Parton Distributions Functions, Part 1

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A. Introduction
B. Properties of the PDFs
C. Results of CTEQ Global Analysis
D. Uncertainties of the PDFs
E. Applications to LHC Physics

A. Introduction. QCD and High-Energy Physics

QCD is an elegant theory of the strong interactions – the gauge theory of color transformations. It has a simple Lagrangian

$$\mathcal{L} = \overline{\psi}\left[i\,\partial\!\!\!/ - g A\!\!\!/ - m\right]\psi - \frac{1}{4}F_a^{\mu\nu}F_{a\mu\nu}$$

$$F_a^{\mu\nu} = \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - g f_{abc} A_b^\mu A_c^\nu$$

$$A\!\!\!/ = \gamma_\mu A_a^\mu T_a$$

(sums over flavor and color are implied)

Parameters: g; m_1, m_2, m_3, …, m_6

$$\mathcal{L} = \overline{\psi}\,[i\,\partial\!\!\!/ - g A\!\!\!/ - m]\,\psi - \frac{1}{4} F_a^{\mu\nu} F_{a\mu\nu}$$

However, the calculation of experimental observables is quite difficult, for 2 reasons:

(i) there are divergent renormalizations; the theory requires

- regularization
- perturbation theory
- a singular limit
  - $\Lambda \to \infty$ (GeV)
  - or, a $\to$ 0 (fm)
  - or, n$\to$ 4

(ii) quark confinement;
the asymptotic states are color singlets, whereas the fundamental fields are color triplets.

Nevertheless, certain cross sections can be calculated reliably ---
--- inclusive processes with large momentum transfer (i.e., short-distance interactions)

The reasons that QCD can provide accurate predictions for short-distance interactions are

- ❑ asymptotic freedom

$$\alpha_s(Q^2) \sim \textit{const.}/\ln(Q^2/\Lambda^2) \quad \text{as} \quad Q^2 \to \infty$$

- ❑ the factorization theorem

$$d\sigma_{hadron} \sim \text{PDF} \otimes C$$

where C is calculable in perturbation theory.

The PDFs provide a connection between quarks and gluons (the partons) and the nucleon (a bound state).

Global Analysis of QCD and Parton Distribution Functions

$$d\sigma_{hadron} \sim \text{PDF} \otimes C \qquad \text{(sum over flavors implied)}$$

The symbol $\sim$ means "asymptotically equal as $Q \to \infty$"; the error is $O(m^2/Q^2)$ where Q is an appropriate (high) momentum scale.

The C's are calculable in perturbation theory.

The PDFs are not calculable today, given our lack of understanding of the nonperturbative aspect of QCD (binding and confinement). But we can determine the PDFs from Global Analysis, with some accuracy.

The symbol $\otimes$ means "convolution", $f \otimes g(x) = \int_x^1 f(\xi) g(x/\xi) d\xi$.

A

next

B. Properties of the PDFs -- Definitions

First, what are the Parton Distribution Functions? (PDFs)

The PDFs are a set of 11 functions,

$f_i(x,Q^2)$ where $\begin{cases} 0 \leq x \leq 1 & \text{longitudinal momentum fraction} \\ Q >\sim 2\ GeV & \text{momentum scale} \end{cases}$

$i = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ parton index

$f_0 = g(x,Q^2)$ the gluon PDF
$f_1 = u(x,Q^2)$ the up-quark PDF
$f_{-1} = \bar{u}(x,Q^2)$ the up-antiquark PDF
$f_2 = d$ and $f_{-2} = \bar{d}$
$f_3 = s$ and $f_{-3} = \bar{s}$
etc.

Exercise:
What about the top quark?

Second, what in the *meaning* of a PDF?

We tend to think and speak in terms of

"Proton Structure"

$u(x,Q^2)\ dx$ = the mean number of up quarks with longitudinal momentum fraction from x to $x + dx$, appropriate to a scattering experiment with momentum transfer Q.

$u(x,Q^2)$ = the up-quark density in momentum fraction

This heuristic interpretation makes sense from the LO parton model. More precisely, taking account of QCD interactions, $d\sigma_{\text{proton}}$ = PDF $\otimes$ C .

$f_i(x,Q^2)$ = the *density* of parton *i*
w.r.t. longitudinal momentum fraction x

Momentum Sum Rule

$$\int_0^1 x f_i(x, Q^2) dx =$$

longitudinal momentum fraction, carried by parton type i

$$\sum_{i=0}^{\pm 5} \int_0^1 x f_i(x, Q^2) dx = 1.$$

Flavor Sum Rules (for the proton)

$$\int_0^1 \left[u(x, Q^2) - \overline{u}(x, Q^2)\right] dx \ = \ 2$$

valence up quark density,

$$\int_0^1 \left[d(x, Q^2) - \overline{d}(x, Q^2)\right] dx \ = \ 1$$

valence down quark density,

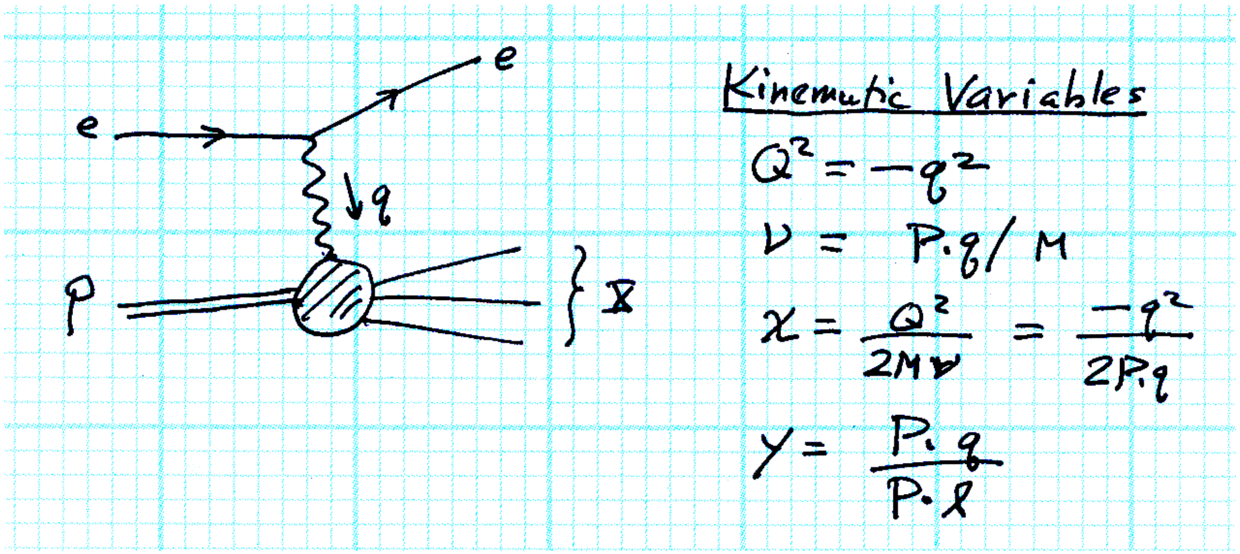
$$\int_0^1 \left[s(x, Q^2) - \overline{s}(x, Q^2)\right] dx \ = \ 0$$

valence strange quark density

Example.

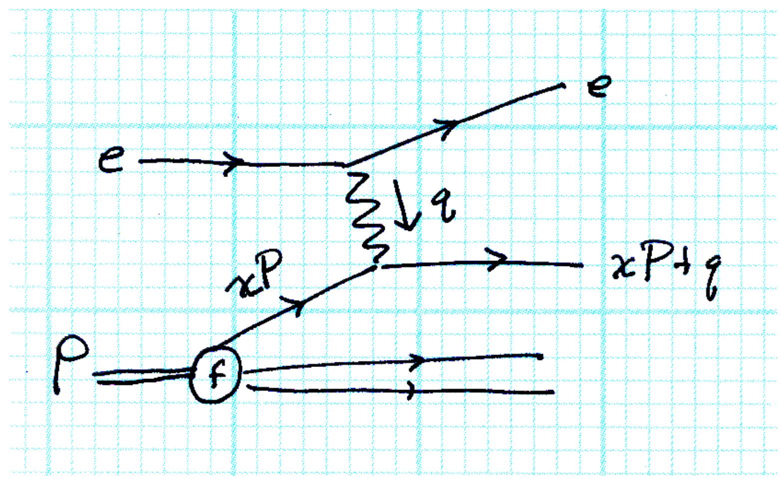
DIS of electrons by protons; e.g., HERA experiments

$$e + p \rightarrow e + X$$

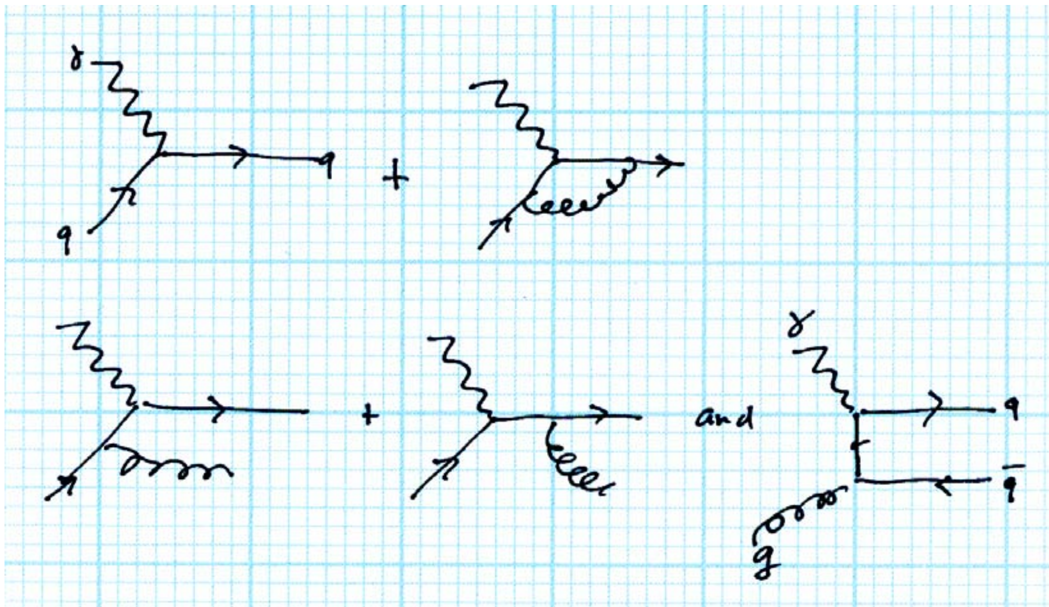


in lowest order of QCD

(summed over flavors!)



But QCD radiative corrections must be included to get a sufficiently accurate prediction.
The NLO approximation will involve these interactions …



From these perturbative calculations, we determine the coefficient functions $C_i^{(NLO)}$, and hence write

$$\sigma_{ep} \sim \sum_i (PDF)_i^{(NLO)} \otimes C_i^{(NLO)}$$

Approximations available today: LO, NLO, NNLO

The Factorization Theorem

For short-distance interactions,

$$\sigma_{ep} \sim \sum_{\mathrm{i}} (\mathrm{PDF})_{\mathrm{i}} \otimes C_{\mathrm{i}}$$

$$\sigma_{pp} \sim \sum_{\mathrm{i,j}} (\mathrm{PDF})_{\mathrm{i}}\ (\mathrm{PDF})_{\mathrm{j}} \otimes \otimes\ C_{\mathrm{ij}}$$

and the PDFs are universal !

We can write a formal, field-theoretic expression,

Correlator function

$$f_i(x,\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dy^- \, e^{iy^- p^+} \times \langle p \,|\overline{\psi}_i(0,y^-,0_T)\gamma^+\psi_i(0,0,0_T)|\, p\,\rangle$$

although we can't evaluate it because we don't know the bound state |p>.

Exercise.

Suppose the parton densities for the proton are known,

$$f_i\left(x, Q^2\right) \quad \text{for} \quad \left\{ i = 0, \pm 1, \pm 2, \ldots, \pm 5 \right\}$$

(A) In terms of the $f_i(x,Q^2)$, write the 11 parton densities for the neutron, say, $g_i(x,Q^2)$.

(B) In terms of the $f_i(x,Q^2)$, write the 11 parton densities for the deuteron, say, $h_i(x,Q^2)$.

B. Properties of the PDFs – Q^2 evolution

Evolution in Q

The PDFs are a set of 11 functions,

$f_i(x,Q^2)$ where $\begin{cases} 0 \le x \le 1 \\ Q >\sim 2\ GeV \end{cases}$ longitudinal momentum fraction; momentum scale

$f_i(x,Q^2)$ = the density of partons of type *i,* carrying a fraction *x* of the longitudinal momentum of a proton, when resolved at a momentum scale Q.

