Theory of Hadronic B Decays

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- Summary and Outlook

M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda:
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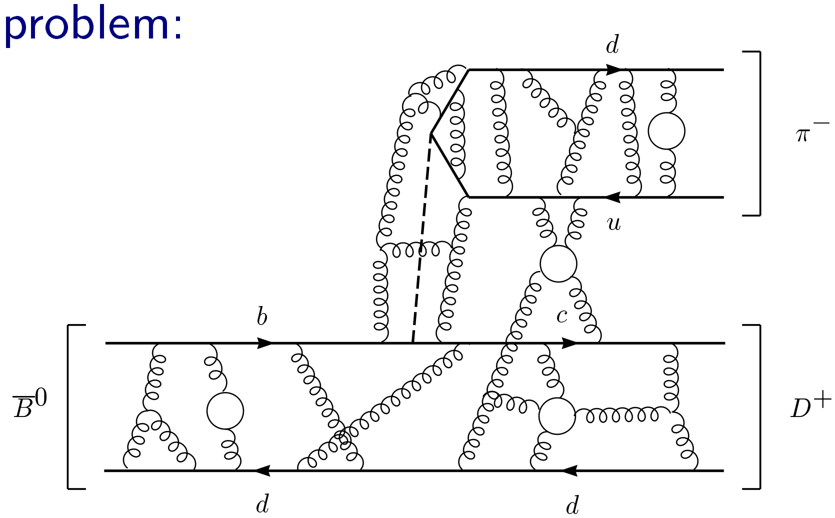
Introduction

* theoretical description of hadronic weak decays is difficult due to non-perturbative hadronic dynamics
* this affects interpretation of B factory data, studies of CP violation, and searches for New Physics
* the problem:



* hard gluon effects can be calculated and lead to an effective weak Hamiltonian:

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_i \lambda_i^{\rm CKM}\, C_i(\mu)\, O_i(\mu)$$

* difficulty is to calculate hadronic matrix elements of local operators $O_i(\mu)$

Dynamical approaches:

* Lattice QCD → Maiani–Testa no-go theorem (difficulty with FSI)
* QCD sum rules → too complicated, similar problems
* hard-scattering formalism → misses leading soft contributions to $B \to M$ decay form factors
* large-energy effective theory → not applicable to exclusive processes (collinear singularities)

Phenomenological approaches:

* "naive" factorization and generalizations

⇒ quite successful, but not a systematic treatment

Classification schemes:

* flavor topologies (trees, penguins, . . .)
* SU(3) or isospin amplitudes
* Wick contractions (charming penguins, . . .)

⇒ useful, but no dynamical insight

"Naive" Factorization

* consider $\bar{B}^0 \to D^+\pi^-$ as an example:

$$\mathcal{A}_{\bar{B}^0\to D^+\pi^-} \sim \left(C_1 + \frac{C_2}{N_c}\right)\langle D^+\pi^-|(\bar{d}u)(\bar{c}b)|\bar{B}^0\rangle + \frac{C_2}{2}\,\langle D^+\pi^-|(\bar{d}t_a u)(\bar{c}t_a b)|\bar{B}^0\rangle \stackrel{\rm fact.}{\to} \left(C_1 + \frac{C_2}{N_c}\right) \underbrace{\langle \pi^-|(\bar{d}u)|0\rangle}_{\sim f_\pi} \underbrace{\langle D^+|(\bar{c}b)|\bar{B}^0\rangle}_{\sim F^{B\to D}}$$

hence:

$$\mathcal{A}_{\bar{B}^0\to D^+\pi^-} \sim G_F V_{cb} V_{ud}^*\, f_\pi\, F^{B\to D}(m_\pi^2)\, a_1$$

with

$$a_1 = C_1(\mu) + \frac{C_2(\mu)}{N_c}$$

* similarly, define parameter $a_2 = C_2 + C_1/N_c$, and further parameters $a_3, \dots, a_{10}$ for more complicated decays

Problem: a_i are renormalization-scale and -scheme dependent in "naive" factorization!

* "generalized (naive)" factorization:

$$a_1 = C_1(m_b) + \xi_1\, C_2(m_b)$$
$$a_2 = C_2(m_b) + \xi_2\, C_1(m_b)$$

etc.

