An Introduction to Cosmology

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UNITS

• The most natural units to use:
  \( \hbar = c = 1 \)

• Consequence:
  mass = energy = GeV
  length = time = 1/GeV

If \( \hbar = c = \frac{M_{pl}}{\sqrt{8\pi}} = 1 \), dimensionless.

Sometimes normal units will be used in the talk.
Plan

• Lecture 1 (Basics)
  • Basic Intro to cosmology and its problems
  • Inflation
  • Baryogenesis/Leptogenesis

• Lecture 2 (Connecting High Energy and Cosmo)
  • Electroweak Baryogenesis
  • Dark Matter
  • Outlook and Conclusions
The Very Basic
What is cosmology?

- Study of the origin and large scale structure of the universe
  - Large scale > 10 kpc (= 30,000 lyr ; galaxy size).
  - Largest scale observed (around 10,000 Mpc).
- Traditionally: gravitational and thermal history
  - Far away galaxies seem to receding away from us with a velocity proportional to its distance. (universe is not static or stationary -- history)
  - There is a thermal background radiation at 2.7 degrees Kelvin. (thermal history)
Observational foundations

- Hubble Expansion (redshift of galaxies, quasar, supernovae, etc. as a function of brightness)
- Homogeneous and isotropic $T=2.72^\circ\text{K}$ background $\gamma$
- Light element abundances (absorption/emssion spectra)
- Galaxy surveys (distribution of visible matter)
- Lensing (distribution of invisible clumped matter)
- Temperature fluctuations (primordial, SZ effect, etc.)
- Diffuse gamma ray, X-ray, etc.
- Cosmic rays (neutrino, positron, antiproton, ultra-high energy, etc.)
Theory

• Einstein equations (Equivalence principle)

\[ S = \frac{-1}{16\pi} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} L_M \]

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = 8\pi T_{\mu \nu} \quad \rightarrow \text{Put in known fields} \]

(more later . . .)

• Boltzmann Equations

\[ \left[ p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta \gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \right] f (x^\alpha, p^\alpha) = C [f] \]

Collision term;
Approximation
Homogeneity and Isotropy

• “on the average” Homogeneity and isotropy

\[ ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Theta^2 + r^2 \sin^2 \Theta \, d\varphi^2 \right] \]

- characterizes the curvature of space at a fixed time

\[ 3R = 6 \frac{k}{a^2} < 0.01 \, H^2 \sim (10^{-44} \text{GeV})^2 \]

• Stress Tensor: Perfect fluid

\[ T_{\mu\nu} = (\rho(t) + P(t)) u_{\mu} u_{\nu} + P(t) g_{\mu\nu} \quad \text{e.g.} \quad T_{00} = \rho \quad T_{11} = a^2 P \]

• Open Problem: Is the naïve averaging of the background density correct?
Background

• Einstein

\[ H^2 + \frac{k}{a^2} = \frac{\rho}{3} \]

\[ H \equiv \frac{a'(t)}{a(t)} \]

\[ \frac{M_{pl}}{\sqrt{8\pi}} = \frac{1}{\sqrt{G_N}8\pi} = 1 \]

\[ 2[H'(t)]^2 + 3H^2 + \frac{k}{a^2} = -P \]

expansion rate

combine: \[ \frac{a''(t)}{a(t)} = -\frac{1}{6}(\rho + 3P) \]

ordinary matter: decelerate

alternate: \[ d(\rho a^3) = -P d(a^3) \]

perfect fluid energy conservation and adiabatic flow

• Notation and examples

\[ w \equiv \frac{P}{\rho} = \text{equation of state} \]

Matter dominated \[ \left\{ \rho \propto \frac{1}{a^3} \quad , \quad w = 0 \right\} \rightarrow a \propto t^{2/3} \]

Radiation dominated \[ \left\{ \rho \propto \frac{1}{a^4} \quad , \quad w = \frac{1}{3} \right\} \rightarrow a \propto t^{1/2} \]
Basic picture emerging

- $T_{\text{00}}$ contains the following fraction of the total:
  - 73% “dark energy” defined by its negative pressure
  - 22% cold dark matter
  - 4.4% in baryons (protons and neutrons)
  - 0.6% neutrinos
  - 0.005% in photons

- The universe is spatially flat to about 1%.

- $a(t)$ is expanding with $H \equiv \frac{d}{dt} \ln a(t) = 70$ km/s/Mpc.

- Energy density was homogeneous and isotropic to 1 part in $10^5$ about 15 billion years ago.
Explicit Stress-Energy Components

• Popular Model

\[ \rho_y = \rho_y(t_0) \left( \frac{a_0}{a} \right)^4 \quad P_y = \frac{\rho_y}{3} \]

\[ \rho_{b,c} = \rho_{b,c}(t_0) \left( \frac{a_0}{a} \right)^3 \quad P_{b,c} = 0 \]

\[ \rho_\nu = \rho_\nu(t_0) \left( \frac{a_0}{a} \right)^3 \quad P_\nu = 0 \]

\[ \rho_\Lambda = \rho_\Lambda(t_0) \quad P_\Lambda \approx -\rho_\Lambda \]

\[ \rho_{\text{tot}} = \rho_y + \rho_\nu + \rho_{b,c} + \rho_\Lambda \]

\[ P_{\text{tot}} = P_y + P_\nu + P_{b,c} + P_\Lambda \]

\[ (\rho_c \equiv 3 H_0^2 \sim 10^{-46} \text{ GeV}^4) \]

\[ \Omega_X = \frac{\rho_X}{\rho_c} \]

\[ \Omega_\gamma \sim 10^{-5} \]

\[ \Omega_b \approx 0.044 \]

\[ \Omega_c \approx 0.22 \]

\[ \Omega_\nu \sim 0.01 \]

\[ \Omega_\Lambda \approx 0.73 \]

\[ \Omega_{\text{tot}} \approx 1.0 \]

energy conservation

noninteracting particle

negative pressure
Temperature of the Universe

- **Equilibrium Thermodynamics**

\[
\rho = \frac{g}{(2\pi)^3} \int f(E) E^2 \, d^3 p
\]

\[
P = \frac{g}{(2\pi)^3} \int \frac{p^2}{3E} f(E) \, d^3 p
\]

\[
E = \sqrt{p^2 + m^2}
\]

\[
f(E) = \frac{1}{\exp \left( \frac{E - \mu}{T} \right) + 1}
\]

can fall exponentially with temperature

- **Photon Temperature** = “temperature of universe”

\[
\rho_Y = \frac{\pi^2}{30} 2T^4
\]

\[
\rho_R \equiv \frac{\pi^2}{30} g_*(T) T^4
\]

- **Entropy (conservation gives T history)**

\[
s = \frac{(\rho + p)}{T} \equiv \frac{2\pi^2}{45} g_*(T) T^3
\]

**Early Universe (T>1 MeV)**

\[g_*(T) \approx g_{*S}(T)\]

SM only: \[g_*(T > 300 \text{ GeV}) \approx 107\]

**today (for massless neutrinos):**

\[T \approx 2.34 \times 10^{-4} \text{ eV}\]

\[g_{*S} \approx 3.9\]

\[g_* \approx 3.36\]

\[g_Y = 2 \rightarrow \text{ photons dominate and can be measured!}\]
Equilibrium?