The DGLAP, or RG, Evolution Equations …

- We know how the f_i vary with Q.
- That follows from the *renormalization group*.
- It’s calculable in perturbation theory .

The DGLAP Evolution Equations

V.N. Gribov and L.N. Lipatov, Sov J Nucl Phys 15, 438 (1972); G. Altarelli and G. Parisi, Nucl Phys B126, 298 (1977);
Yu.L. Dokshitzer, Sov Phys JETP 46, 641 (1977).

$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dz}{z}\, P(z,\alpha_s(Q^2))\, q\left(\frac{x}{z},Q^2\right)$$

for a NON-SINGLET distribution,

e.g., $q = q_i - q_j$

$$P(z,\alpha_s) = P^{(0)}(z) + \frac{\alpha_s}{2\pi} P^{(1)}(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 P^{(2)}(z)$$

Solve the 11 coupled equations numerically.
For example, you could download the program HOPPET.
G. P. Salam and J. Rojo, *A Higher Order Perturbative Parton Evolution Toolkit*; download from http://projects.hepforge.org/hoppet
… a library of programs written in Fortran 90.

A Higher Order Perturbative Parton Evolution Toolkit (HOPPET)

G. P. Salam and J. Rojo
LPTHE,
UPMC – Univ. Paris 6,
Université Paris Diderot – Paris 7,
CNRS UMR 7589,
75252 Paris cedex 05, France

Abstract

This document describes a Fortran 95 package for carrying out DGLAP evolution and other common manipulations of parton distribution functions (PDFs). The PDFs are represented on a grid in x–space so as to avoid limitations on the functional form of input distributions. Good speed and accuracy are obtained through the representation of splitting functions in terms of their convolution with a set of piecewise polynomial basis functions, and Runge-Kutta techniques are used for the evolution in Q. Unpolarised evolution is provided to NNLO, including heavy-quark thresholds in the $\overline{\text{MS}}$ scheme, and longitudinally polarised evolution to NLO. The code is structured so as to provide simple access to the objects representing splitting functions and PDFs, making it possible for a user to extend the facilities already provided. A streamlined interface is also available, facilitating use of the evolution part of the code from F77 and C/C++.

Some informative results obtained using HOPPET

Starting from a set of “benchmark input PDFs”, let’s use HOPPET to calculate the evolved PDFs at selected values of Q.

For the ***input*** (not realistic but used here to study the evolution qualitatively):

$Q_0^2 = 2\ \mathrm{GeV}^2$ →

$$
\begin{aligned}
xu_v(x) &= 5.107200x^{0.8}(1-x)^3\,,\\
xd_v(x) &= 3.064320x^{0.8}(1-x)^4\,,\\
x\bar{d}(x) &= 0.1939875x^{-0.1}(1-x)^6\,,\\
x\bar{u}(x) &= x\bar{d}(x)(1-x)\,,\\
xs(x) &= x\bar{s}(x) = 0.2(x\bar{d}(x)+x\bar{u}(x))\,,\\
xg(x) &= 1.7x^{-0.1}(1-x)^5\,,
\end{aligned}
$$

Output tables

The expected output from the program is:

Evaluating PDFs at Q = 100.000 GeV

x	u-ubar	d-dbar	2(ubr+dbr)	c+cbar	gluon
1.0E-05	3.1907E-03	1.9532E-03	3.4732E+01	1.5875E+01	2.2012E+02
1.0E-04	1.4023E-02	8.2749E-03	1.5617E+01	6.7244E+00	8.8804E+01
1.0E-03	6.0019E-02	3.4519E-02	6.4173E+00	2.4494E+00	3.0404E+01
1.0E-02	2.3244E-01	1.3000E-01	2.2778E+00	6.6746E-01	7.7912E+00
1.0E-01	5.4993E-01	2.7035E-01	3.8526E-01	6.4466E-02	8.5266E-01
3.0E-01	3.4622E-01	1.2833E-01	3.4600E-02	4.0134E-03	7.8898E-02
5.0E-01	1.1868E-01	3.0811E-02	2.3198E-03	2.3752E-04	7.6398E-03
7.0E-01	1.9486E-02	2.9901E-03	5.2352E-05	5.6038E-06	3.7080E-04
9.0E-01	3.3522E-04	1.6933E-05	2.5735E-08	4.3368E-09	1.1721E-06

The Running Coupling of QCD

$$\frac{d\alpha_s}{d\ln Q^2} = \beta(\alpha_s) = -b_0\alpha_s^2 - b_1\alpha_s^3 - b_2\alpha_s^4$$

$$b_0 = (33 - 2n_f)/12\pi$$

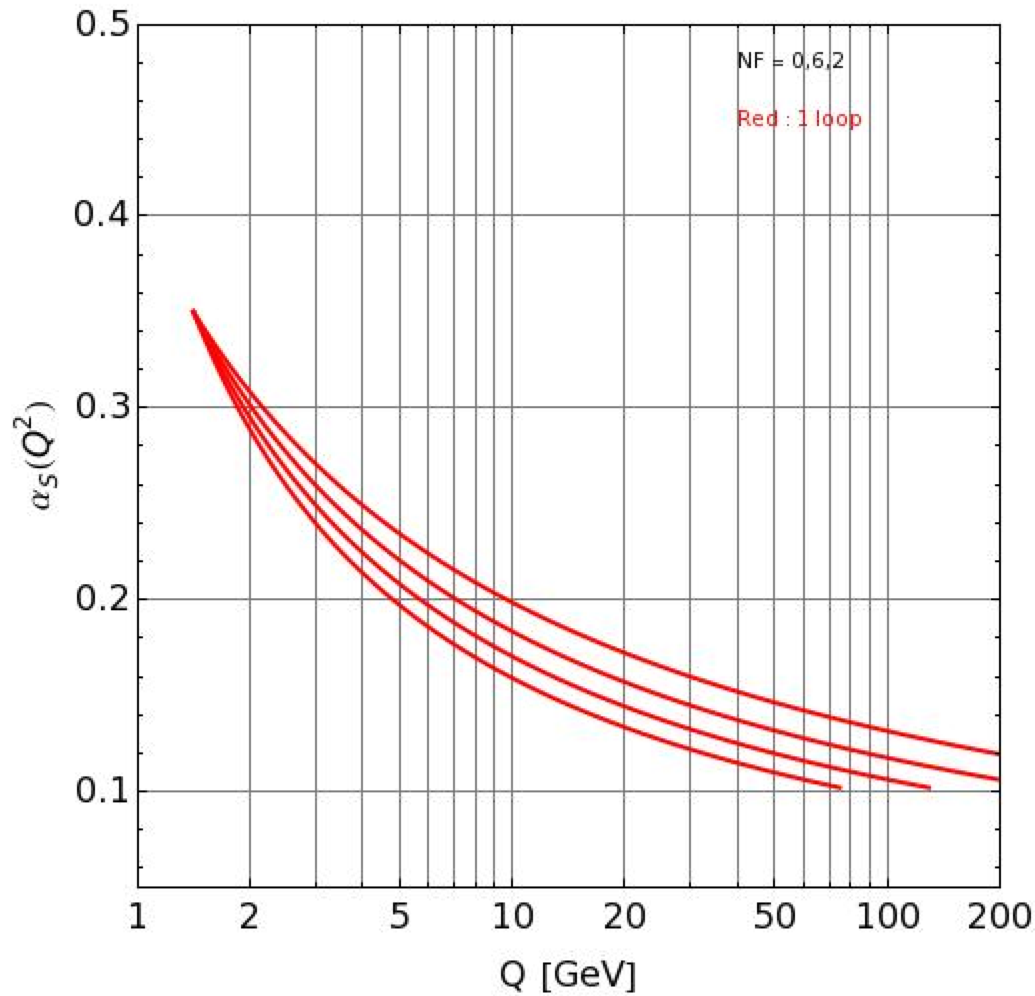
$$b_1 = (153 - 19n_f)/24\pi^2$$

$$b_2 = (77139 - 15099n_f + 325n_f^2)/3456\pi^3$$

The QCD Running Coupling Constant

Evolution of α_S as a function of Q, using
• the 1-loop beta function,
• with NF = number of massless flavors = 0, 2, 4, 6.

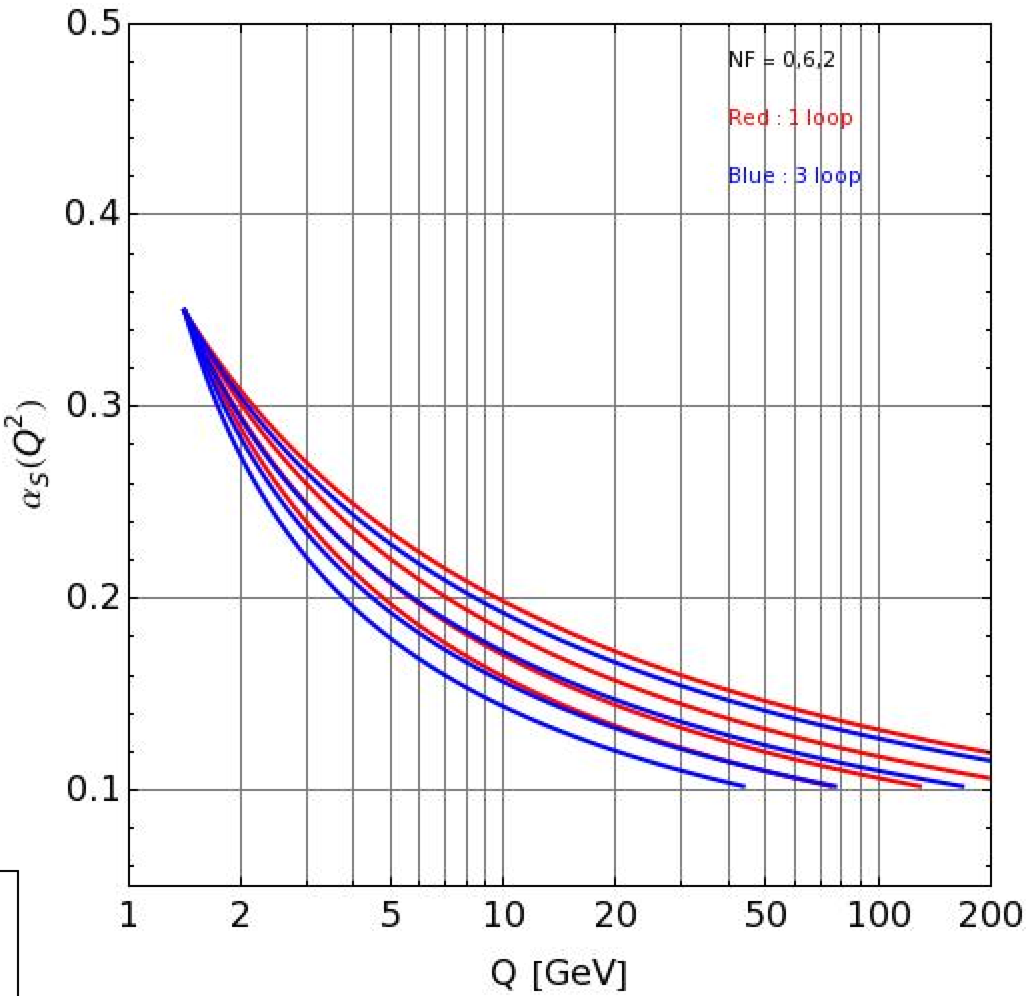
For Global Analysis, we need an accurate $\alpha_S(Q^2)$.



The QCD Running Coupling Constant

Evolution of α_S as a function of Q, using

- the 1-loop beta function (red) and the 3-loop beta function (blue),
- with NF = number of massless flavors = 0, 2, 4, 6.



