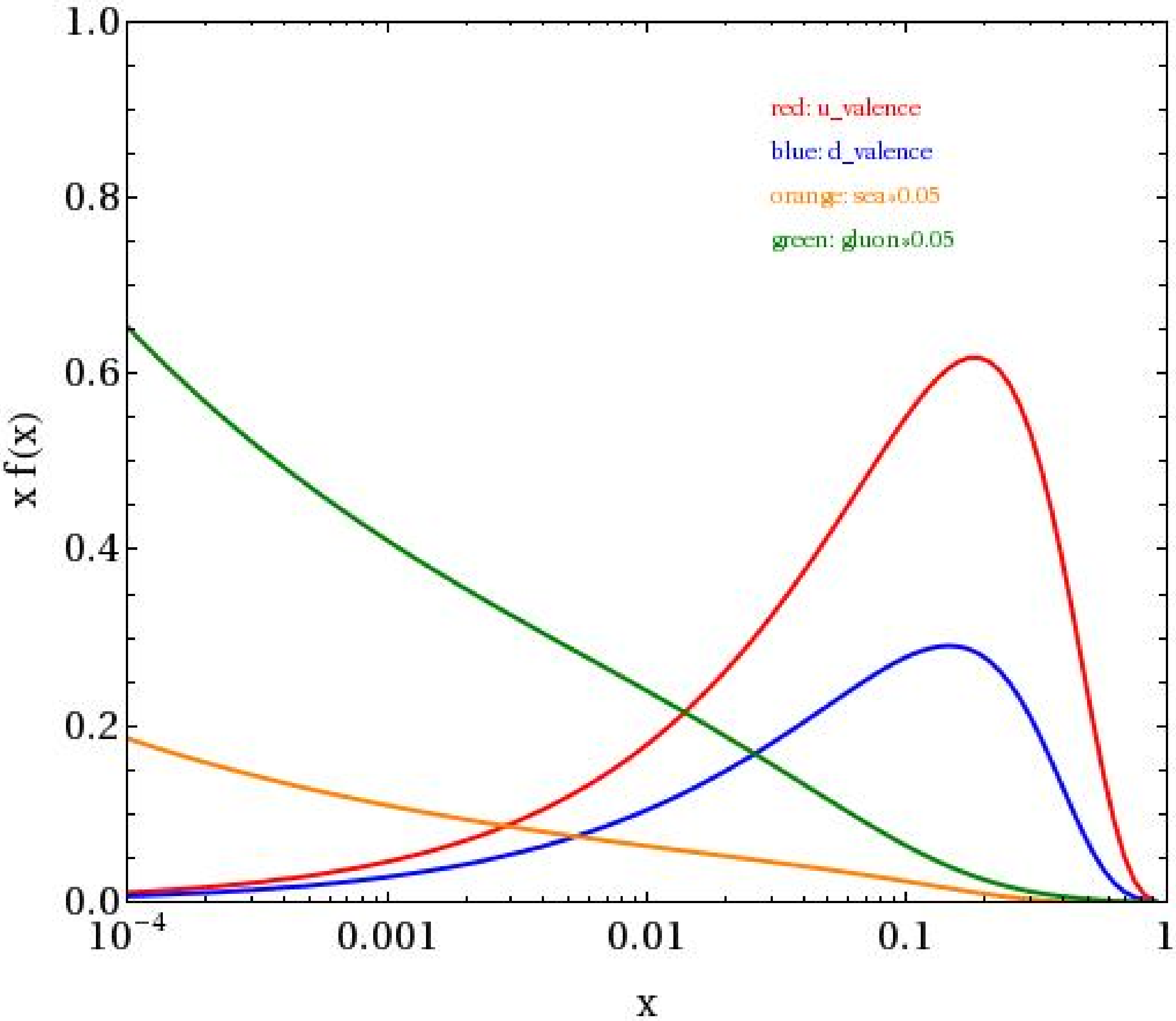
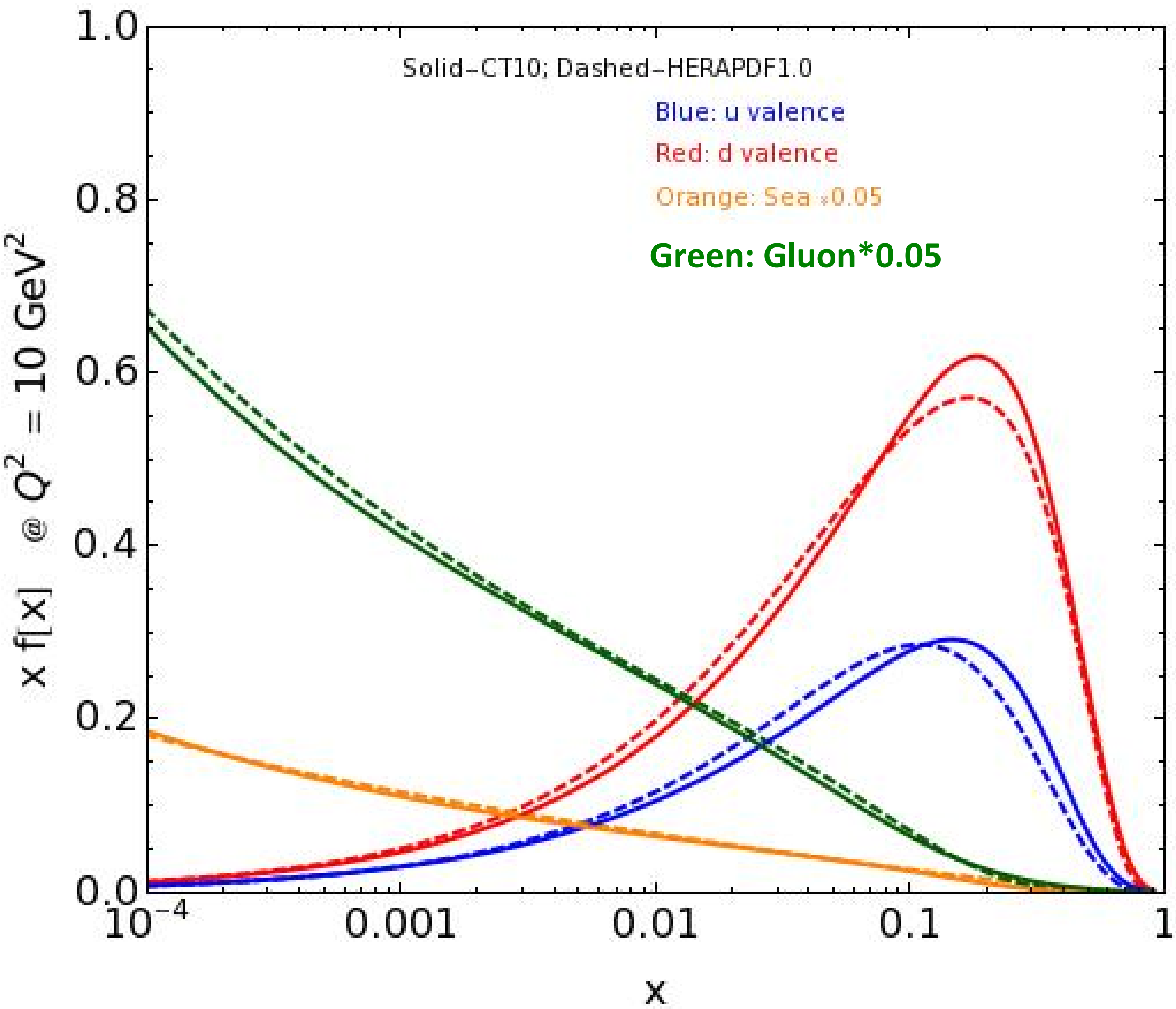