• Equilibrium conditions: \( \Gamma > H \)

  • Kinetic equilibrium: \( X \{ Y \} \rightarrow X \{ Z \} \) Y and Z in equilib with photon
    • maintains same temperature
    • particle number does not change
  • Chemical equilibrium: \( X \{ Y \} \rightarrow \{ Z \} \) Y and Z in equilib with photon
    • maintains same temperature
    • particle number changes
    • particle number is determined by temperature

• Boltzmann equations govern approach to equilibrium

• Out of equilibrium:
  • Kinetic: decoupled
  • Chemical: freeze out
Decoupled Species Temperature

• Theorem: Let $T_x$ be the temperature of particle $\chi$ after decoupling and $T_y$ be the temperature of the photon after decoupling. Suppose $\chi$ decouples at temperature $T_D$.

\[
\frac{T_x}{T_y} = \left[ \frac{g_s(T_y)}{g_s(T_D)} \right]^{1/3} \quad \frac{\Gamma}{H} < 1
\]

[Proof] Separate conservation of entropy:

\[
s_x(t_D)a^3(t_D) = s_x(t_0)a^3(t_0)
\]

Total conservation of entropy:

\[
s_{tot}(t_D)a^3(t_D) = s_{tot}(t_0)a^3(t_0)
\]
Exercise

• Suppose a dark matter species $X$ decoupled at temperature of 120 MeV. Compute the dark matter temperature today assuming 3 neutrino species are massless.

answer:

$$T_X = \left( \frac{43}{11} \frac{4}{57} \right)^{1/3} \quad T_\gamma \approx 1.5 \times 10^{-4} \text{ eV}$$
History of the Universe

Key:
- w, z bosons
- quark
- electron
- neutrino
- photon
- meson
- baryon
- ion
- star
- galaxy
- black hole
- 10^15 degrees
- 10^12 degrees
- 10^9 degrees
- 6000 degrees
- 18 K
- 3 K
- 15 x 10^9 years (today)
- 3 x 10^5 years
- 1 min
- 10^-10 sec
- 10^-5 sec
- 10^-34 sec
- 10^-43 sec
- 10^32 degrees
- 10^27 degrees
- 10^15 degrees
- BIG BANG

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07/22/2004
Problems of cosmology

• What is the composition of dark energy?
• What is the composition of CDM?
• Why more baryons than antibaryons?
• If inflation solves the cosmological initial condition problems, what is the inflaton?
• Classical singularities of general relativity?
• Why is the observed cosmological constant small when SM says it should be big?
• Origin of ultra-high energy cosmic rays?
Rest of the Lectures

• First, a general introduction to these problems.

• Second focus on selected topics.
  • Inflation
  • Baryogenesis
  • Dark Matter

• Collaborative scorecard between problems of physics beyond the standard model and cosmology.
What is dark energy?

\[
\frac{\dot{a}}{a} = \frac{-4\pi}{3 M_{pl}^2} (\rho + 3P) > 0 \quad \rightarrow P < -\frac{1}{3} \rho
\]

Recall that normal gas of matter has positive pressure.

\[
P(x) = \frac{1}{3} \sum_N \frac{p_N^2}{\sqrt{p_N^2 + m_N^2}} \delta^3(x - x_N)
\]

Field energy can have negative pressure (like inflation).

\[
P = \frac{1}{2} \left( \frac{d\phi}{dt} \right)^2 - V(\phi)
\]

- Why is the energy density nearly coincident with the matter density today?
- If a dynamical field explains the coincidence, how can such a small mass scale (cosmological time scale) be protected?
What is cold dark matter?

- Definition: dark matter that is nonrelativistic at the time of matter radiation equality.
- Can it be all in cold baryons not emitting light?
  - BBN (chemistry of producing elements heavier than hydrogen) says no.
  - Microlensing (gravitational deflection of light from compact objects) agrees with this picture.
- CDM neutrinos would overclose the universe.
- Physics beyond the standard model necessary!
Why more baryons than antibaryons?

- The absorption spectra measurements, CMB, and BBN agree
  \[
  \frac{n_B}{n_\gamma} \sim 10^{-10}
  \]

- Naturalness of small dimensionless number?

- According to SM, \( \frac{n_B}{n_\gamma} = 10^{-18} \) at \( T > 100 \) GeV.

- Probability that the small number is from mere thermal fluctuation is very very small.
What is the inflaton?

- CMB data looks like that expected from inflation
  i.e.
  1. “no” spatial curvature
  2. scale invariant spectra on “superhorizon” scales
- Similar in negative pressure characterization as dark energy; no known particle can produce this
Singularities of GR

• Hawking-Penrose-Geroch theorem: As long as there is nonzero spacetime curvature somewhere and energy is positive, Einstein's theory will develop a singularity. (a classical self-destruction)

• Evidence for black holes exist. Is there a singularity behind the apparent horizon?

• Big bang singularity naively exists: i.e.

\[
R = -6 \left( \frac{a''(t)}{a(t)} + \left[ \frac{a'(t)}{a(t)} \right]^2 + \frac{k}{a^2(t)} \right) \sim \frac{1}{t^2} \rightarrow \infty
\]
A small cosmological constant?

- Due to SM quantum fluctuations
  \[
  \rho_\Lambda \sim M^4
  \]

- Possible values of \( M \)
  - Planck scale \( 10^{18} \) GeV
  - GUT scale \( 10^{16} \) GeV
  - See-saw scale \( 10^{13} \) GeV

- On the other hand we observe
  \[
  \rho_\Lambda \sim (10^{-12} \text{GeV})^4
  \]
Ultra-high energy cosmic rays

- There is a GZK cutoff at $10^{19.8}$ eV due to efficient
  \[ \gamma p \rightarrow \pi p \]
- Proton cannot ravel more than 40 Mpc.
- Events above $10^{19.8}$ eV measured (possibly).
- No energetic extragalactic sources within 40 Mpc.
- Primary? Source & acceleration mechanism?
Inflation

Motivation: Mostly initial condition problems.

1. Flatness Problem

\[ \frac{k}{H^2 a^2} = \Omega - 1 \quad \text{time dependent} \]

today: \[ \frac{k}{H_0^2 a_0^2} = \Omega_0 - 1 < 10^{-2} \]

early universe: \[ \frac{k}{H_e^2 a_e^2} = \frac{k}{H_0^2 a_0^2} \left( \frac{a_r}{a_e} \right)^2 < (10^{-2})(10^{-4})(10^{-6})^2 = 10^{-18} \]

at nucleosynthesis Why small?

Initial spatial curvature had to be finely tuned for universe to be this old and flat. Why?
More Motivation

2. Horizon/causality problem: Why homogeneous and isotropic on “acausal scales?"

real singularity

naïve horizon (with $w > -\frac{1}{3}$):

$$d_H = a(t) \int_0^t \frac{dt'}{a(t')} = \frac{t}{1 - \frac{2}{3(1 + w)}}$$

$$\sim \frac{1}{H}$$

causal signals travel beyond naïve horizon
Unwanted Relics

3. Unobserved relic problem

e.g. Suppose the SM is embedded in a larger theory with gauge group \( G_1 \)

\[
G_1 \rightarrow G_2 \rightarrow \ldots \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y
\]

Monopoles arise whenever \( \Pi_2(G_i/G_j) \neq I \)

\[
n_M \sim H^3 \sim T_c^6 / M_{pl}^3
\]

\[
n_M / s \sim \frac{T_c^3}{M_{pl}^3} \sim \left( \frac{10^{14}}{10^{19}} \right)^3 \sim 10^{-15}
\]

\[
m_M \sim 10^{16} \text{ GeV} \rightarrow \Omega_M \sim 10^{11} \quad \text{unacceptably large!}
\]
Inflation

\[ ds^2 = g_{\mu \nu} \, dx^\mu \, dx^\nu = dt^2 - a^2(t) \, dx^i \, dx^j \, \delta_{ij} \]

- **Inflationary solution:** Blow up a small flat patch into the entire universe
  - Flat patch becomes the entire universe (solves flatness)
  - Lengthen the time it takes to reach the singularity (horizon)

\[
d_H = a(t) \int_0^t \frac{dt'}{a(t')} \quad \frac{d}{dt} \left[ \frac{x}{a(1/H)} \right] > 0 \quad \rightarrow \quad \frac{d^2 a}{d t^2} > 0
\]

causal signals travel beyond naïve horizon

\[ x \equiv \text{comoving coordinate separation} \]

- **Dilutes unwanted relics**
- **Prediction:** scale invariant density perturbations

\[
\frac{\delta \rho}{\rho} = \frac{1}{(2\pi)^3} \int d^3 k \Delta_k e^{-i \vec{k} \cdot \vec{x}} \quad \text{power:} \left( \frac{\delta \rho}{\rho} \right)_{\text{hor}} \sim k^{3/2} \frac{|\delta_k|}{\sqrt{2\pi}} \sim 10^{-5}
\]
Qualitative description of inflaton $\phi$

single field inflationary models:

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]$$

How to choose the potential and initial conditions?