⇒ phenomenological parameters $\xi_i \equiv 1/(N_c^{\rm eff})_i$ account for "non-factorizable" effects

⇒ to maintain predictive power, assume that ξ_i are process-independent and universal for all operators with same chirality ($\rightarrow$ only two parameters ξ_{LL} and ξ_{LR})

* no theoretical justification for these assumptions!

QCD Factorization Theorem

consider decays $\bar{B} \to M_1 M_2$, where M_1 is meson which absorbs light spectator quark of B meson (can be heavy or light), and M_2 is “emission” particle (must be light)

* idea is to factorize and compute hard contributions and to parameterize soft and collinear ones, exploiting presence of large scale $m_b \gg \Lambda_{\rm QCD}$
* hard and IR contributions are separated on basis of Feynman diagrams, assuming that soft effects not visible from diagrams do not destroy power counting (no known counter example)
* since a light final-state meson carries energy and momentum $\sim m_b/2$, it can be described by light-cone distribution amplitudes (LCDAs) if all constituents have large momenta
* probability for asymmetric parton configurations where some partons are soft can be estimated from endpoint behavior of asymptotic LCDAs; if unsuppressed, such endpoint contributions require introducing additional non-perturbative parameters

Results:

1a. non-factorizable contributions, i.e. effects not associated with $B \to M_1$ form factor or M_2 wave function, are dominated by hard gluon exchange and can be calculated $\to$ convolution of hard scattering kernels with LCDAs

b. non-factorizable soft exchanges are power suppressed, because $(q\bar{q})$ pair that forms M_2 is produced as a small color dipole (Bjorken's color transparency argument)

c. non-factorizable collinear exchanges cancel at leading power

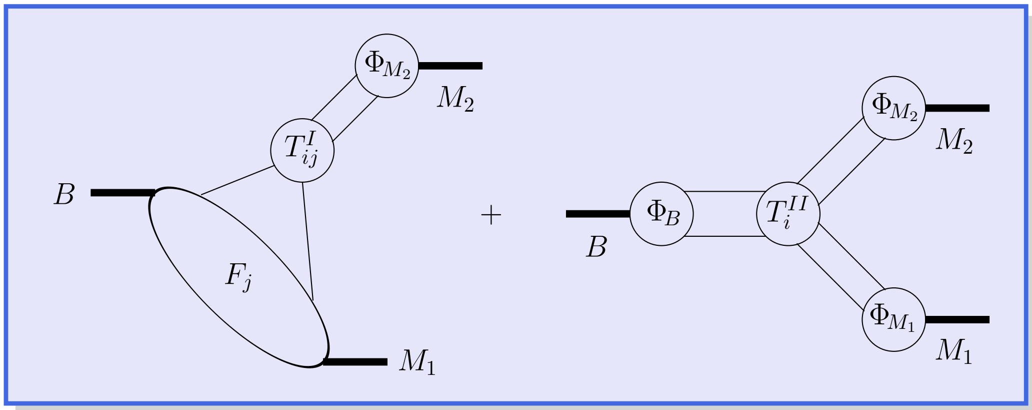
2. $B \to M_1$ form factor is dominated by soft gluon exchange by power counting (for both heavy and light M_1), since B meson contains a soft spectator quark ($\to$ ignores possibility that resummation of Sudakov logarithms suppresses soft contributions, which however appears unrealistic for B mesons)

3. non-factorizable hard gluon exchange between M_2 and B-meson spectator quark is a leading effect if M_1 is light

4. annihilation topologies and higher Fock states of M_2 give power-suppressed contributions

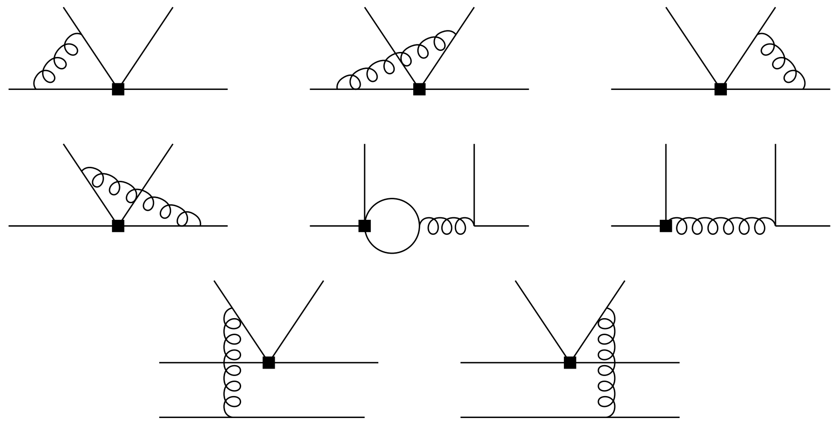
Factorization theorem:

$$\langle M_1 M_2|O_i|\bar{B}\rangle = F_j^{B\to M_1}\, T_{ij}^{\rm I} \otimes \Phi_{M_2} + T_i^{\rm II} \otimes \Phi_B \otimes \Phi_{M_1} \otimes \Phi_{M_2} + \text{power suppressed contributions}$$



- if M_1 is heavy, the second term is power suppressed and should be dropped
- factorization does not hold if M_2 is a heavy-light meson, but it works for an onium state such as J/ψ
- validity of factorization theorem demonstrated by explicit 1-loop calculation; general arguments support factorization to all orders in perturbation theory

Soft and collinear cancellations at 1-loop order:



* IR poles for non-factorizable vertex diagrams in $\bar{B}^0 \to D^+\pi^-$:

$$D_1 \sim \left(\frac{m_b}{\mu}\right)^{-2\epsilon} \left[-\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\left(\ln\frac{2xv_b\cdot p_\pi}{m_b} - 1\right)\right]$$

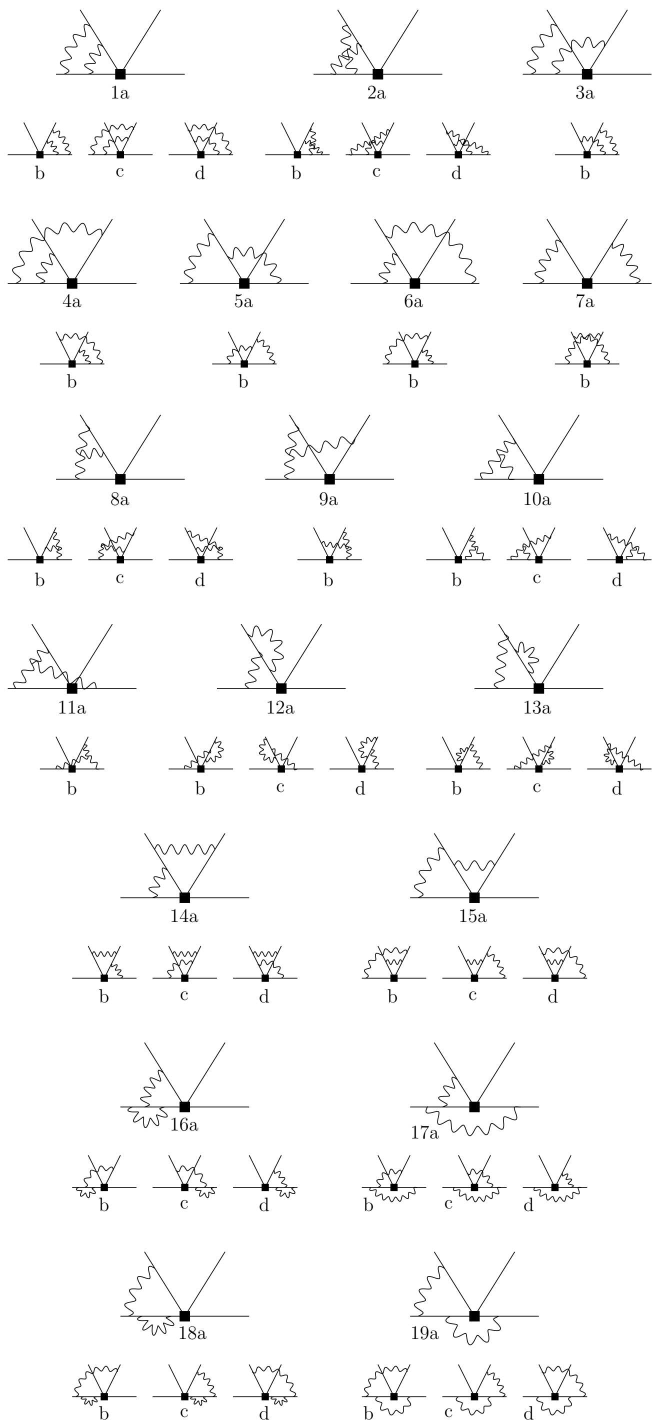
$$D_2 \sim \left(\frac{m_b}{\mu}\right)^{-2\epsilon} \left[+\frac{1}{\epsilon^2} - \frac{2}{\epsilon}\left(\ln\frac{2(1-x)v_b\cdot p_\pi}{m_b} - 1\right)\right]$$