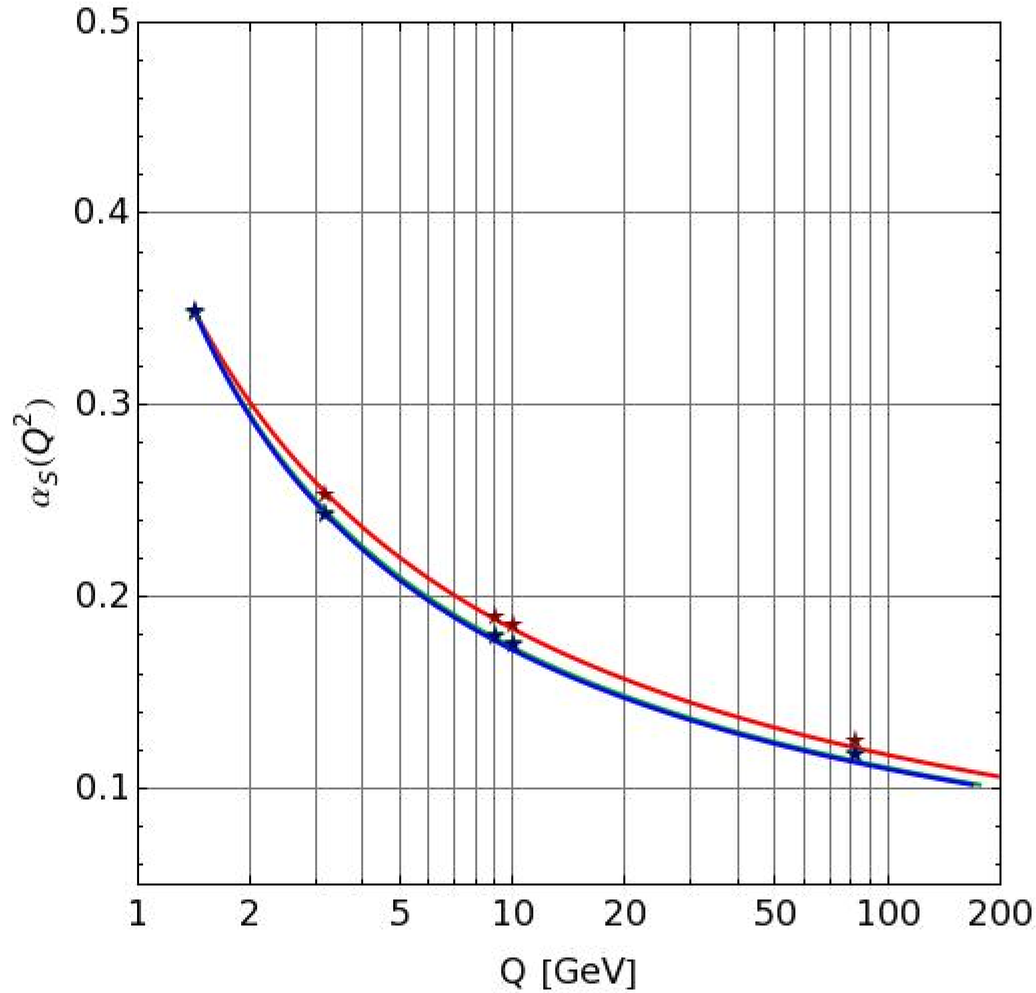
For Global Analysis, we need an accurate $\alpha_S(Q^2)$.

The QCD Running Coupling Constant

Red curve: 1-loop beta function; NF = number of massless quark flavors = 4.

Red points: 1-loop beta function from HOPPET.

The blue curve and blue points, are the same for the 3-loop beta function.

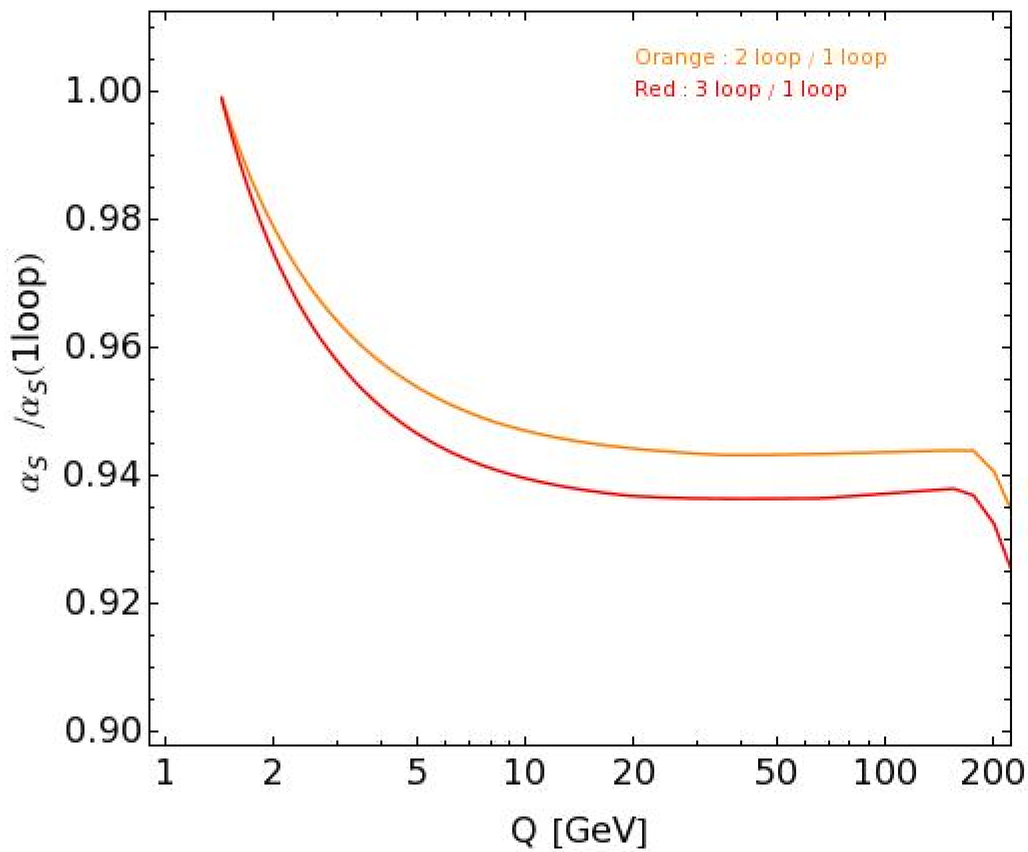


The QCD Running Coupling Constant

How large are the 2-loop and 3-loop corrections for $\alpha_S(Q^2)$?

Orange:
2-loop / 1-loop

Red:
3-loop / 1-loop



Exercise: What does it mean?

Asymptotic Freedom

Why does QCD have this property?

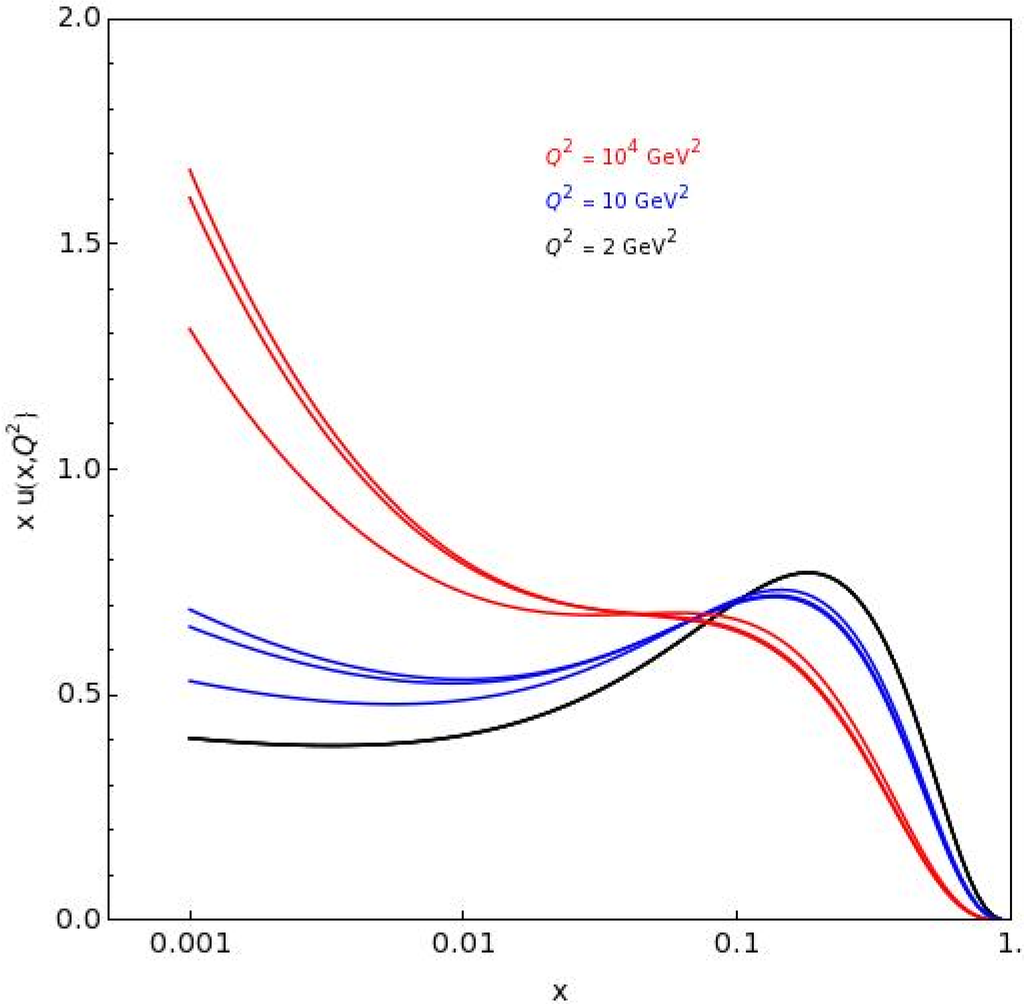
How does the u-quark PDF evolve in Q?

Examples from HOPPET

U-quark PDF evolution :

Black : $Q = Q_0 = 1.414$ GeV
Blue : $Q = 3.16$ GeV
(1-loop, 2-loop, 3-loop)
Red : $Q = 100.0$ GeV
(1-loop, 2-loop, 3-loop)

(Benchmark PDFs of
A. Vogt)



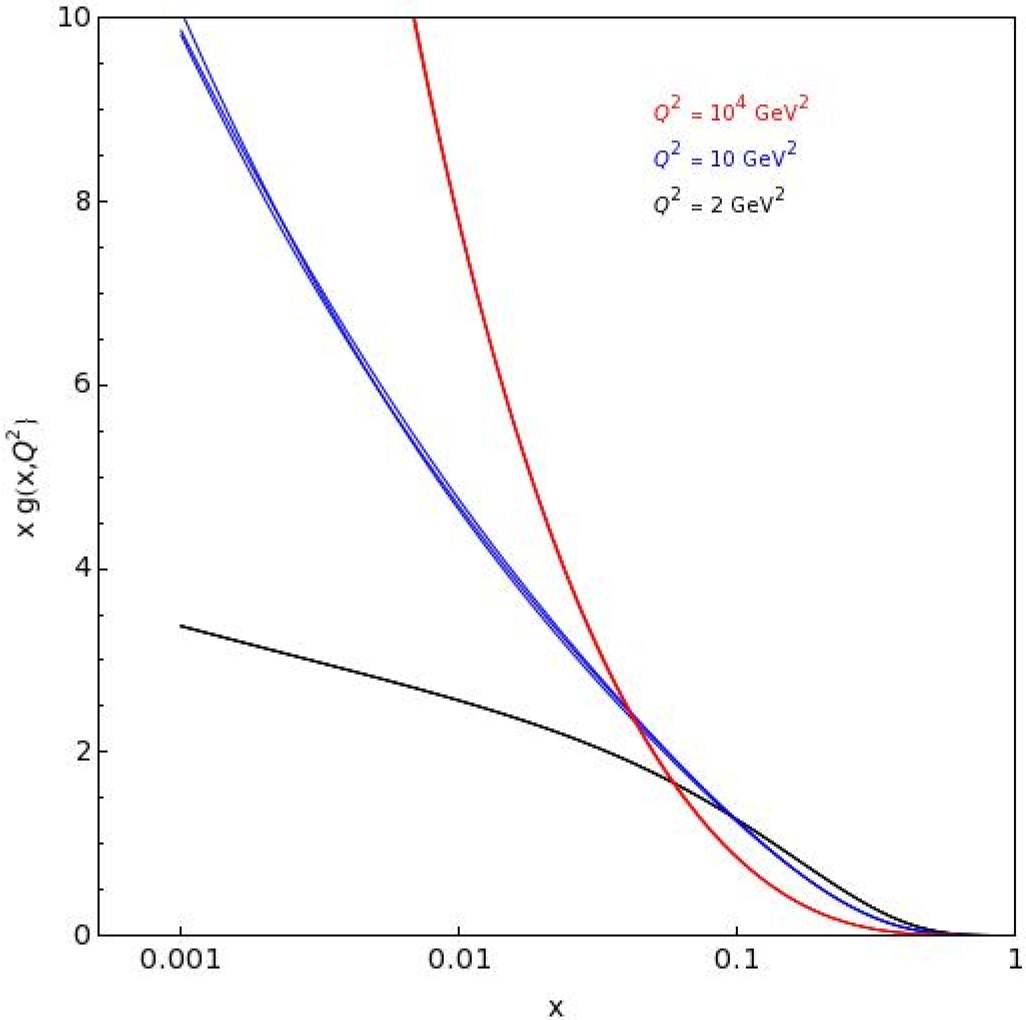
How does the gluon PDF evolve in Q?