- $\frac{d^2 a}{dt^2} > 0$ for about 60 e-folds. $\frac{a(t_f)}{a(t_i)} = e^{N_{\text{efold}}}$
- Inflation must end.
- Spatial inhomogeneities of $\phi$ must be sufficiently small to be consistent with cosmology (too big = too many black holes, too small = not enough structure).
- After inflation ends, the universe must reheat to $T > 10 \text{ MeV}$.
- After inflation ends, unwanted relics must not be created (e.g. low enough temperature).
Magic of negative pressure

- Horizon problem

\[
\frac{d^2 a}{dt^2} / a = \frac{-4\pi}{3 M_{pl}^2} (\rho + 3P) > 0 \quad \Rightarrow P < -1/3 \rho \quad \text{(just like dark energy)}
\]

\[
\rho = \frac{1}{2} \left( \frac{d \phi}{dt} \right)^2 + V(\phi) \quad \Rightarrow \left( \frac{d \phi}{dt} \right)^2 < V(\phi)
\]

\[
P = \frac{1}{2} \left( \frac{d \phi}{dt} \right)^2 - V(\phi)
\]

- 60 efolds desired:

\[
\frac{da}{dt} / a = H \quad \Rightarrow \quad a \propto e^{\int dt H}
\]

\[
\Rightarrow \frac{d^2 \phi}{dt^2} \ll H \frac{d \phi}{dt} \quad \text{slow roll inflation}
\]
Quantitative Single Field

Slow Roll approximation \( 3H \frac{d\phi}{dt} \approx -V'(\phi) \quad H^2 \approx \frac{V}{3} \)

- Negative Pressure and 60 e-folding

\[ \epsilon \equiv \frac{1}{2} \left( \frac{V'(\phi)}{V} \right)^2 \approx -\frac{dH}{dt} \frac{1}{H^2} \ll 1 \quad \eta \equiv \frac{V''(\phi)}{V} \ll 1 \quad N(\phi(t_i)) \equiv \left| \int_{\phi(t_i)}^{\phi(t_f)} \frac{d\phi}{\sqrt{2\epsilon}} \right| > 60 \]

- End of inflation: \( \epsilon(\phi(t_f)) \approx 1 \) with \( V(\phi_{\text{min}}) \approx 0 \) at the minimum of the potential

- Density perturbation amplitude: \( \sqrt{\mathcal{P}_k^\zeta} \approx \sqrt{\frac{V}{24\pi^2 \epsilon(\phi_{60})}} \approx 10^{-5} \)

scale invariance nearly automatic! \( |2\eta(\phi_{60}) - 6\epsilon(\phi_{60})| < 0.2 \) never Planckian

indicates source of fine tuning
Standard Reheating

- Inflaton field decays: e.g.

\[ L = \frac{1}{2} [(\partial \phi)^2 + m^2 \phi^2 + L_i] \]

\[ L_i = -\lambda \phi \bar{\psi} \psi \]

\[ \Gamma_{\text{tot}} = \frac{3 \left( \Sigma (m^2) \right)}{m} \]

\[ \partial_t^2 \phi + (3H + \Gamma_{\text{tot}}) \partial_t \phi + \left( m^2 + \frac{\Gamma_{\text{tot}}^2}{4} \right) \phi \approx 0 \]

use following approx: \[ \rho_\phi \approx \frac{1}{2} \left( (\partial_t \phi)^2 + m^2 \phi^2 \right) \]

\[ \frac{(\partial_t \phi)^2}{2} \approx \frac{m^2 \phi^2}{2} \]

\[ \Gamma^2 / 4 \ll m^2 \]

estimate:

\[ \rho_R = \frac{\pi^2}{30} g^* (T) T^4 \]

\[ T_{RH} \approx 0.2 \left( \frac{200}{g^*} \right)^{1/4} \sqrt{\frac{\Gamma_{\text{tot}}}{M_{\text{pl}}}} \]

⇒ reheating temperature as a function of time
Why 60 efolds?

- Largest scale that we see homogeneous and isotropic:
  \[ L = a_0 \int dx = a_0 \int_{a_{dec}}^{a_0} \frac{da}{H a^2} \approx a_0 \int_{a_{dec}}^{a_0} \frac{da}{H_0 (a_0/a)^{3/2} a^2} \approx \frac{2}{H_0} \equiv a_0 X \]

- Inflation can take place only if homogeneous (small patch of comoving coordinate size > X became the observable universe):

(also for curvature) \( \frac{1}{H_I} > X a_I \) \[ \Rightarrow \frac{1}{H_I} > a_0 X \underbrace{\left( \frac{a_I}{a_0} \right)}_{\text{sufficiently small}} \]

need enough efolds

\[
\begin{align*}
\frac{a_I}{a_0} &= \frac{a_I}{a_e} \frac{a_e}{a_0} \equiv e^{-N} \frac{a_e}{a_0} \\
\max \left( \frac{A_e}{a_0} \right) &\approx \frac{a_{RH}}{a_0} = \left( \frac{g^* S(t_{RH})}{g^* S(t_0)} \right) \\
\frac{T_0}{T_{RH}} &\approx \frac{T_0}{T_{RH}} \\
H_I &\approx \frac{T_{RH}^2}{\sqrt{3}} \\
\ln \left( \frac{T_{RH}}{T_0} \right) + \frac{1}{2} \ln \left( \Omega_{R0} \right) &\approx 60 + \ln \left( \frac{T_{RH}}{10^{15} \text{ GeV}} \right) < N
\end{align*}
\]
Single Scalar Field Computation

Action: \[ S = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} R + \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \]

perturb: \[ \phi = \phi_0(\tau) + \delta \phi(x) \quad dx^\mu dx^\nu g_{\mu \nu}^{(0)} = a^2(\tau)(d \tau^2 - d \vec{X}^2) \]

gauge degree of freedom: Freedom of slicing the spacetime (splitting perturbed versus unperturbed) infinitesimally gauge invariant (same as longitudinal gauge)

\[ v = a \left( \delta \phi - \frac{\phi_0(\tau)}{(a'/a)} \phi \right) \]

\[ S = \int d \tau d^3 x \frac{1}{2} \left[ v'^2 - (\nabla v)^2 + \frac{a'/a}{a\phi_0'} \left[ \partial^2 \left( \frac{a \phi'}{a'/a} \right) \right] v^2 \right] \]

\[ \Re \equiv -\left( \frac{a'/a}{a\phi_0'} \right) v \quad \Re = \Phi \quad \text{constant on constant } \phi \text{ hypersurface (comoving)} \]

Seed formation of galaxies!
Power Spectrum

quantize: \( v(\tau, \mathbf{x}) = \int \frac{d^3 k}{(2\pi)^{3/2}} \left[ a_k v_k(\tau) + a_k^\dagger v_k^*(\tau) \right] e^{i\mathbf{k} \cdot \mathbf{x}} \)

\( [a_k, a_k^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}') \)

\[ \langle \Re(\tau, \mathbf{x}) \Re(\tau, \mathbf{y}) \rangle = \int \frac{dk}{k} \frac{\sin(k|\mathbf{x} - \mathbf{y}|)}{k|\mathbf{x} - \mathbf{y}|} P_\Re \quad \Re \equiv -\left( \frac{a'/a}{a\phi_0'} \right)v \quad P_\Re = \frac{k^3}{2\pi^2 \left( \frac{a'/a}{a\phi_0'} \right)^2} |v_k|^2 \]

\( v_k'' + \left[ k^2 - \frac{2 + p}{\eta^2} \right] v_k = 0 \quad \text{boundary condition:} \quad v_k \rightarrow_{\eta \rightarrow -\infty} \frac{e^{-ik\eta}}{\sqrt{2k}} \)

\( p = 3(3\epsilon - \eta) \)