$$D_3 \sim \left(\frac{m_c}{\mu}\right)^{-2\epsilon} \left[-\frac{1}{\epsilon^2} + \frac{2}{\epsilon}\left(\ln\frac{2(1-x)v_c\cdot p_\pi}{m_c} - 1 - i\pi\right)\right]$$

$$D_4 \sim \left(\frac{m_c}{\mu}\right)^{-2\epsilon} \left[+\frac{1}{\epsilon^2} - \frac{2}{\epsilon}\left(\ln\frac{2xv_c\cdot p_\pi}{m_c} - 1 - i\pi\right)\right]$$

$\Rightarrow$ pairwise cancellation of soft singularities; complete cancellation of soft and collinear singularities in sum of the four diagrams

* for heavy-light final state such as $D\pi$, cancellation of soft and collinear divergences proved at 2-loop order:



Implications:

* obtain approach that allows for a systematic, model-independent calculation of corrections to "naive" factorization, which emerges as leading term in heavy-quark limit

* possibility to compute systematically logarithmic corrections to "naive" factorization solves problem of scale and scheme dependences (scale and scheme dependences of hard scattering kernels compensate those of Wilson coefficients)

* non-factorizable corrections are process dependent and hence non-universal, in contrast with basic assumption of "generalized" factorization models

* strong FSI and rescattering phases are calculable and are perturbative or power suppressed (soft rescattering vanishes in the heavy-quark limit)

Final State Interactions

* unitarity implies that:

$$\text{Im}\mathcal{A}_{B\to M_1 M_2} \sim \sum_n \mathcal{A}_{B\to n}\mathcal{A}^*_{n\to M_1 M_2}$$

* can discuss FSI using hadronic or partonic language (justified by dominance of hard contributions)
* in heavy-quark limit, arbitrarily large number of hadronic intermediate states contribute, and their average is described by the partonic language (quark–hadron duality)
* hadronic description makes it difficult to observe systematic cancellations, which usually occur in inclusive sums over intermediate states (cf. $e^+e^- \to$ hadrons)
* while each particular intermediate state n cannot be described in a partonic language, the inclusive sum is accurately represented by a small $(q\bar{q})$ color dipole which interacts little with its environment
* resulting physical picture is very different from elastic rescattering and Regge phenomenology

Application: $\bar{B}^0 \to D^+\pi^-$

- obtain explicit, renormalization-scheme invariant expression for parameter a_1 at next-to-leading order in α_s and leading power in $\Lambda_{\rm QCD}/m_b$:

$$a_1 = C_1(\mu) + \frac{C_2(\mu)}{N_c} + \frac{C_2(\mu)}{N_c}\,\frac{C_F\alpha_s}{4\pi}\left[\underbrace{12\ln\frac{m_b}{\mu} - B}_{\text{cancels scale and scheme dep.}} + \Delta_{D\pi}\left(\frac{m_c}{m_b}\right)\right]$$

with

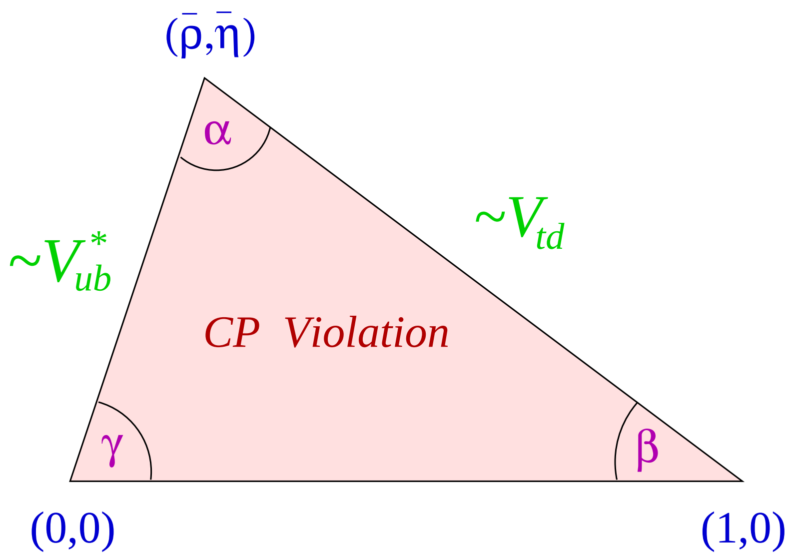
$$\Delta_{D\pi}(z) = \frac{1}{f_\pi}\int_0^1 \mathrm{d}x\,\Phi_\pi(x)\left[g(x,z) + ih(x,z)\right]$$