Examples from HOPPET

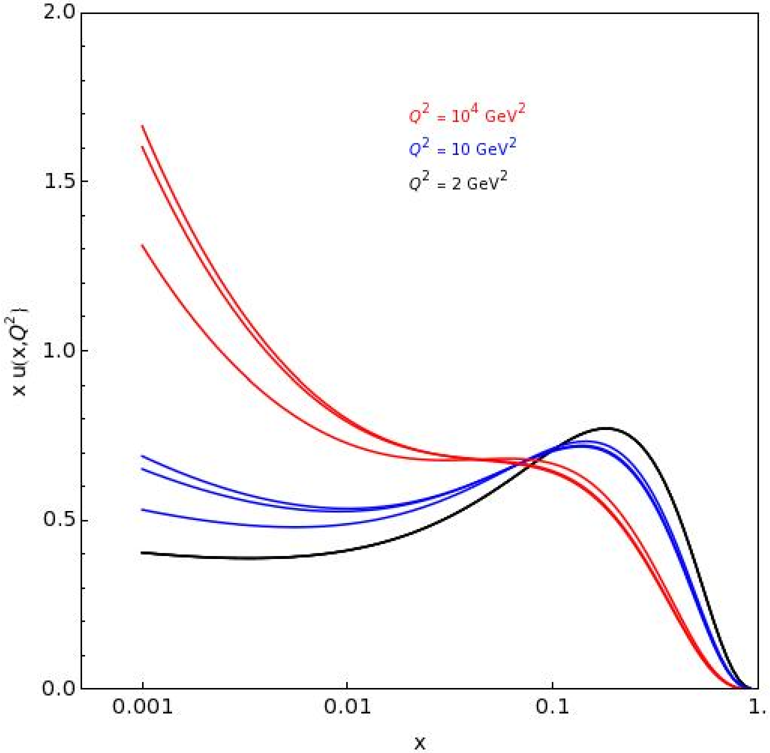
Gluon PDF evolution :

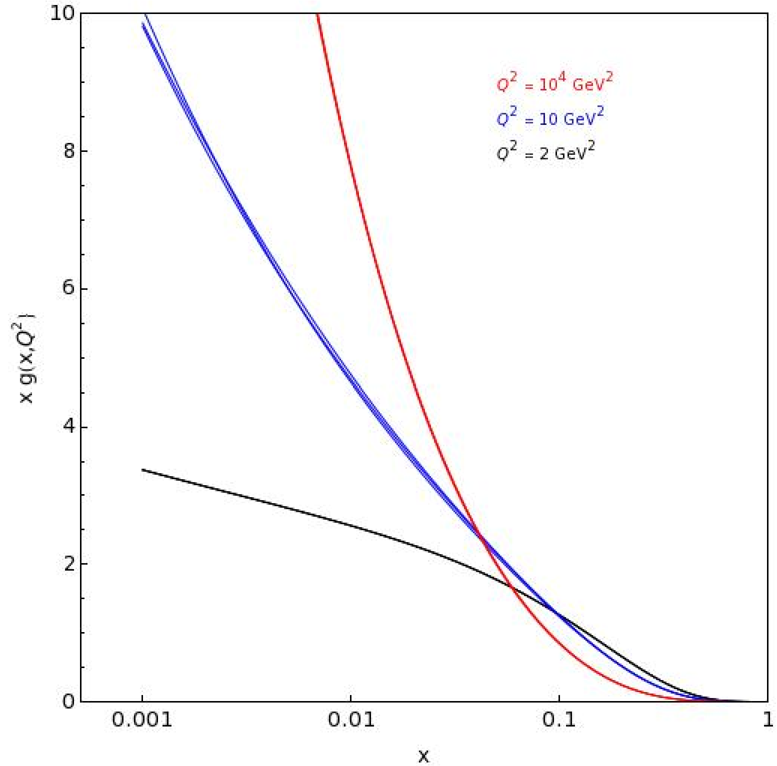
Black : Q = Q0 = 1.414 GeV
Blue : Q = 3.16 GeV
(1-loop, 2-loop, 3-loop)
Red : Q = 100.0 GeV
(1-loop, 2-loop, 3-loop)

(Benchmark PDFs of A. Vogt)



HOPPET – DGLAP evolution of PDFs





- The "structure of the proton" depends on the resolving power of the scattering process. As Q increases …

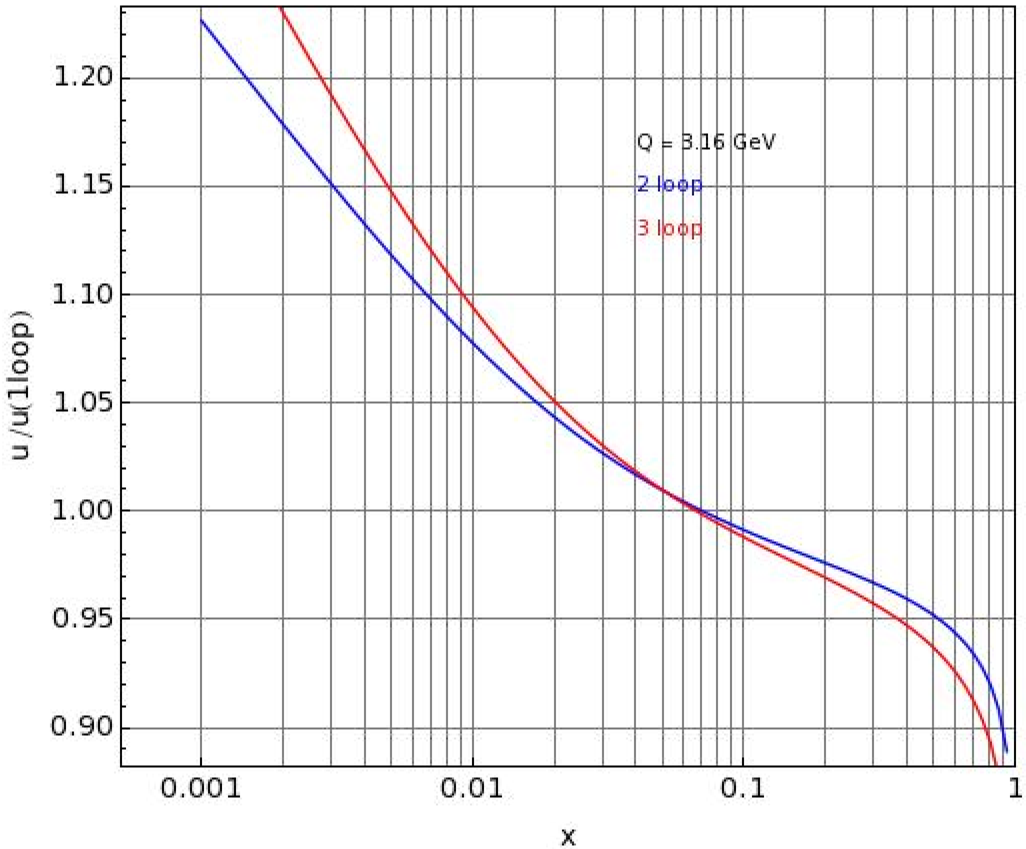
  PDFs decrease at large x

  PDFs increase at small x

  as we resolve the gluon radiation and quark pair production.
- The momentum sum rule and the flavor sum rules hold for all Q.
- These graphs show the DGLAP evolution for LO, NLO, NNLO Global Analysis.

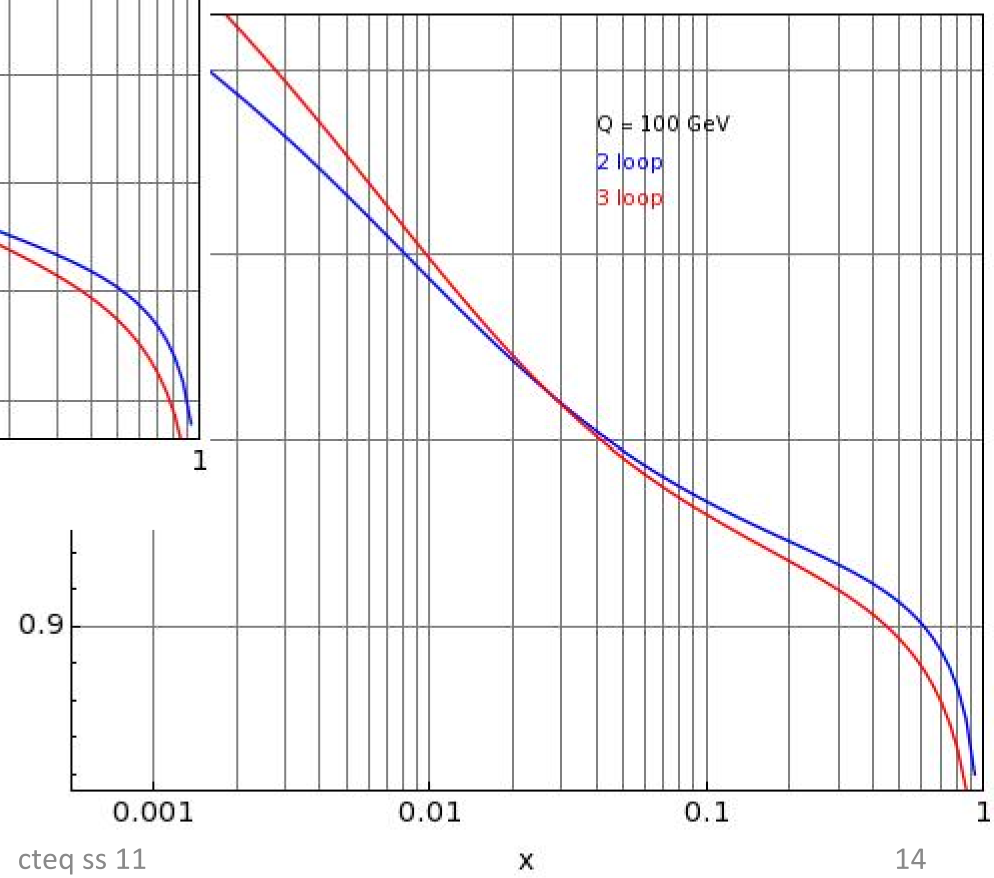
How large are the NLO and NNLO corrections?

Examples from HOPPET



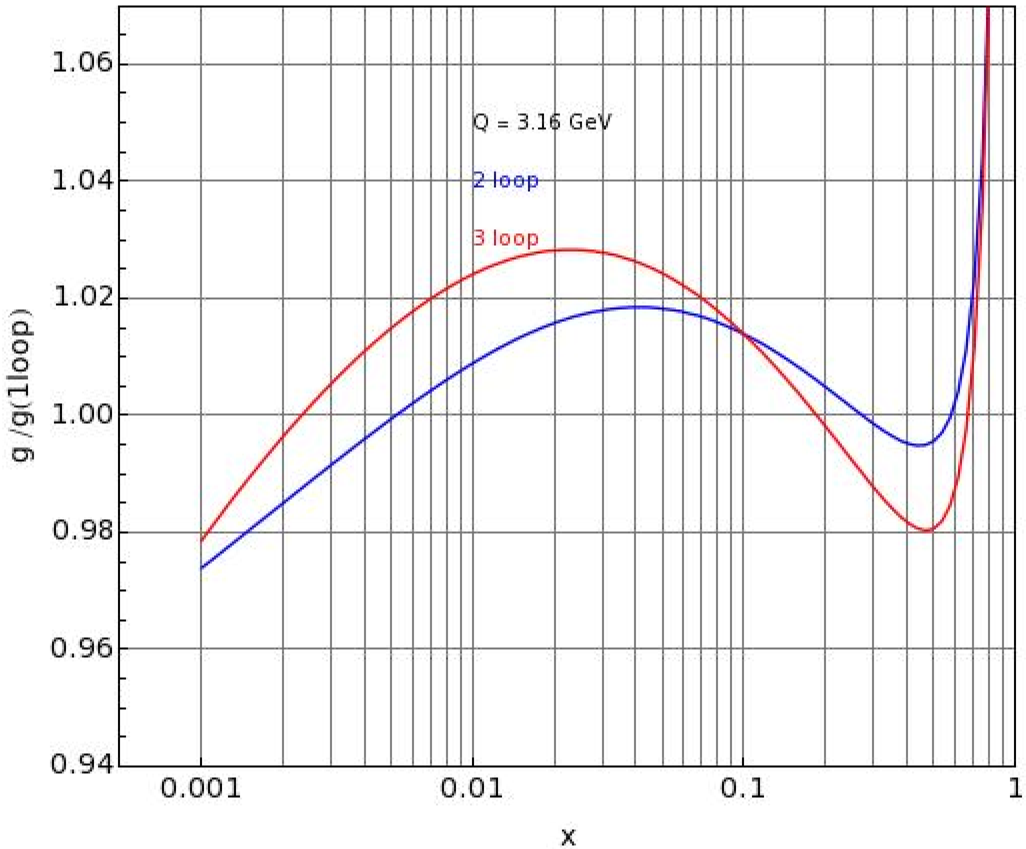
U-quark PDF at Q = 3.16 GeV;
blue ratio   u(2-loops)/u(1-loop)
red ratio   u(3-loops)/u(1-loop)