\( v_k = \sqrt{\frac{\pi}{2}} \sqrt{-\tau} H_v^{(1)}(-k\tau) e^{i\frac{\pi}{2}(\frac{1}{2} + \nu)} \quad \nu \equiv \frac{\sqrt{9 + 4p}}{2} \)

\( H_v^{(1)}(-k\tau) \sim_{kl(aH) \rightarrow 0} \frac{-i\sqrt{2}}{\pi \sqrt{\left(-k\tau\right)^{3/2}}} \Rightarrow \sqrt{P_k^{\xi}} \approx \sqrt{\frac{V}{24\pi^2 \epsilon(\phi_{60})}} \sim 10^{-5} \)
Quantum to “Classical” Transition

On large scales: \( \frac{k}{(a H)} \to 0 \)

\[
v_k'' - \frac{a'/a}{a \phi_0'} \partial^2_{\eta} \left( \frac{a \phi_0'}{a'/a} \right) v_k \approx 0
\]

growing mode: \( v_k \sim A_k \frac{a \phi_0'}{a'/a} \)

\[
\alpha_k \frac{a \phi_0'}{a'/a} = a_k v_k(\eta) + a_{-k}^{dagger} v_k^*(\eta)
\]

\( \alpha_k \) are constants

\( \mathcal{R}_k = -\alpha_k \)  Constants on superhorizon scales!

\[ [\alpha_k, \alpha_l^{dagger}] = 0 \]

\( \lim_{k/(aH) \to 0} \mathcal{R}_k \) are classical random variables!

Spectral index: \( P_{s} \propto k^{n_s-1} \)

\[
n_s = 2 \eta - 6 \epsilon
\]

\[
\frac{d n_s}{d \ln k} = 16 \epsilon \eta - 24 \epsilon^2 - 2 \xi
\]

\[
\xi = \frac{V' V'''}{V^2}
\]

Running of spectral index measurement = measuring potential
Gravity waves

Tensor perturbations:
\[ \delta g^{(T)}_{\mu \nu} = \begin{pmatrix} 0 & 0 \\ 0 & h_{ij} \end{pmatrix} \]

\[ P_T(k) \equiv \frac{k^3}{8\pi^2} (|h_{+k}|^2 + |h_{\times k}|^2) \propto k^{n_T - 1} \]

\[ P_T = \frac{H^2}{(2\pi)^2} = 2\epsilon P_R \]

\[ r \equiv \frac{P_T}{P_R} \]

\[ n_T - 1 = -2\epsilon = -r \]

Consistency relation gives evidence for single field inflation.

In multifield inflation (more realistic):
\[ 1 - n_T > r \]
What is this good for?

fixes the boundary condition to the Boltzmann equation

\[ \Theta \equiv \frac{\Delta T}{T} \]
\[ \Theta = \sum_{l=0}^{\infty} \Theta_l(\eta) e^{i \vec{k} \cdot \vec{x}} P_l(\vec{k} \cdot \vec{y}) \]
\[ \Theta(t_i) \propto \Phi(t_i) \]

\[ \partial_t^2 \Theta_0 + \frac{\partial_t R}{1 + R} \partial_t \Theta_0 + k^2 c_s^2 \Theta_0 = F \]

\[ c_s^2 = \frac{1}{3} \frac{1}{1 + R} \]
\[ R \approx \frac{3 \rho_b}{4 \rho_y} \]

\[ \frac{\Theta_l(\eta)}{2l + 1} \sim (\Theta_0 + \Psi)(\eta) j_l(k(\eta - \eta_0)) + \ldots \]

\[ \langle \Theta(\eta_0, \vec{x}, \gamma) \Theta(\eta_0, \vec{x}, \gamma') \rangle = \sum_l C_l P_l(\gamma \cdot \gamma') \]

\[ \frac{(2l + 1)}{4\pi} C_l = \frac{1}{(2\pi^2)} \int \frac{dk}{k} \frac{k^3 |\Theta_l|^2}{2l + 1} \]

“acoustic oscillations”
“Recent” Developments

- Transferring power from isocurvature to curvature perturbations.
  - Can be used to generate density perturbations during reheating (Gruzinov and Zaldarriaga 2003)
  - Helps to relax constraints on inflationary models, but loss of predictivity.

- Stringy models of inflation: no inherently stringy insights.

- Uncertainties in the boundary condition determining the quantum gravity giving rise to nonstandard vacuum.
  
  Unlikely, arbitrary, and lacks compelling motivation thus far.
Future Prospects

- Running of the spectral index will be better known with future experiments such as Planck.

- Polarization:
  - B polarization comes from tensor and lensing (contaminant as far as inflation is concerned).
  - B polarization has no contribution from scalar perturbations.
  - Measuring tensor is important for checking consistency condition (to know if it really is inflation!)
  - Unfortunately, typically less than 1% of the scalar spectrum

- Theoretical Problems
  - What is the inflaton? Are there truly natural models?
  - Stability of de Sitter space and back reaction.
  - More observables to experimentally ascertain inflation.
Baryogenesis
Observation

• In solar system much more baryons than antibaryons explained by $pp \rightarrow 3p + \bar{p}$
  
  $n_p / n_{\bar{p}} < 3 \times 10^{-4}$

  $n_{4_{He}} / n_{4_{He}} < 3 \times 10^{-8}$ (?)

• Dominance of matter clear on scales < 10 Mpc: bound on $\gamma$ from $p \bar{p} \rightarrow \pi^0 \rightarrow 2\gamma$.

• Other constraints: distortion of CBR, diffuse $\gamma$-ray.

[e.g. Cohen and de Rujula 1997] + void -
Is there a problem?

- SM contains nonperturbative baryon number violating operators that erase B+L
- These become efficient when \( T > T_c \sim 100 \text{ GeV} \)
- Otherwise, an aesthetic initial condition problem
- Starting from \( n_B \equiv n_b - n_{\bar{b}} = 0 \) initial conditions why
  \[ \eta \equiv \frac{n_B}{n_y} \approx 6 \times 10^{-10} \]
  naively, \( \eta_{\text{naive}} \sim 10^{-18} \) and not separated.
Illustration of Sakharov Criteria

- Suppose "X" carrying 0 baryon number can decay only into "a" carrying baryon number $b_a$ and "b" carrying baryon number $b_b$.

- Branching ratios:
  \[ r = \frac{\Gamma(X \rightarrow a)}{\Gamma_X} \quad \text{"CP":} \quad \bar{r} = \frac{\Gamma(\overline{X} \rightarrow \overline{a})}{\Gamma_X} \]

  \[ 1 - r = \frac{\Gamma(X \rightarrow b)}{\Gamma_X} \quad \text{"CP":} \quad 1 - \bar{r} = \frac{\Gamma(\overline{X} \rightarrow \overline{b})}{\Gamma_X} \]

- Baryon produced:
  \[ \Delta B_X = r b_a + (1 - r) b_b \]
  \[ \Delta B_{\overline{X}} = -\bar{r} b_a - (1 - \bar{r}) b_b \]

  \[ \Delta B = \Delta B_X + \Delta B_{\overline{X}} = (b_a - b_b)(r - \bar{r}) \]

- Out of equilibrium: otherwise, the other direction produces
  \[ (b_a - b_b)(\bar{r} - r) \]
Boltzmann

• Phase space evolution (useful B-genesis, dark matter, CMB):

\[
\left[ p^\alpha \frac{\partial}{\partial x^\alpha} - \Gamma^\alpha_{\beta\gamma} p^\beta p^\gamma \frac{\partial}{\partial p^\alpha} \right] f(x^\alpha, p^\alpha) = C[f]
\]

\[
\partial_t \int d^3 p f + 3 H \int d^3 p f = \int \frac{d^3 p}{E} C[f]
\]

\[
n(t) = \frac{g}{(2\pi)^3} \int d^3 p f
\]

\[
\frac{g_x}{(2\pi)^3} \int \frac{d^3 p_x}{E_x} C[f] = -\int d \Pi_x d \Pi_a d \Pi_b d \Pi_c (2\pi)^4 \delta^{(4)}(p_x + p_a - p_b - p_c) \times
\]

\[
\left[ |M|^{2}_{X + a \rightarrow b + c} f_x f_a (1 \pm f_b) (1 \pm f_c) - |M|^{2}_{b + c \rightarrow X + a} f_b f_c (1 \pm f_x) (1 \pm f_a) \right]
\]

\[
d \Pi_x \equiv \frac{g_x}{(2\pi)^3} \frac{d^3 p_x}{2 E_x}
\]