process-dependent, non-universal correction

- confirms earlier result by Politzer and Wise (1991)
- QCD factorization theorem does not hold for the color-suppressed decay $\bar{B}^0 \to \pi^0 D^0$, because in this case "emission" particle is heavy

Application: Extraction of γ in $B \to \pi K$

* rare B decays into two light mesons are important for studies of CP violation and the CKM paradigm
* unitarity triangle:



* information on γ and the Wolfenstein parameters $(\bar\rho, \bar\eta)$ can be obtained from measurements of CP-averaged decay rates in $B \to \pi K$ and $B \to \pi\pi$
* define:

$$R_* = \frac{\text{Br}(B^\pm \to \pi^\pm K^0)}{2\text{Br}(B^\pm \to \pi^0 K^\pm)}$$

$$\bar\varepsilon_{3/2} = R_1 \tan\theta_C \left[\frac{2\text{Br}(B^\pm \to \pi^\pm \pi^0)}{\text{Br}(B^\pm \to \pi^\pm K^0)}\right]^{1/2}$$

and ($x_t = (m_t/m_W)^2$):

$$\delta_{\rm EW} = R_2\,\frac{\cot^2\theta_C}{\sqrt{\bar\rho^2+\bar\eta^2}}\,\frac{\alpha_W x_t}{8\pi}\left(1+\frac{3\ln x_t}{x_t-1}\right)$$

* solve for $\cos\gamma$ in terms of a strong-interaction phase ϕ:

$$\cos\gamma = \delta_{\rm EW} - \frac{X_R + \frac12\bar\varepsilon_{3/2}(X_R^2 - 1 + \delta_{\rm EW}^2)}{\cos\phi + \bar\varepsilon_{3/2}\delta_{\rm EW}}$$

where

$$X_R = \frac{\sqrt{R_*^{-1}} - 1}{\bar\varepsilon_{3/2}}$$

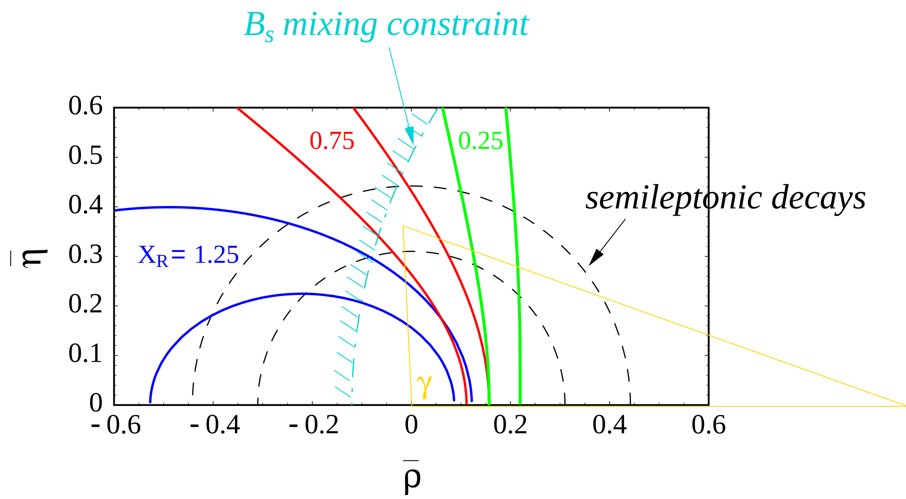
* theory input (R_1, R_2 account for SU(3) breaking):

$$R_1 = 1.22 \pm 0.05$$
$$R_2 = 0.92 \pm 0.09$$
$$|\phi| < 25^\circ \quad (\text{HQL: } \phi \approx -11^\circ)$$

