say, from 929.9 to 930.1 kHz, a bandwidth of 0.2 kHz or 200 Hz, no 400-Hz signal would be present in the receiver.

Let us Fourier analyze the amplitude-modulated carrier:

\[ v(t) = V_0(1 + M \cos \omega_m t) \cos \omega_c t \]  

(3.26)

where \( \omega_m = 2 \pi f_m \), \( f_m = 400 \text{ Hz} \), \( \omega_c = 2 \pi f_c \), \( f_c = 930 \text{ kHz} \), and \( M \) is the degree of modulation, \( 0 \leq M \leq 1 \). \( M \) is 1.0 in Fig. 3.8(b). The modulated carrier wave \( v(t) \) is an even function, so only cosine terms will be present in the Fourier expansion. Also, we can see that \( a_0 = 0 \), because the average dc level is zero. Thus

\[ v(t) = \sum_{n=1}^{\infty} a_n \cos n \omega_c t \]  

(3.27)

where

\[ a_n = \frac{2}{T} \int_{-T/2}^{T/2} v(t) \cos n \omega_c t \, dt \]  

(3.28)

Choose \( T = 2 \pi / \omega_m = T_m \); this is the shortest time interval over which \( v(t) \) is periodic. Substituting (3.26) in (3.28) yields, with \( \omega = \omega_m \),

\[ a_n = \frac{4}{T_m} \int_0^{T_m/2} V_0(1 + M \cos \omega_m t) \cos \omega_c t \cos n \omega_m t \, dt \]
SEC. 3.3 Fourier Analysis

(a) circuit

\[ \omega_c = \frac{1}{\sqrt{LC}} \]

(b) output of diode before passing through the RC low-pass filter

\[ T_c = \frac{2\pi}{\omega_c} \]

(c) filtered diode output (original audio signal)

RC filter that passes only the audio envelope at \( \omega_m \), provided

\[ T_m \gg RC \gg T_c \quad \text{where} \quad T_m = \frac{2\pi}{\omega_m} \quad \text{and} \quad T_c = \frac{2\pi}{\omega_c} \]

Thus, the filter output is the same as the original audio modulation of the transmitted wave. Another type of amplitude modulation widely used in long-range noncommercial radio communication is single sideband (SSB). In this technique only one set of sidebands is actually transmitted, not the carrier
wave frequency is

\[ \omega = \omega_c + \Delta \omega \cos \omega_m t \]  \hspace{1cm} (3.39)

where \( \Delta \omega \) is the maximum frequency swing (the \textit{deviation}) of the carrier. The FM carrier wave is then

\[ v_c(t) = V_0 \cos \omega t = V_0 \cos[\omega_c t + \Delta \omega \cos \omega_m t] \]  \hspace{1cm} (3.40)

The carrier frequency swings from \( \omega_c + \Delta \omega \) to \( \omega_c - \Delta \omega \), as shown in Fig. 3.13. In commercial FM radio \( \Delta \omega / 2 \pi \) is limited to 75 kHz, so the maximum carrier-frequency swing is \( \pm 75 \text{ kHz} \) around \( \omega_c / 2 \pi \); for example, if \( \omega_c / 2 \pi = f_c = 100 \text{ MHz} \), then the swing is from 100.075 to 99.925 MHz. Commercial FM stations are assigned frequency \( f_c \) every 200 kHz, so their signals do not overlap. Notice that the frequency swing of the carrier is determined by the \textit{amplitude} (\( \Delta \omega \)) of the audio modulation, whereas the audio modulation \textit{frequency} (\( \omega_m \)) determines how often the carrier frequency swings up and

\[ \begin{array}{c}
\text{(a) unmodulated carrier} \\
\text{(b) modulating signal (audio)} \\
\text{(c) frequency modulated carrier}
\end{array} \]

\textbf{FIGURE 3.13} FM radio waveforms.