• Simplification

• Chemical equilibrium of others:  
  e.g.  
  \( f_b = f_b^{eq}, \quad f_c = f_c^{eq} \)

• Kinetic equilibrium of all states:  
  e.g.  
  \( f_x = F(t) f_x^{eq}, \quad f_a = A(t) f_a^{eq} \)
Interference

- CP violation involves a complex parameter in the Lagrangian:

\[
L = |m|^2 (|\phi_1|^2 + |\phi_2|^2) - |m| (e^{i\rho} \psi_1 \psi_2 + e^{-i\rho} \overline{\psi_2} \overline{\psi_1}) \\
+ |M_3| (e^{i\theta} \lambda^a \lambda^a + e^{-i\theta} \overline{\lambda^a} \overline{\lambda^a}) + |m_{LR}|^2 (e^{i\phi} \phi_2 \phi_1 + e^{-i\phi} \phi_1^* \phi_2^*)
\]

- In this Lagrangian, there is only one physical phase (phase that cannot be removed by field redefinition).

\[
\delta_{\text{phys}} = \phi - \theta \lambda - \rho
\]

- CP violation = interference of transition amplitudes:

\[
|M|^2 = |M_1 + M_2 e^{i\delta_{\text{phys}}}|^2 = |M_1|^2 + |M_2|^2 + 2 \Re (M_1 M_2 e^{-i\delta_{\text{phys}}})
\]

\[
|M^{CP}|^2 = |M_1 + M_2 e^{-i\delta_{\text{phys}}}|^2 = |M_1|^2 + |M_2|^2 + 2 \Re (M_1 M_2 e^{i\delta_{\text{phys}}})
\]
Cutting

• Recall in the simple example

\[ \Delta B = \Delta B_x + \Delta B_{\bar{x}} = (b_a - b_b) (r - \bar{r}) \]

\[ |M|^2 - |M^{CP}|^2 = \Re \left( M_1 M_2 e^{i\delta_{phys}} - M_1 M_2 e^{-i\delta_{phys}} \right) \]

This is 0 unless the non-CP violating part develops an imaginary part due to virtual states going on shell.

• Diagrammatically

Since the real part of this should be taken:
Thermal Leptogenesis

- Have only perturbatively significant B-L violating operators.
- Generate L as we have been discussing.
- Convert L into B through the B+L violating sphaleron.

\[ B = \left( \frac{8 N_f + 4 N_H}{22 N_f + 13 N_H} \right) (B - L) \]

- Theoretical attractiveness: L-violating operators natural in seesaw neutrino masses
- “uncomfortable” aspect: in gravity mediated SUSY breaking models, gravitino bound strongly constrains it.
Boltzmann Eq.

\[ Y_i = \frac{n_i}{s} \]

\[
\frac{z}{Y_{\psi}^{eq}} \frac{d Y_{\psi}}{dz} = -\frac{1}{H} \sum_{a,i,j,...} \left[ \frac{Y_{\psi} Y_{a}^{eq} ...}{Y_{\psi}^{eq} Y_{a}^{eq} ...} \bar{Y}^{eq} (\psi + a + ... \rightarrow i + j + ...) \right] 
\]

\[
- \left[ \frac{Y_{i} Y_{j}^{eq} ...}{Y_{i}^{eq} Y_{j}^{eq} ...} \bar{Y}^{eq} (i + j + ... \rightarrow \psi + a + ...) \right] 
\]

\[ z = \frac{m_{\psi}}{T} \]

decay \quad \bar{Y}^{eq} = \frac{K_1(z)}{K_2(z)} \Gamma \]

scatter \quad Y^{eq} (\psi + a \rightarrow i + j + ...) = \frac{T}{64\pi^4 n_{\psi}^{eq}} \int_{(m_{\psi} + m_{a})^2}^{\infty} ds \hat{\sigma} (s) \sqrt{s} K_1 \left( \frac{\sqrt{s}}{T} \right) \]

\[
= \langle \sigma v \rangle n^{eq} 
\]

\[ \hat{\sigma} (s) = \frac{2[s - (m_{\psi} + m_{a})^2][s - (m_{\psi} - m_{a})^2]}{s} \sigma (s) \]

(same equation is applicable to dark matter.)
Leptogenesis Estimate

1) Assume temperature of the universe is high enough
→ right-handed neutrinos are in equilibrium (fixes initial cond.)
Typically, CP conserving reactions control this.

2) Temperature falls:
\[ \langle \sigma \nu \rangle n_{\nu_R}(T) < \frac{T^2}{M_{pl}} \]
→ right handed neutrinos go out of equilibrium

3) When the right handed neutrino abundance falls below L density, the lepton number freezes out.
\[
\eta \approx \frac{\delta_{CP} m_{\nu} M}{g_* v^2} \sqrt{\frac{M}{T_c}} e^{-M/T_c} \quad m_{\nu} \sim 10^{-1} \text{ eV}, \quad M \sim 10^9 \text{ GeV}, \quad g_* \sim 100, \quad m_W \sim 100 \text{ GeV}
\]
\[
\sim 10^{-10} \quad \sqrt{\frac{M}{T_c}} e^{-M/T_c} \sim 0.1 \quad \text{(out of equilibrium temperature)}
\]
End of Lecture 1

• General Cosmology
  • Edward Kolb and Michael Turner, THE EARLY UNIVERSE.
  • Scott Dodelson, MODERN COSMOLOGY

• Inflationary references
  • Lidsey, Liddle, Kolb, Copeland Barreiro, and Abney Rev. Mod. Phys 69, 373 (1997).
  • Lyth and Riotto, hep-ph/9807278.
  • Hu and Sugiyama, astro-ph/9411008.

• General Baryogenesis

• Cosmology related to supersymmetry
  • Chung, Everett, Kane, King, Lykken, and Wang hep-ph/0312378.
People and References for EW baryogenesis

- Incomplete list of ewbgenesis people:

- “Randomly” selected “overview” references
  - hep-ph/0312378
  - hep-ph/0208043
  - hep-ph/0006119
  - hep-ph/9901362
  - hep-ph/9901312
  - hep-ph/9802240
EW Motivation

• In minimal SM, **EW phase transition** is inevitable!

\[ T > T_c \sim m_h \quad \text{EW symmetry restoration} \]

• An exciting era:

  probing at LHC and its microphysics

  Nearly everything at \( T_c \) associated with SM measurable

• Almost no cosmological probe to this era

  • Explaining the **baryon asymmetry of the universe**
  • Establishing thermal equilibrium for WIMPs close
Why worry about electroweak baryogenesis scenario instead of leptogenesis?