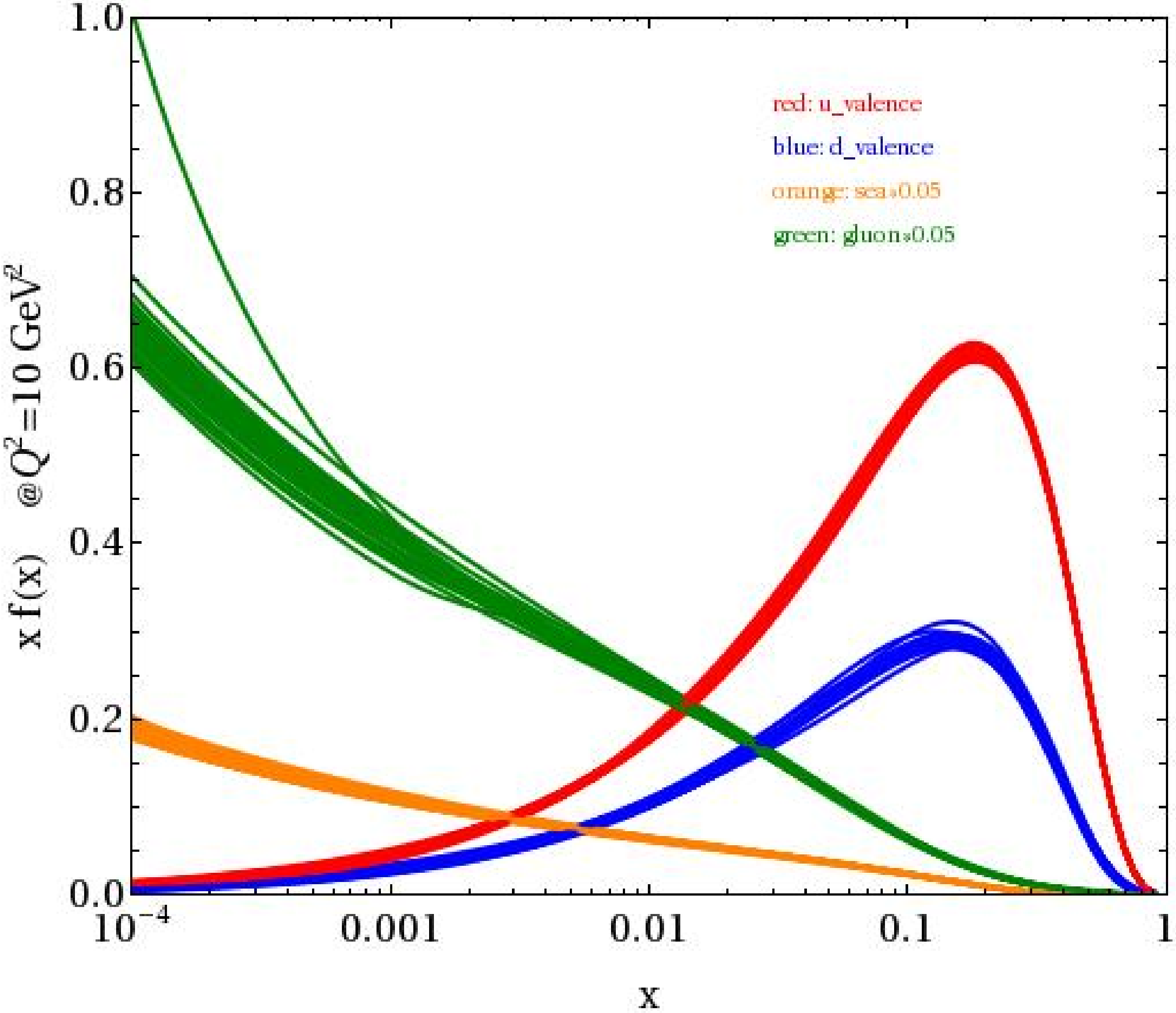
Prologue to Lecture 2