U-quark PDF at Q = 100.0 GeV;
blue ratio   u(2-loops)/u(1-loop)
red ratio   u(3-loops)/u(1-loop)



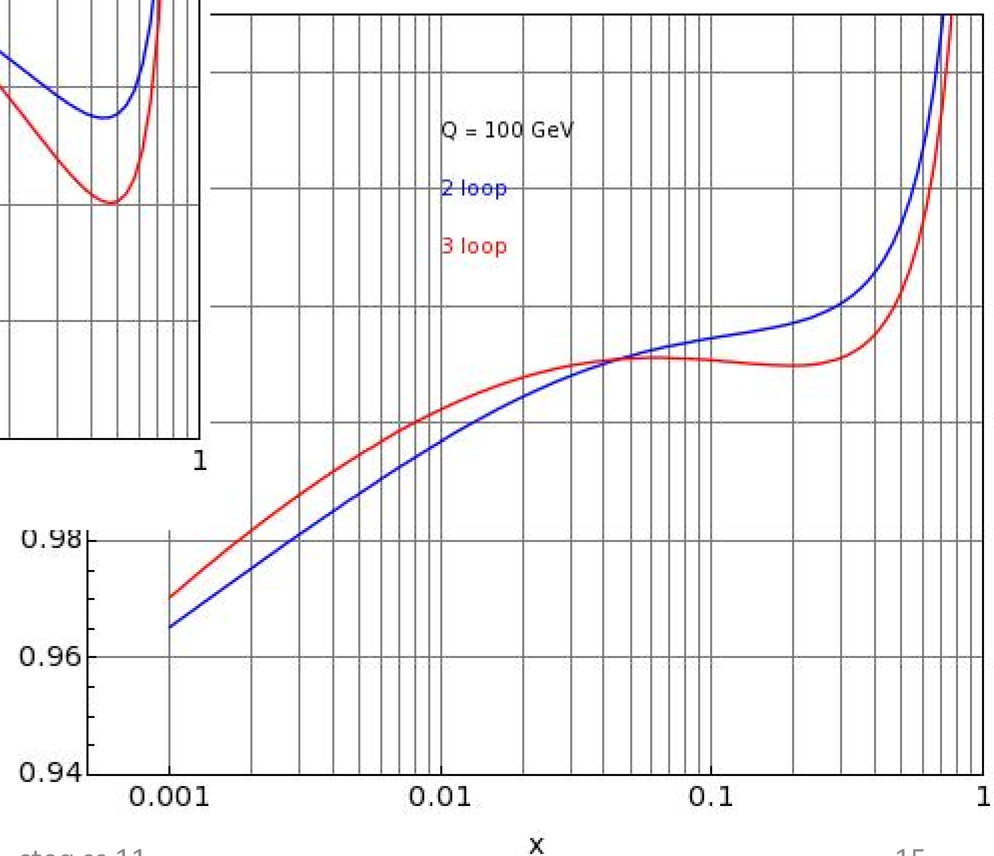
How large are the NLO and NNLO corrections?

Examples from HOPPET



Gluon PDF at Q = 3.16 GeV;
blue ratio g(2-loops)/g(1-loop)
red ratio g(3-loops)/g(1-loop)

Gluon PDF at Q = 100.0 GeV;
blue ratio g(2-loops)/g(1-loop)
red ratio g(3-loops)/g(1-loop)

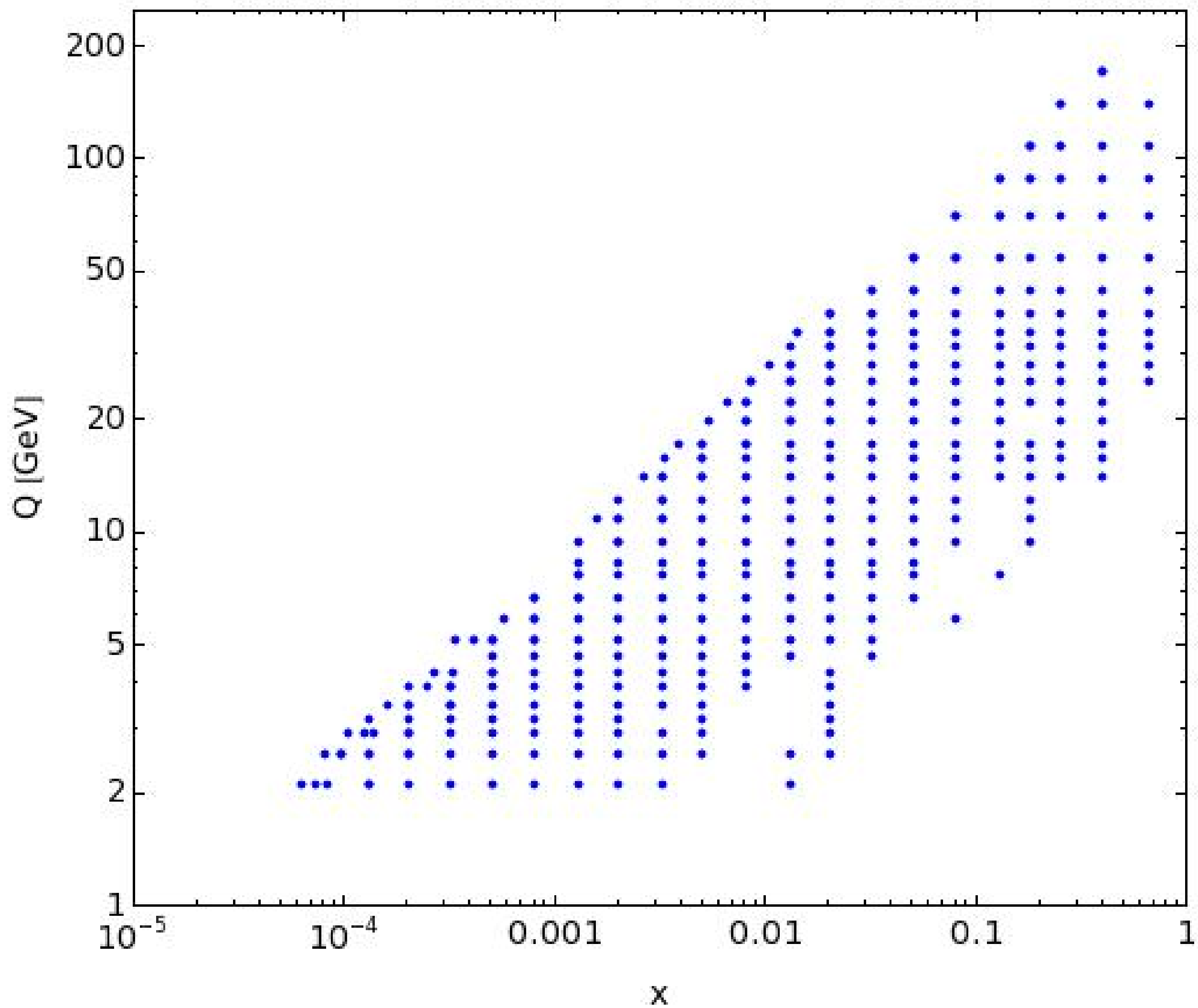


C. Some Results from the CT10 Global Analysis

code#	***EXPT***	***Chi2/N***	***EXPT***
159	Hera-I	1.18	Combined HERA1 NC+CC DIS (2009)
101	BcdF2pCor	1.12	BCDMS F2 proton (CERN-EP 89-06)
102	BcdF2dCor	1.05	BCDMS F2 deuteron (CERN-EP 89-170)
103	Nmcf2pCor	1.7	NMC F2 (Nucl Phys B483, 3, (1997)
104	NmcRatCor	1.01	NMC F2d/F2p (Nucl Phys B483, 3, (1997)
108	cdhswf2	0.73	P Berge et al Z Phys C49 187 (1991)
109	cdhswf3	0.69	P Berge et al Z Phys C49 187 (1991)
110	ccfrf2.mi	1.04	CCFR F2 (PMI): Phys.Rev.Lett.86:2742-2745 (2001) Yang
111	ccfrf3.md	0.37	CCFR xF3: Phys. Rev. Lett. 79: 1213 (1997) Shaevitz&Seli
201	e605	0.78	E605 dimuon yield PRD, s*dsig/drtaudy (nbarnGeV**2/r
203	e866f	0.45	E866 final: hep-ex/0103030 -> pd / 2pp
225	cdfLasy	0.79	W production: decay lepton asymmetry CDF Run-1
505	cdf1jtCorB	1.64	Run 1b 1800 GEV central jet xsecs to be used with the C
515	d0jetR1B	0.74	D0 inclusive jet xsecs (nb/GeV); Run IB; PRL86,1707(200
140	HN+67F2c	1.28	H1 96/97 data on F2c - e+p; hep-ex/0108039 Ref: Phys.
143	HN+90X0c	1.55	H1 99/00 rsigmac for c-cbar, e+p; hep-ex/0507081,0411
145	HN+90X0b	0.78	H1 99/00 NC rsigmab for b-bbar, e+p; hep-ex/0507081,0
156	ZN+67F2c	0.9	ZEUS 96/97 data on F2c - e+p; hep-ex/9908012
157	ZN+80F2c	0.77	ZEUS 98/00 F2c from e+ p ; hep-ex/0308068
124	NuTvNuChXN	0.89	NuTev Neutrino Dimuon Reduced xSec--corrected for NL
125	NuTvNbChXN	0.83	NuTev Neutrino Dimuon Reduced xSec--corrected for NL
126	CcfrNuChXN	1.25	Ccfr Neutrino Dimuon Reduced xSec--corrected for NLO
127	CcfrNbChXN	0.77	Ccfr Neutrino Dimuon Reduced xSec--corrected for NLO
504	cdf2jtCor2	1.56	(run II: cor.err; ptmin & ptmax)
514	d02jtCor2	1.14	(run II: cor.err; ptmin & ptmax)
204	e866ppxf	1.24	DY pp: Q^3 dSig/dQ dxf
260	ZyD02a	0.57	Z rapidity dist. (D0 TeV II-a)
261	ZyTeV2	1.74	Z rapidity dist. (CDF TeV II)
227	cdfLasy2	1.45	W production: decay lepton asymmetry CDF Run-2

HERA Combined Data
positron – proton Neutral Current DIS

- ❑ Positron-proton Deep Inelastic Scattering
- ❑ Q and x are the kinematic variables for Deep-Inelastic Scattering.
- ❑ The HERA combined data set



HERA Combined Data

H1 and ZEUS Collaborations, JHEP 01 (2010) 109
"Combined measurement and QCD analysis of the inclusive $e^{\pm}p$ scattering cross sections at HERA."