- **Leptogenesis**
  - Computationally simpler: spatially homogeneous
  - Neutrino mass suggests such scenario if see saw invoked (lepton num violation & dim 5 operator suppression scale)
  - May depend on near-future-lab-immeasurable phase:
    \[ m_\nu = (U_{\text{MNS}} \sqrt{(m_\nu)_{\text{diag}}} R)(R^T \sqrt{(m_\nu)_{\text{diag}}} U_{\text{MNS}}^\dagger) \]
  - Squeezed by gravitino bound

- **EW Baryogenesis is physics at 100 GeV**
  - Almost everything about it can be lab probed in principle
  - In SM and MSSM, EW phase transition occurred!
Aspects of MSSM

soft susy breaking (definition: does not introduce quadratic divergence)

\[-L_{soft} = \frac{1}{2} [M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B}] + \epsilon_{\alpha \beta} [-b H_d^\alpha H_u^\beta - H_u^\alpha \tilde{Q}_i \tilde{A}_{uij} \tilde{U}_j^c + H_d \tilde{Q}_i \tilde{A}_d \tilde{D}_j^c + H_d \tilde{L}_i \tilde{A}_{uj} \tilde{E}_j^c + h.c.]
\]

supersymmetric Yukawa and mass term

\[W = \epsilon_{\alpha \beta} [-\tilde{H}_u^\alpha \tilde{Q}_i \tilde{Y}_{ui}^\beta \tilde{U}_j^c + \tilde{H}_d^\alpha \tilde{Q}_i \tilde{Y}_{dj}^\beta \tilde{D}_j^c + \tilde{H}_d^\alpha \tilde{L}_i^c \tilde{Y}_{ei}^\beta \tilde{E}_j^c - \mu \tilde{H}_d^\alpha \tilde{H}_u^\beta]
\]

Chargino mass matrix

\[L_c = \frac{-1}{2} (\chi^+ \chi^-) \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix} (\chi^+) \quad \chi^+ \equiv \begin{pmatrix} \tilde{W}_+^c \\ \tilde{H}_u^c \end{pmatrix} \quad X \equiv \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \sin \beta & \mu \end{pmatrix}\]
Sakharov conditions

recall: 1) B-violation, 2) CP violation, 3) out of equilibrium

1) Baryon number violation: SU(2) sphaleron

\[ O_{B+L} = C \bar{h}_L h^4 \prod_i (q_L, q_L, q_L, l_L) \]

e.g. 1 generation \[ \bar{u}_L \rightarrow d_L d_L \nu_e \]

unbroken phase: \[ \Gamma_{EW} = (k \alpha_w) \alpha_w^4 T^4 \quad k \alpha_w \sim O(1) \]

broken phase: \[ \Gamma \approx 2.8 \times 10^5 T^4 \left( \frac{\alpha_w}{4\pi} \right)^4 \kappa \zeta^7 \exp(-\zeta) \quad 10^{-4} < \kappa < 10^{-1} \]

\[ \zeta = \frac{E_{sph}(T)}{T} \]

\[ E_{sph} \sim \frac{2 m_w}{\alpha_w} \sim \frac{8\pi \langle H \rangle}{g} \]
Sakharov conditions

recall: 1) B-violation, 2) CP violation, 3) out of equilibrium

2) CP violation:

In SM:

$$
\delta_{CP} = \left( \frac{g_w}{2 m_W} \right)^{12} \left( m_t^2 - m_u^2 \right) \left( m_t^2 - m_c^2 \right) \left( m_c^2 - m_u^2 \right) \left( m_b^2 - m_d^2 \right) \left( m_b^2 - m_s^2 \right) \left( m_s^2 - m_d^2 \right) j \sim 10^{-22}
$$

$$
j = \Im \left[ V_{cs} V_{us}^* V_{ud} V_{cd}^* \right] \sim 10^{-4}
$$

Too small.

In MSSM, soft SUSY breaking phases: e.g.

$$
\Im \left( M_2 \mu \right)
$$

$$
\Delta L = \frac{1}{2} M_2 \, \overline{W}_R^{\dagger} \, \overline{W}_L + \mu \, \tilde{h}_R^{\dagger} \, \tilde{h}_L + h.c.
$$
Out of equilibrium

3) Phase transition:

- $T > 100$ GeV, symmetry is restored.
- $T < 100$ GeV, symmetry broken.

$$V(H) = D(T^2 - T_0^2)H^2 - E T H^3 + \frac{\lambda(T)}{4} H^4$$

$\langle H \rangle = z$

$\langle H \rangle = 0$

$\psi\rightarrow\beta_w$

$T > T_c$

$V(H)$

$T < T_c$

$z$

$H$

(Attractive, because almost no new assumption!)
EW B creation step 1

1. Pick up CP/chiral asymmetry

\[ \langle H \rangle = z \]

sphalerons inactive

\[ n_b^L - n_b^R \neq 0 \]

\[ n_b - n_b = n_b^L + n_b^R - n_b^L - n_b^R = 0 \]

e.g. 1 generation

\[ \overline{u}_L u_R \quad B = 0 \]
EW B creation step 2

\[ \langle H \rangle = z \rightarrow \langle H \rangle = 0 \]

sphalerons active

\[ \Delta (n_b^L - n_{\bar{b}}^L) \]

sphalerons inactive

\[ n_b - n_{\bar{b}} = n_b^L + n_b^R - n_{\bar{b}}^L - n_{\bar{b}}^R \neq 0 \]

e.g. 1 generation

\[ u_R \rightarrow u_R \]

\[ \bar{u}_L \rightarrow d_L d_L \nu_e \]

\[ \bar{u}_L d_L \bar{\nu}_e \]

\[ B = 1 \]

\[ B = 0 \]
EW B creation step 3

\[ \langle H \rangle = z \]

sphalerons inactive

\[ n_b - n_{\bar{b}} = n_b^L + n_{\bar{b}}^R - n_b^L - n_{\bar{b}}^R \neq 0 \]

sphalerons active

\[ \langle H \rangle = 0 \]

\[ \beta_w \]
Computational Steps

• Diffusion equations for (s)quarks and higgs(inos): relatively fast process

• Make assumptions about certain processes (Yukawa and strong sphaleron) being in equilibrium due to large interaction rate.

• Solve for SU(2) charged left handed fermions

• Integrate sphaleron transition sourced by above.
Schematically

- One of massaged diffusion equations

\[ v_w \partial_z n_H = D_h \partial_z^2 n_h + \Gamma_Y \left[ \frac{n_Q}{k_Q} - \frac{n_T}{k_T} - \frac{n_H + \rho^n}{k_H} \right] - \Gamma_h \frac{n_H}{k_H} + S_H \]

- Source term = CP violating, Higgs field gradient
  - flow of current w/ background force
  - \( S_H > S_Q \)
  - \( n_B \sim \frac{c_1 \Gamma_{EW}}{v_w} \int_{-\infty}^{0} dz \ n_L(z) \exp(\frac{c_2 z \Gamma_{EW}}{v_w}) \)
  - \( n_L(z) \sim \frac{\exp(\frac{f_1 z}{f_3})}{f_2} \int_{0}^{\infty} dx \ S_H(x) \exp(-f_2 x) \)
Uncertainties

- **Source uncertainty (off diagonal term)**

  \[ S_H \sim D_h \frac{\bar{z} (M_2 \mu)}{M_2^2 + \mu^2} \partial_z^2 [(v_1 \partial_z v_2 - v_2 \partial_z v_1)(F_1(z) \partial_z m_1 + F_2(z) \partial_z m_2)] \]

  \[ m_A < 300 \text{ GeV} \quad |M_2| \approx |\mu| \]

  Partially addressed by Carena et al in hep-ph/0208043
  Some contend the problem persists hep-ph/0312110

- **Damping rates in the diffusion equations**

- **Overall uncertainties in final baryon asymmetry**
Resonance

- Consider mass matrix

\[ M(z) = \begin{pmatrix} m_1 & \epsilon(z) \\ \epsilon(z) & m_2 \end{pmatrix} \rightarrow \Lambda = \begin{pmatrix} \lambda_1(z) & 0 \\ 0 & \lambda_2(z) \end{pmatrix} \]

\[ \frac{\partial_z \lambda_{\pm}}{\lambda_{\pm}} \approx \frac{\pm 4 \epsilon \partial_z \epsilon}{\sqrt{(m_1 - m_2)^2 + 4 \epsilon^2 (m_1 + m_2 \pm \sqrt{(m_1 - m_2)^2})}} \]

\[ S_H \sim D \frac{x (M^2 \mu)}{M^2 + \mu^2} \partial_z^2 [(v_1 \partial_z v_2 - v_2 \partial_z v_1)(F_1(z) \partial_z m_1 + F_2(z) \partial_z m_2)] \]
Source of source discrepancy