* experimental input: values for X_R and $\bar\varepsilon_{3/2}$

⇒ preliminary CLEO data:
$\bar\varepsilon_{3/2} = 0.21 \pm 0.06$ and $X_R = 0.7 \pm 1.0$

$\Rightarrow$ allowed region in the $(\bar\rho,\bar\eta)$ plane for different values of X_R:



* measurement of X_R implies non-trivial constraint on γ and Wolfenstein parameters $(\bar\rho,\bar\eta)$, which is largely complementary to constraints arising from semileptonic B decays and B–$\bar{B}$ mixing

* a value close to present value $X_R \approx 0.7$ would imply $\gamma \approx 90^\circ$, corresponding to $\bar\rho \approx 0$ and $\bar\eta \approx 0.4$

* a value $X_R > 1$ would be incompatible with the B_s–$\bar{B}_s$ mixing bound, indicating New Physics

$\Rightarrow$ simple, yet powerful measurement at B factories!

Application: Extraction of α in $B \to \pi\pi$

* time-dependent, mixing-induced CP asymmetry in $B_d \to \pi^+\pi^-$ decays:

$$A_{\rm CP}(t) = \frac{\Gamma(B^0(t) \to \pi^+\pi^-) - \Gamma(\bar{B}^0(t) \to \pi^+\pi^-)}{\Gamma(B^0(t) \to \pi^+\pi^-) + \Gamma(\bar{B}^0(t) \to \pi^+\pi^-)}$$

$$= -S \cdot \sin(\Delta M_B\, t) + C \cdot \cos(\Delta M_B\, t)$$

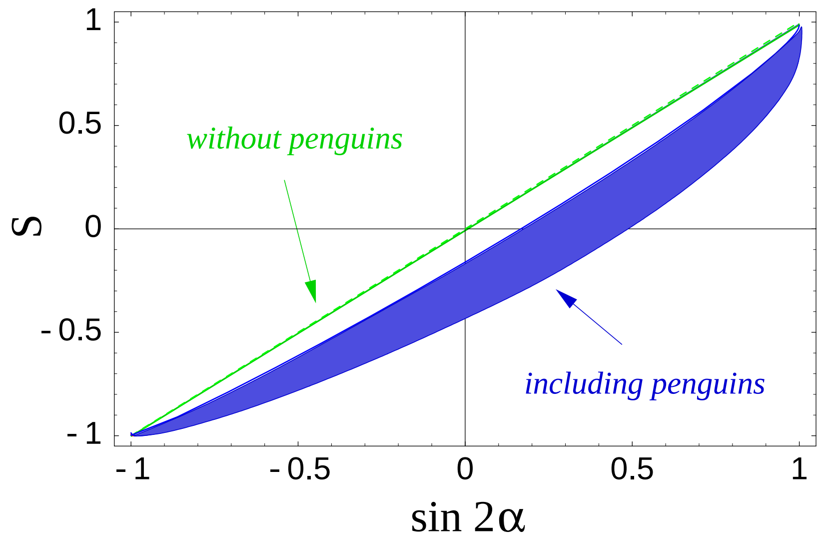
* without "penguin pollution":

$$S = \sin 2\alpha\,, \qquad C = 0$$

free of hadronic uncertainties

* interference of tree and (subdominant) penguin topologies introduces hadronic uncertainties, which can be estimated by applying the QCD factorization theorem to the $B \to \pi\pi$ decay amplitudes

Result:



⇒ can be used to convert a measurement of S into a range for $\sin 2\alpha$

Summary and Outlook

* QCD factorization theorem provides systematic, model-independent description of most two-body, non-leptonic B decays in heavy-quark limit

* wide range of applications relevant to CP-violation studies and searches for New Physics at B factories

* numerical results presented here are *preliminary!*

* much conceptual work remains to be completed:

  → proof of factorization beyond 1-loop order for decays with two light final-state mesons

  → understand and estimate power corrections in $\Lambda_{\rm QCD}/m_b$, especially chirally-enhanced ones

  → control large logarithms in B-meson LCDA entering second term in factorization formula

* work in progress . . .