REDUCED CROSS SECTIONS

$$\sigma^{\pm}_{r,NC} = \frac{d^2\sigma^{e^{\pm}p}_{NC}}{dx\, dQ^2} \cdot \frac{Q^4 x}{2\pi\alpha^2 Y_+}$$

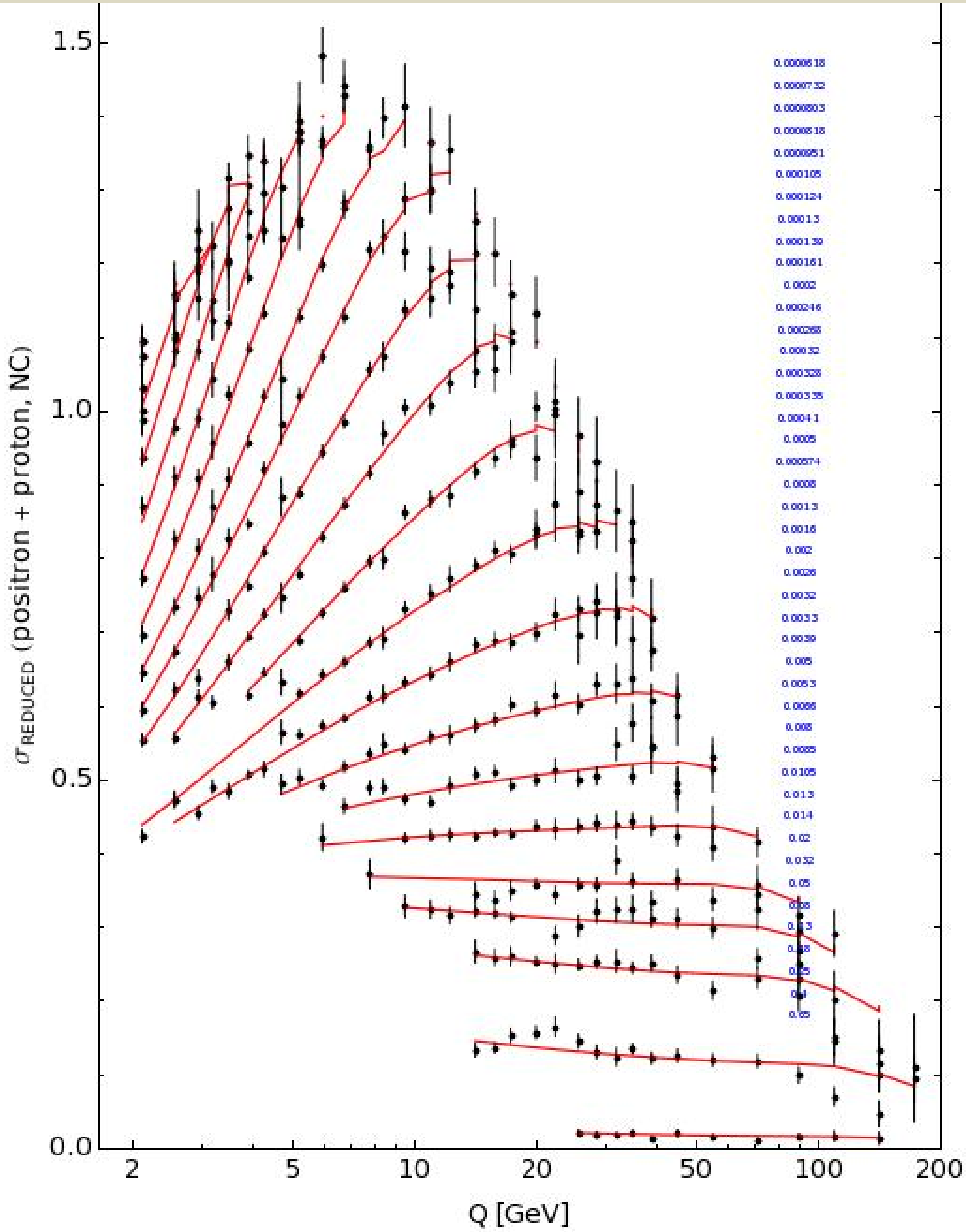
$$= F_2 \mp \frac{Y_-}{Y_+} xF_3 - \frac{y^2}{Y_+} F_L$$

$$\sigma^{\pm}_{r,CC} = \frac{2\pi x}{G_F^2} \left[\frac{M_W^2 + Q^2}{M_W^2}\right]^2 \frac{d^2\sigma^{e^{\pm}p}_{CC}}{dx\, dQ^2}$$

$$Y_{\pm} = 1 \pm (1-y)^2$$

This graph shows the REDUCED CROSS SECTION as a function of momentum transfer Q, for individual values of x.

(Q and x are the kinematic variables for Deep-Inelastic Scattering.)



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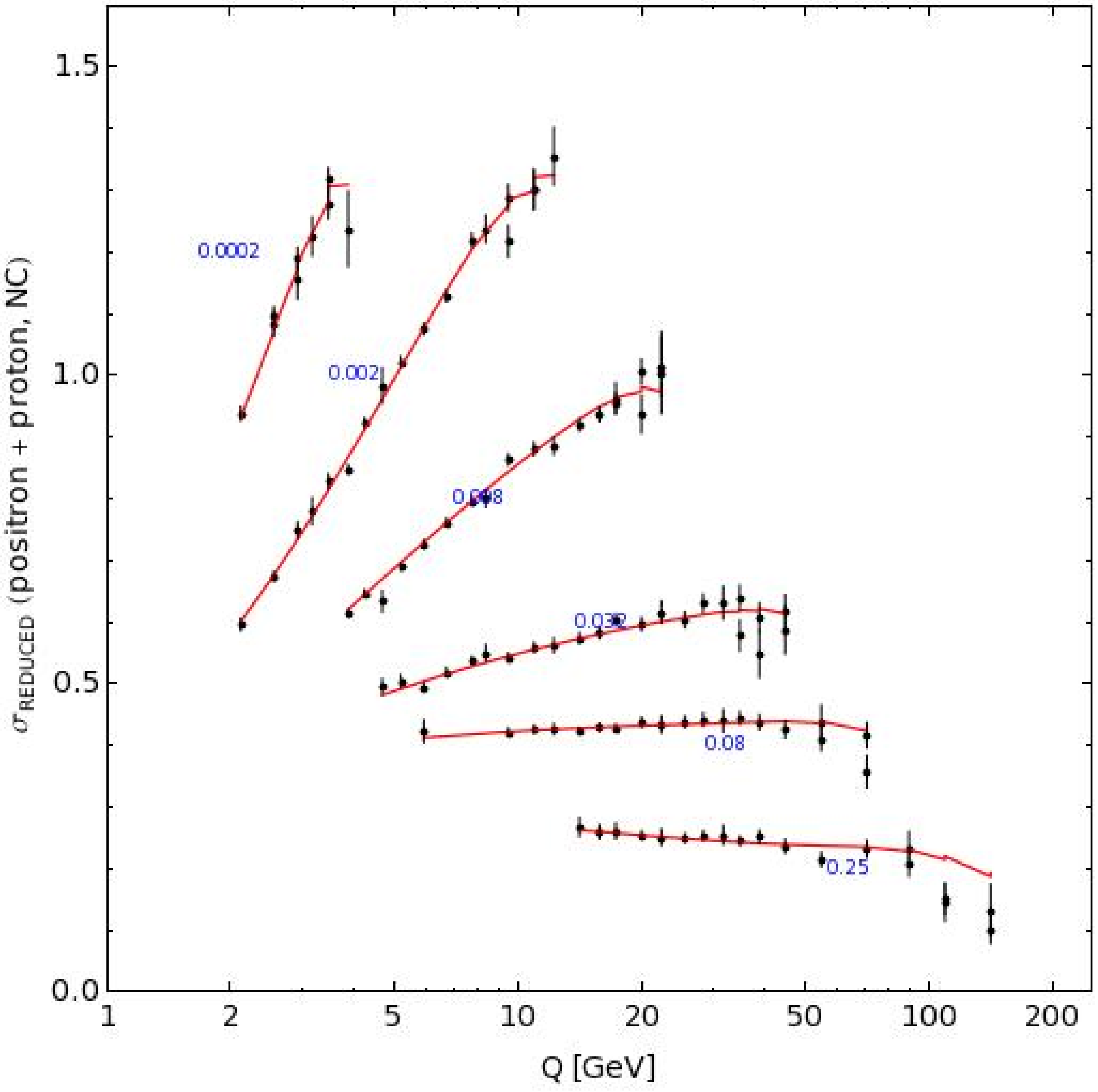