- CQSW has “interaction” term; never diagonal
  \[ L_{\text{int}} = (x - z)^\mu [\psi_R^{\dagger} M_\mu \psi_L + \text{h.c.}] \]
- CKJPSW uses WKB-like approximation
  \[ (\partial_z^2 + f^2)\psi = 0 \]
  \[ (\partial_z^2 + f_d^2)\bar{\psi} = 0 \quad f_d^2 = U f^2 V^{\dagger} \quad \bar{\psi} = V \psi \]
  \[ \bar{\psi} \sim \frac{1}{\sqrt{2 f_d}} \exp (i \int dz f_d) \]
- Dispute unsettled
sketch of parameter region

- $120 \text{ GeV} < m_\tau < m_t$
- $0.2 \ m_Q \leq A_t \leq 0.4 \ m_Q$
- $m_h < 115 \text{ GeV}$
- $\tan \beta > 4$
- $m_Q > 1 \text{ TeV}$
- $\mu, M_{1,2} < m_Q$
- $\Im [\mu \ M_{1,2}] / T_c^2 > 0.05$
- $\mu, M_{1,2} < 2 \ T_c$

strong enough phase transition
- charge and color breaking minima
  - if $m_h > 114 \text{ GeV}$ (suggestive)
- sufficient diffusion
- sufficient CP violation
- sufficient density processed by the sphaleron
sketch of parameter region

- $120 \text{ GeV} < m_\ell < m_t$
- $0.2 m_Q \leq A_t \leq 0.4 m_Q$
- $m_h < 115 \text{ GeV}$
- $\tan \beta > 4$
- $m_Q > 1 \text{ TeV}$
- $\mu, M_{1,2} < m_Q$
- $\exists [\mu M_{1,2}] / T_c^2 > 0.05$
- $\mu, M_{1,2} < 2 T_c$

- strong enough phase transition
- charge and color breaking minima
- if $m_h > 114 \text{ GeV}$ (suggestive)
- sufficient diffusion
- sufficient CP violation
- sufficient density processed by the sphaleron
sketch of parameter region

- $120 \text{ GeV} < m_t < m_{1/2}
- $0.2 m_Q \leq A_t \leq 0.4 m_Q$
- $m_h < 115 \text{ GeV}$
- $\tan \beta > 4$
- $m_Q > 1 \text{ TeV}$
- $\mu, M_{1,2} < m_Q$
- $\Im(\mu M_{1,2})/T_c^2 > 0.05$
- $\mu, M_{1,2} < 2 T_c$
- strong enough phase transition
- charge and color breaking minima
- if $m_h > 114 \text{ GeV}$ (suggestive)
- sufficient diffusion
- sufficient CP violation
- sufficient density processed by the sphaleron
sketch of parameter region

- $120 \text{ GeV} < m^- < m_t$
- $0.2 \ m_Q \leq A_t \leq 0.4 \ m_Q$
- $m_h < 115 \text{ GeV}$
- $\tan \beta > 4$
- $m_Q > 1 \text{ TeV}$
- $\mu, M_{1,2} < m_Q$
- $\Im [\mu M_{1,2}] / T_c^2 > 0.05$
- $\mu, M_{1,2} < 2 \ T_c$

- strong enough phase transition
- charge and color breaking minima
- if $m_h > 114 \text{ GeV}$ (suggestive)
- sufficient diffusion
- sufficient CP violation
- sufficient density processed by the sphaleron
Strong enough phase transition

- To protect the baryon number

\[ \Delta V = -ET \phi^3 + \frac{\lambda}{4} \phi^4 \quad \frac{E}{\lambda(T_c)} \sim \frac{\phi}{T_c} \geq 1.3 \]

- In the MSSM, \( m_Q^2 \gg m_U^2, m_t^2 \)

\[ \Delta V = \frac{-T}{2\pi} \left[ m_U^2 + \Pi_{\tilde{t}_R}(T) + 0.15 M_Z^2 \cos 2\beta + m_t^2 \left( 1 - \frac{(A_t - \mu/\tan \beta)^2}{m_Q^2} \right)^{3/2} \right] \]

\[ m_t^2 \approx m_U^2 + 0.15 M_Z^2 \cos 2\beta + m_t^2 \left( 1 - \frac{(A_t - \mu/\tan \beta)^2}{m_Q^2} \right) \]

\[ m_{\tilde{t}} < m_t \quad \text{and} \quad m_H^2 \approx \lambda v^2 \]

\[ A_t < 0.4 m_Q \quad \text{and} \quad m_H < 115 \text{ GeV} \]
Source Term

- Interaction Lagrangian derivative

\[ \Delta L = (x - z)^\alpha \left[ \psi_R^\dagger M_\alpha \psi_L + \psi_L^\dagger M_\alpha^\dagger \psi_R \right] \]

\[ U M V^\dagger = \begin{pmatrix} m_1(z) & 0 \\ 0 & m_2(z) \end{pmatrix} \]

- The CP violating current is proportional to the CP violating propagator correction.

\[ \Delta S_{RR}(x, y) = \int d^4w S_{RR}^{QR}(x, w)(w - z)^\alpha U(z) M_\alpha(z) V^\dagger(z) S^{LR}(w, y) + h.c. \]

mass suppressed

\[ S_h = D_h \partial_z^2 \frac{1}{2} \lim_{r \to 0} Tr \left[ \Delta S_{RR}(z + \frac{r}{2}, z - \frac{r}{2}) + \ldots \right] \]

\[ M_Q > 1 \text{ TeV} \quad \Rightarrow S_h > S_Q \quad \text{if} \quad M_{1,2}, \mu < M_Q \]
Sufficient CP violation

• Estimate

\[ \eta \sim \frac{(k \alpha_w) \alpha_w^4}{g_s} \delta_{CP} f \sim 10^{-10} \quad \alpha_w^4 \sim 10^{-6} \quad g_s > 10 \]

\[ f < v_w \sim 0.1 \rightarrow \delta_{CP} > 10^{-2} \quad \text{e.g.} \ \delta_{CP} \sim \Im \left( \frac{M_w \mu}{T_c^2} \right) \]

• Importance

• Large top Yukawa coupling \[ O_{B+L} = \tilde{h}_{l_1} \tilde{h}_{l_2} \tilde{w}^4 L_i (q_{_L}, q_{_L}, q_{_L}, l_{_L}) \]

• Higgs mediated CP asymmetry

Chargino, Higgs(ino), neutralinos, (s)quark
EDM

- experimental EDM bounds

\[ |d_e| < 1.6 \times 10^{-27} \text{ e cm} \quad [\text{Regan et al 2002}] \]

\[ |d_n| < 12 \times 10^{-26} \text{ e cm} \quad [\text{Lamoreaux et al 2002}] \]

\[ |d_{Hg}| < 2.33 \times 10^{-28} \text{ e cm} \quad [\text{Romalis et al 2001}] \]

- Theoretical constraints complicated & uncertain

  - e.g. without cancellations,

\[ \text{Arg} \left( M_2 \mu \right) < 0.05 \quad [\text{Chang et al 2002; Pilaftsis 2002}] \]
Sufficient density

- For there to be sufficiently large current

\[ n^p - n^\bar{p} = \frac{g}{2\pi^2} \int_m^\infty dE \frac{E}{\sqrt{E^2 - m^2}} \left[ \frac{1}{1 + \exp[(E - \mu)/T]} - \frac{1}{1 + \exp[(E + \mu)/T]} \right] \]

\[ \approx g \mu \frac{T^2}{6} \]

otherwise

\[ = 2g \left( \frac{m T}{2\pi} \right)^{3/2} \sinh(\mu/T) \exp[-m/T] \]

- Critical temperature

\[ T_c \approx 100 \text{ GeV} \]

\[ \mu, M_{1,2} < 2 T_c \]
EW bgenesis Prospects

• Next generation of colliders can rule out the MSSM electroweak baryogenesis scenario
  • very squeezed parameter space viable
  • ruled out if
    • large Higgs mass or right handed stops
    • more stringent EDM constraints $\Im(M_2 \mu)$
• There is a dispute of the strength of the source term when $|M_2| \approx |\mu|$
Recent collaborative progress
Cosmology and High Energy Physics

- renormalization group flow $\approx$ cosmological time flow.
  - Integrating out degrees of freedom in field theory is most of the time not invertible.
  - Entropy producing events $\Rightarrow$ evolution noninvariant under time reversal.
- What are some recent collaborative efforts between high energy physics and cosmology?
- Any prospects for further success?
Problems of the Standard Model (SM)

- Why is the Higgs field light?
- What is the origin of electroweak symmetry breaking?
- Is it simply an accident that the gauge couplings seem to meet?
- How is gravity incorporated into the SM?
- Why is the CP violation from QCD small?
Lightness of Higgs

- Precision electroweak data & LEP direct search
  \[ 114 \text{ GeV} < m_H < 200 \text{ GeV} \]

- Quantum fluctuations

- Possible values of $\Lambda$
  - Planck scale $10^{18} \text{ GeV}$
  - GUT scale $10^{16} \text{ GeV}$
  - See-saw scale $10^{13} \text{ GeV}$

- What generates low $\Lambda$?

