
to the symmetry-breaking mechanism, such as the squarks and gluinos of supersymmetry or the pseudo-Goldstone bosons of technicolor models. The first comprehensive look at the Higgs channel could come from Tristan if toponium is near 60 GeV, in which case the radiative decay $\theta \to h\gamma$ would allow a thorough search for $m_\theta \lesssim 40$ GeV. When the Z factories have logged about $5 \times 10^6$ produced Z’s they will be able to scan the same mass range in $Z \to h\ell^+\ell^-$ with $\ell = e$ or $\mu$. LEP 200 can carry the search to $\sim 70$ GeV with $e^+e^- \to Zh$. A higher energy $e^+e^-$ collider might be needed to search in the interval $2M_Z > m_h > 2m_t$, minimally with $\sqrt{s} \approx 320$ GeV and $L \geq 10^{32}$ cm$^{-2}$ s$^{-1}$ or with $\sqrt{s} \geq 1$ TeV and $L \geq 10^{33}$ cm$^{-2}$ s$^{-1}$.

The mass range above $2M_Z$ belongs to TeV $e^+e^-$ colliders, when we know how to build them, and to the SSC and LHC, which we already know how to build. At $L = 10^{33}$ cm$^{-2}$ s$^{-1}$ the LHC can scan up to $m_h = 600$ GeV and the SSC up to $m_h = 1$ TeV in the channel $h \to ZZ \to \ell\ell\bar{\nu}\bar{\nu}$ with $\ell = e$ or $\mu$. One TeV is perhaps the largest plausible mass for the standard Higgs boson, but it is not the largest scale that must be probed in a comprehensive search, which must cover the possibility that there is no Higgs boson at all but a dynamical mechanism involving strong interactions above 1 TeV. Unitarity implies in this case that a strong WW scattering continuum will emerge above backgrounds between 1 and 1.8 TeV in the $W^+W^-$, $W^\pm Z$, and $ZZ$ channels, visible at the SSC with tens of events in leptonic channels and perhaps an order of magnitude more in mixed lepton-hadron decays. A larger signal, of order several tens of events, can also be seen in purely leptonic decays below 1 TeV at the SSC from the strong scattering doubly charged continuum, $W^+W^+$ and $W^\pm W^\mp$, that has no $q\bar{q}$ annihilation background. These continuum signals are generic signs of strong interaction symmetry breaking with new quanta at or above 1 TeV. The most likely mass range for the new quanta is 1–2 TeV, in which case they are also visible at the SSC in leptonic decay modes. Since the WW fusion mechanism has four-body phase space, $q\bar{q} \to qqWW$, while the background is two-body, $q\bar{q} \to WW$, not only the signals but also the signal-to-background ratios are rapidly varying functions of energy.

$A \sqrt{s} = 2$ TeV $e^+e^-$ collider with $L \geq 10^{33}$ cm$^{-2}$ s$^{-1}$ appears to be sufficient to detect the 1-TeV Higgs boson but further analysis is needed to decide if it is also sensitive to the WW continuum signals of dynamical symmetry breaking. It could certainly not match the reach of the SSC for WW resonances at $\sim 2$ TeV, though a $A \sqrt{s} = 3$ TeV collider might.

Further careful study is needed to assess and extend the capabilities of both the pp and $e^+e^-$ colliders for the range of possible signals. Progress beyond the electroweak scale depends pivotally on knowing the electroweak symmetry-breaking mechanism. It is gratifying that an experimental program exists able to provide that knowledge within the coming decade.
ELECTROWEAK SYMMETRY BREAKING

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