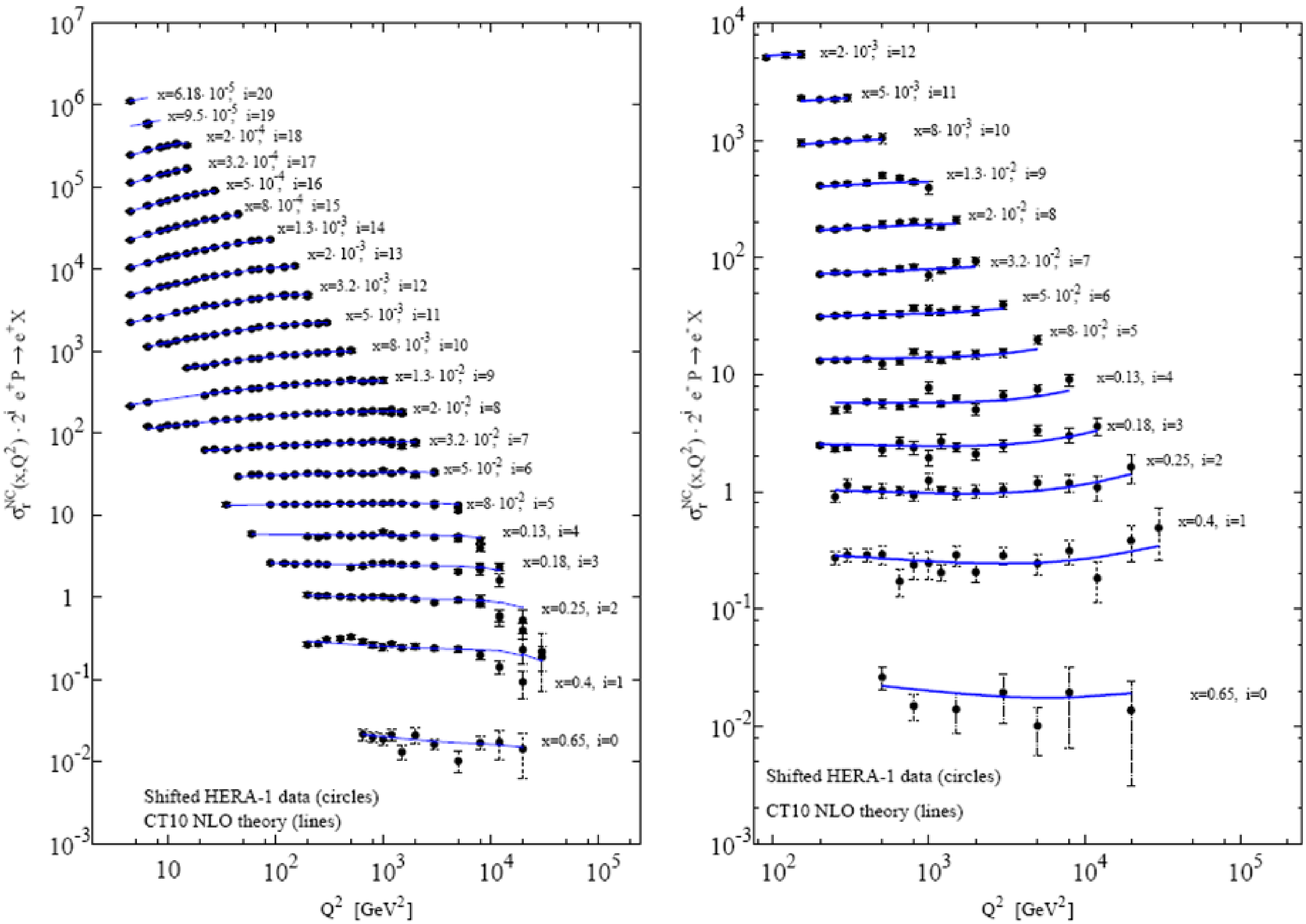
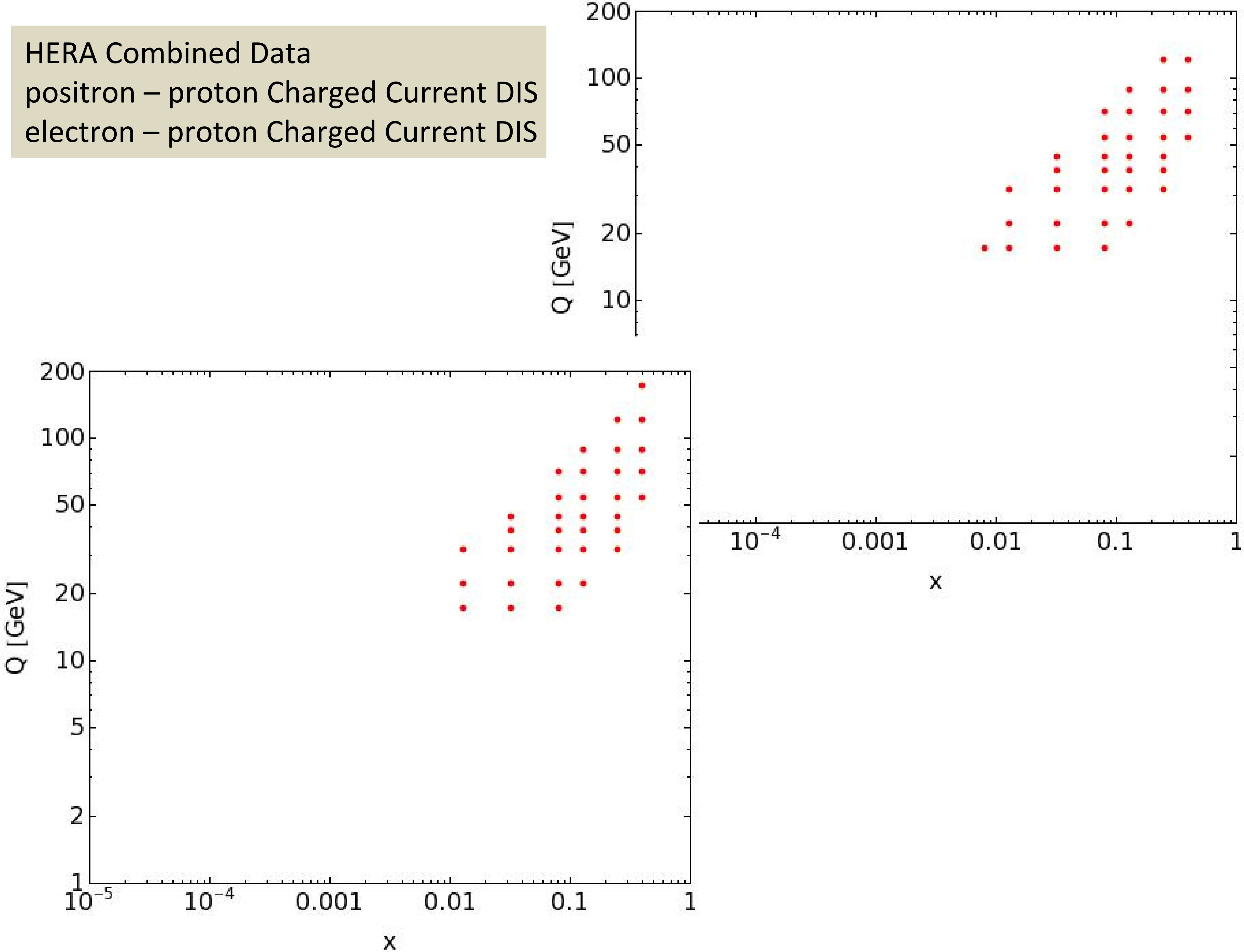
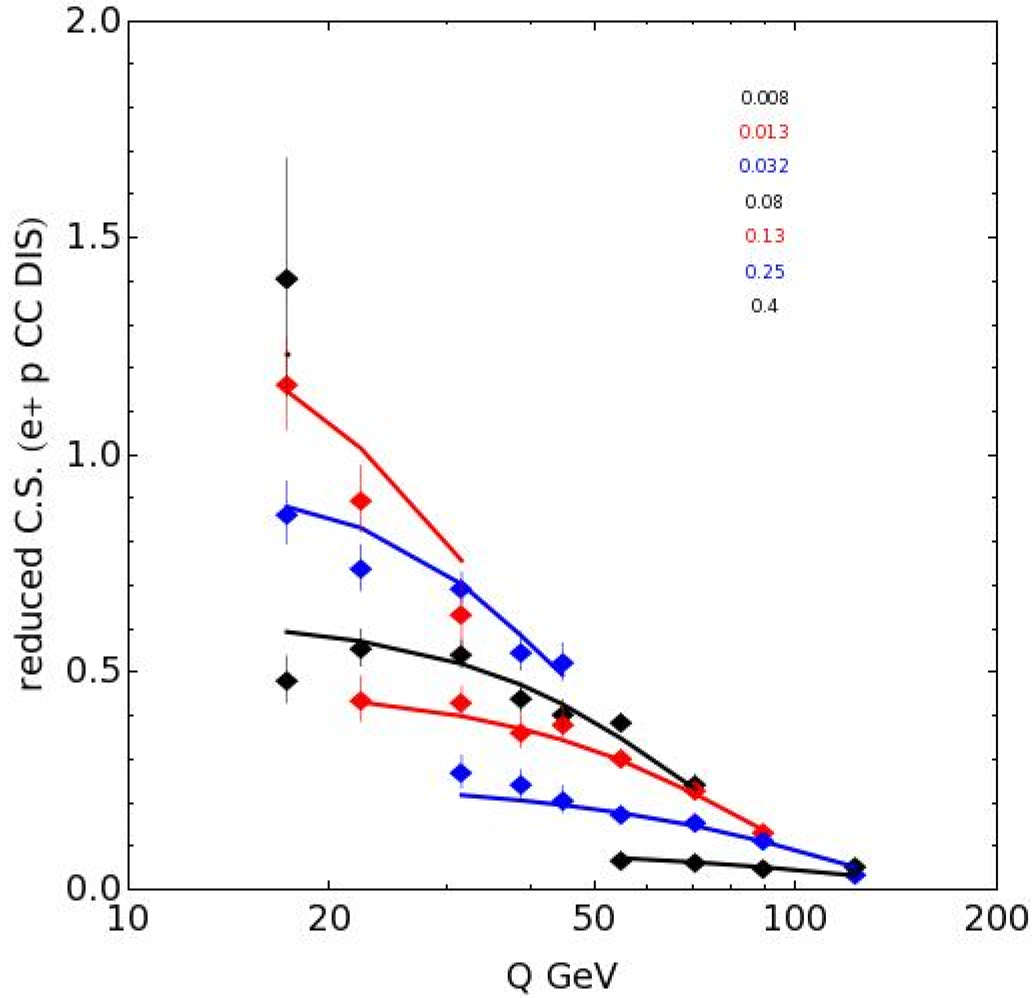


Figure 1: Comparison of CT10 NLO predictions for reduced cross sections in e^+p (left) and e^-p (right) neutral current DIS with the combined HERA-1 data [4], with correlated systematic shifts

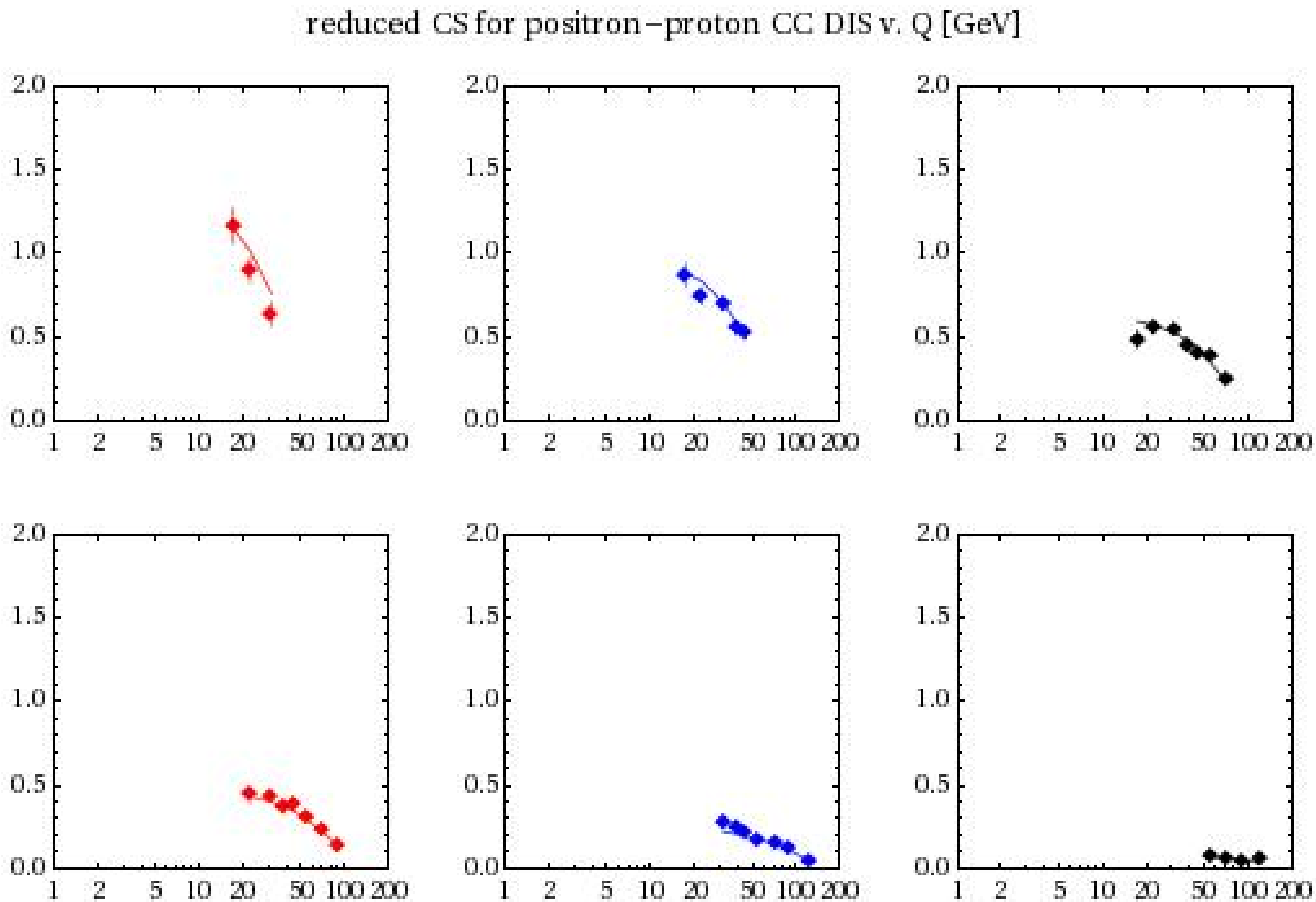
HERA Combined Data
positron – proton Charged Current DIS
electron – proton Charged Current DIS



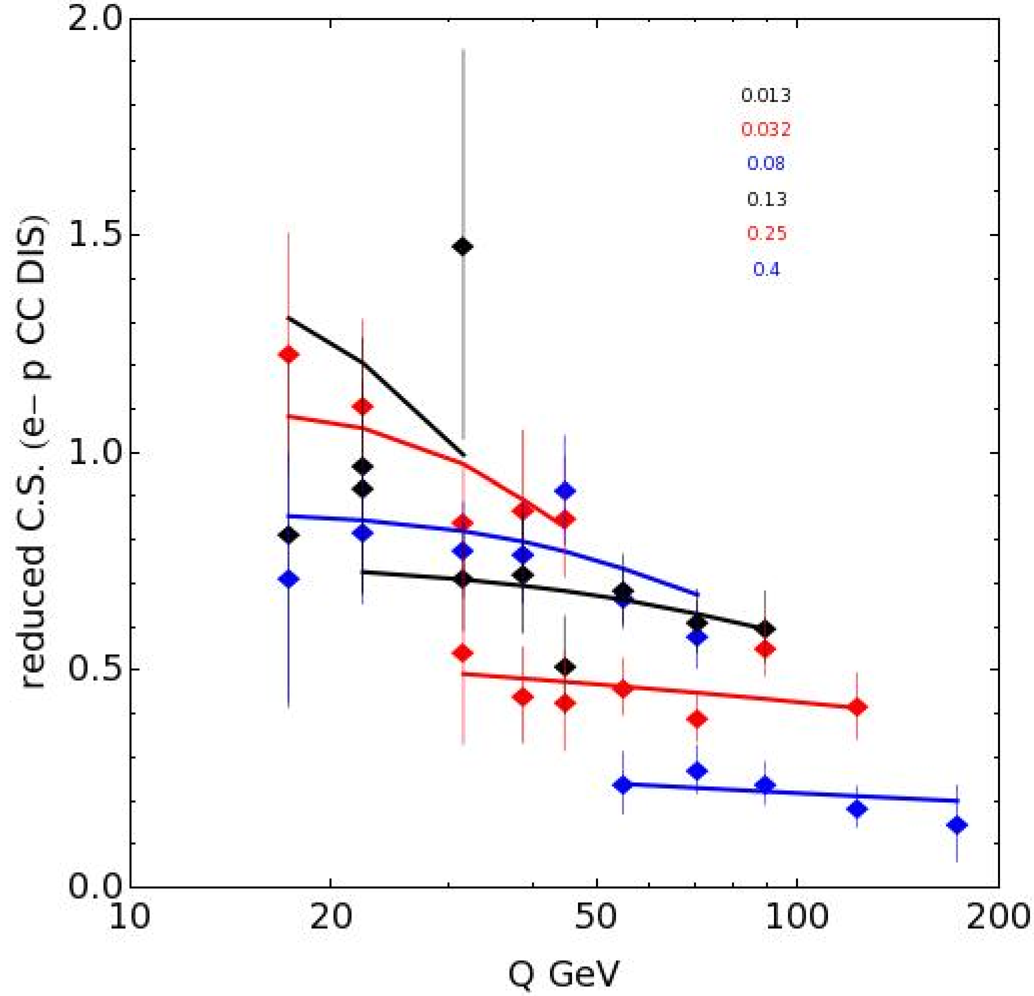
HERA Combined Data
positron – proton Charged Current DIS