- Unnatural if $\Lambda^2 \gg m_H^2$
Origin of the electroweak scale

• The value of the Higgs field \( \langle H \rangle \sim 100 \text{ GeV} \) permeating the universe is much smaller than what we might expect from short distance scales.

• As before, the possible values are
  - Planck scale \( 10^{18} \text{ GeV} \)
  - GUT scale \( 10^{16} \text{ GeV} \)
  - See-saw scale \( 10^{13} \text{ GeV} \)

• Protection from radiative corrections does not mean that the EW scale can be naturally small.
Unification of coupling

• Because of “backreaction” (renormalization) coupling constants depend on energy scale (or length scale).

• Is this an accident?
Incorporating quantum gravity

• All fields are quantized according to the SM.
• Gravity appears in SM. \( S_{\text{SM}} = \int d^4 x \sqrt{-g} L_{\text{SM}} \)

• Quantize gravity as well.

\[
\delta S_{\text{GR}} = \int d^4 x \sqrt{-g} \left( c_{21} R^2 + c_{22} R_{a b} R^{a b} + \ldots + c_{31} R^3 + \ldots \right)
\]

undetermined!

• Gravity becomes non-predictive!
Strong CP problem

- The strong interactions (responsible for holding the nucleus together) contains

\[ L_{CP} = \theta \int d^4x \sqrt{-g} \epsilon^{\mu\nu\alpha\beta} \text{Tr}[F_{\mu\nu} F_{\alpha\beta}] \]

which is the analog of the CP violating term

\[ \int d^4x \sqrt{-g} E \cdot B \]

in Maxwell theory.

- Absence of electric dipole moment of the neutron requires \( \theta < 10^{-9} \).

- The problem: to explain this small number.
Collaborative score card

- ✓ Why is the Higgs field light?
- ✓ What is the origin of electroweak symmetry breaking?
- ✓ Is it simply an accident that the gauge couplings seem to meet?
- ✓ How is gravity incorporated into the SM?
- ✓ Why is the CP violation from QCD small?
- ✓ What is the dark energy?
- ✓ ✓ What is the CDM?
- ✓ Why more baryons than antibaryons?
- ✓ If inflation solves the cosmological initial condition problems, what is the inflaton?
- ✓ Classical singularities of general relativity?
- ✓ Why is the observed cosmological constant small when SM says it should be big?
- ✓ Origin of ultra-high energy cosmic rays?

Many other speculative connections exist.
Not very convincing yet, unfortunately.
Restricting to particle physics.

✓ with SUSY
✓ ✓ with PQ
Supersymmetry (SUSY)

• (N=1 SUSY) a symmetry exchanging bosons and fermions

\[ f \leftrightarrow b \]

examples: SM \quad new \quad particle

\[ \text{electron (spin } \frac{1}{2} \text{)} \leftrightarrow \text{selectron (spin } 0 \text{)} \]

\[ \text{Higgs (spin } 0 \text{)} \leftrightarrow \text{Higgsino (spin } \frac{1}{2} \text{)} \]

\[ \text{graviton (spin } 2 \text{)} \leftrightarrow \text{gravitino (spin } \frac{3}{2} \text{)} \]

• Key feature: “Solves”

• Why is the Higgs field light?

• (partially) Is gauge coupling unification an accident?
Lightness of Higgs

- In SM, the trouble was

\[ m_H^2 = m_H^{(0)^2} + \lambda \Lambda^2 \]

- With SUSY

\[ \delta m_H^2 = -\lambda \Lambda^2 \]

Cancellation of the quantum back reaction! The Higgs mass is stabilized!
Unification of coupling

• “Accident” is more and more looking not like an accident!
Ensuring the proton stability

• General theme in physics: Every new solution has a new set of problems.
• Recall in SM, proton is very stable due to accidental symm.
• The MSSM (minimal supersymmetric standard model) obtained by supersymmeterizing the SM contains baryon number violating operator which leads to proton decays

\[ p \rightarrow \pi^0 + e^+ \]

• To ensure such operators do not appear: conserve R-parity (a new quantum number natural in SUSY).
• Conservation of R-charge forbids an R-charged particle to decay to a non-R-charged final states.

\[ \rightarrow \text{lightest R-charged particle is stable!} \]
LSP Neutralino dark matter

- Direct detection
  - Earth's orbit around the sun
    - annual modulation
  - diurnal modulation can also be sought with direction sensitive detectors (DRIFT)
- theoretical uncertainties:
  - local density of dark matter: 4-5
  - nuclear physics of detector: 2-3
Indirect detection

- collect in the sun by elastic scattering
- $\nu_\mu$ can escape efficiently (no muons in the sun)
- neutrinos produce muons

\[ np \rightarrow W^+ W^- X X \]
\[ \nu_\mu \rightarrow \mu^- \]

\[ \nu_\mu \]
detected
Neutrino telescope reach

- Saturation effect: \( \Gamma_A = \frac{C}{2} \tanh^2 (t \sqrt{C C_A}) \)
- Optimistically,

\[
A_0 = 0 \; \tan (\beta) = 45 \; \mu > 0 \; 40 < m_0 < 3000 \; 40 < m_1/2 < 1000
\]

- Theoretical uncertainty: similar to direct detection (i.e. 10)
Other cosmic rays

• Gamma rays, radiowaves, antimatter,...
• HEAT shows an “excess” of positrons at 10 GeV.
  • Explanation in terms of LSPs speculative
• Greater uncertainty in modelling (as much as $10^3$)
• Need a better dark matter distribution of the galaxy (perhaps by lensing?)
Summary of LSP

- In some region of parameter space (large higgsino component and LSP heavy), the only detection method:
  
  \[ \chi \chi \rightarrow \gamma \gamma \]
  
  \[ \chi \chi \rightarrow \gamma Z \]

- Even if LSP is not the dominant CDM (say 1%), direct detectors and neutrino telescopes can detect CDM.
More Opinions

- string theory (we still do not have the standard model)
- brane world & large extra dimensions (too arbitrary)
- moduli problem (good guidance to restricting string theory related speculations); above is a subset
- long distance modifications of gravity (surprisingly difficult)
- self-interacting dark matter (better simulations)
- CMB and inflation (no connection to SM yet)
- Self-tuning cosmological constant (unsuccessful thus far)
- New aspects of reheating (curvature perturb. not frozen)
- Transplanckian physics (ill motivated)
Outlook

• Supersymmetry is probably the best motivated.
• Dark matter direct and indirect detection experiments look promising and are indispensable for cosmology AND particle physics. Must be combined with collider data to make progress. NEED PROGRESS IN GALACTIC DISTRIBUTION OF DARK MATTER.
• MSSM EW baryogenesis almost ruled out. A good example of how collider data affects cosmology.
• Neutrinos and leptogenesis look promising. (Still plagued by gravitino problem within SUSY.)
• As the scorecard suggests, there is much to still connect.
• CMB physics (polarization) will tell us more about inflation, but still needs connection to particle physics. Hopefully will not remain an island.