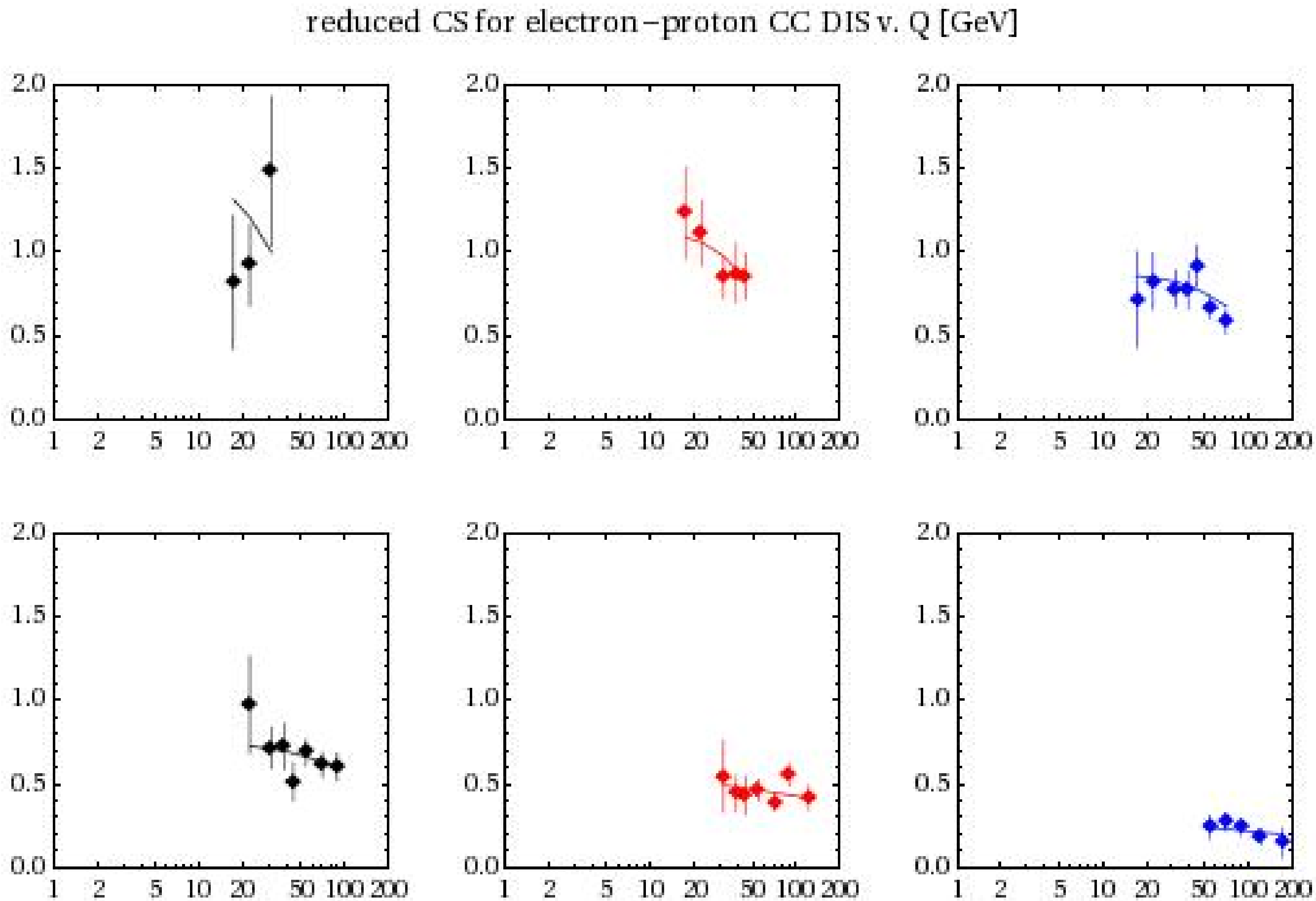
HERA Combined Data
positron – proton Charged Current DIS



HERA Combined Data
electron – proton Charged Current DIS

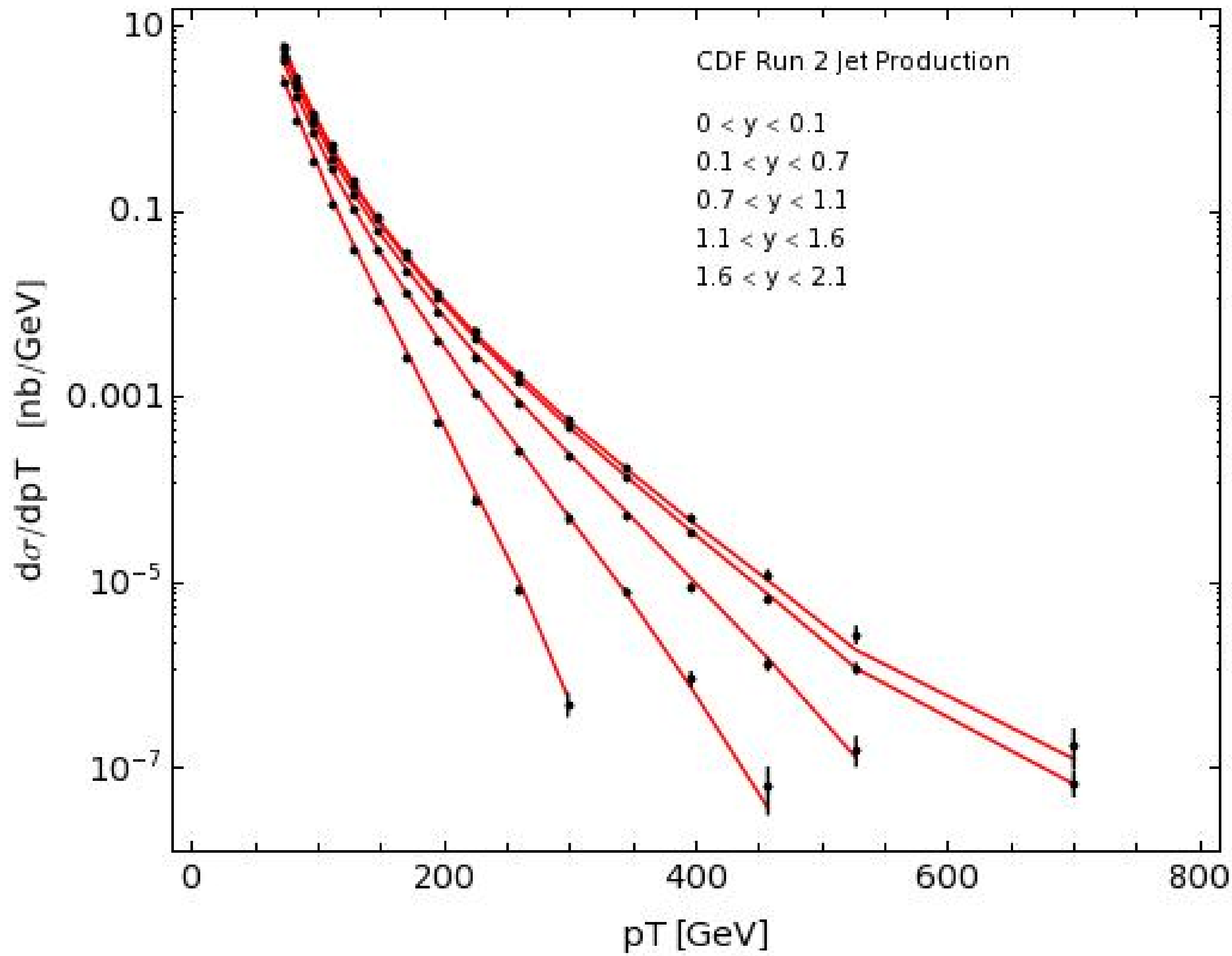


HERA Combined Data
electron – proton Charged Current DIS

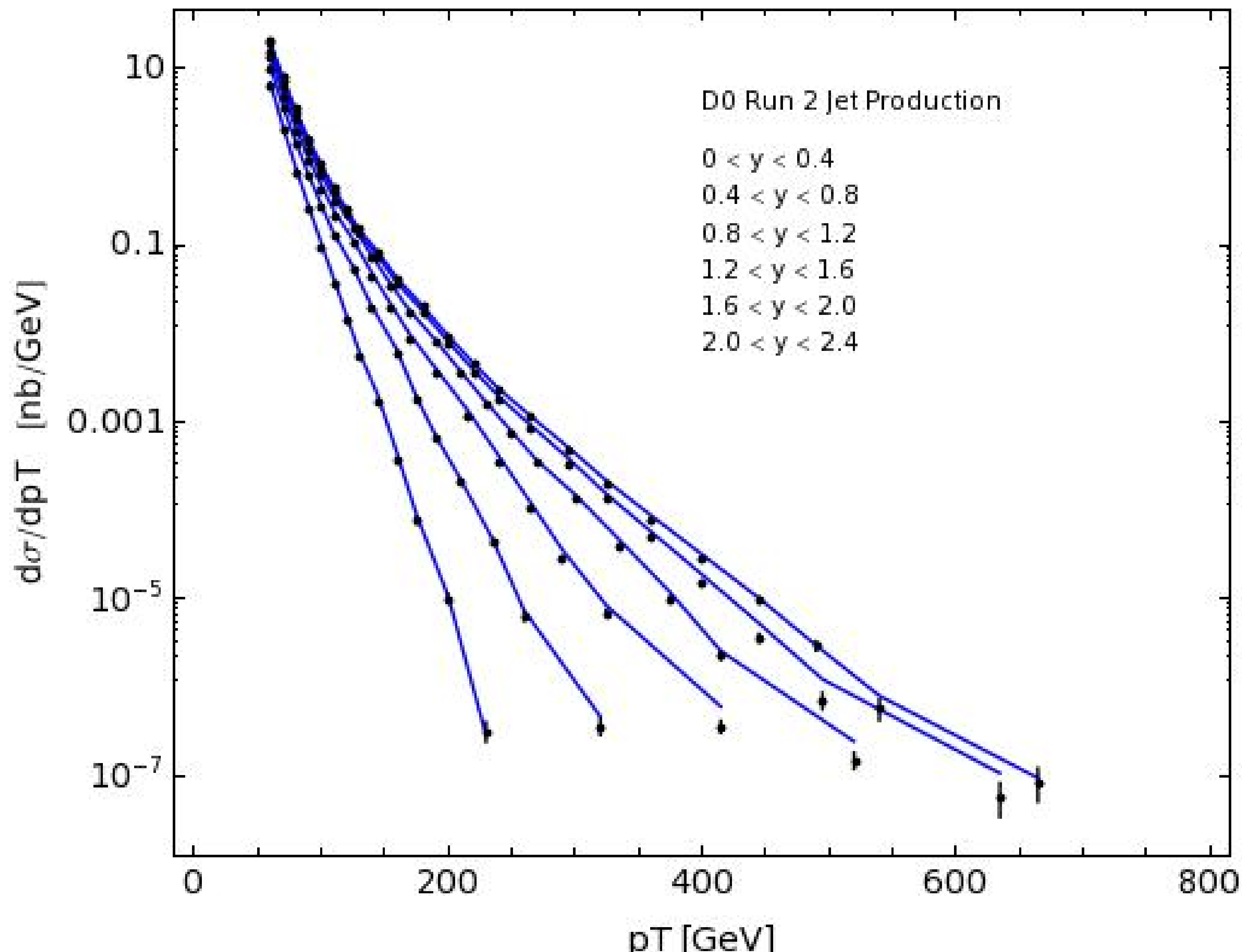


Inclusive Jet Production at the Tevatron

Inclusive Jet Production in Run 2 at the Tevatron Collider - CDF



The red curves are the theoretical calculations with CT10 PDFs.



The blue curves are the theoretical calculations with CT10 PDFs.