1.0
0.8
0.6
0.4
0.2
0.0
x f(x)
red: u_valence
blue: d_valence
orange: sea*0.05
green: gluon*0.05
10^{-4}
0.001
0.01
0.1
1
x



Solid–CT10; Dashed–HERAPDF1.0
Blue: u valence
Red: d valence
Orange: Sea *0.05
Green: Gluon*0.05
x f[x] @ Q^2 = 10 GeV^2
x
1.0
0.8
0.6
0.4
0.2
0.0
10^{-4}
0.001
0.01
0.1
1



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x

D. Uncertainties of Parton Distribution Functions

1. “Errors” and Uncertainties

2. Propagation of Experimental “Errors”

3. PDF Uncertainties

4. The role of α_S in Global Analysis

5. Implications for LHC Physics

Lecture 2: Errors and Uncertainties
in the Global Analysis of QCD

Mark Twain

… from "Chapters from My Autobiography", published in 1906 …

"Figures often beguile me, particularly when I have the arranging of them myself; in which case the remark attributed to Disraeli would often apply with justice and force: ***There are three kinds of lies: lies, damned lies, and statistics."***

“Errors” and Uncertainties

$$\sigma_{ep} = PDF \otimes C$$

Data ; We’re trying to determine this ; Calculation

How accurately can we determine the PDFs?

The accuracy is limited by …

- Experimental “errors” — statistical; systematic
- Theory “errors” — LO, NLO, NNLO; choice of momentum scale; value of α_s
- Parametrization errors

Parametrization

In the CTEQ Global Analysis, we parametrize the PDFs ***$f_i(x,Q^2)$*** at a low Q scale, Q_0 = 1.3 GeV. For example,

$$q_v(x,Q_0) = a_0 x^{a_1} (1-x)^{a_2} \exp\{\ a_3 x + a_4 x^2 + a_5 \sqrt{x}\ \}$$

(q = u or d ; $q_v = q - \bar{q}$)

{ The a_i's are different for different flavors}

Potentially, 6 x (2+2+1+1)=36 parameters. ***(CT10 uses 26 parameters.)***

Then ***$f_i(x,Q^2)$*** is DETERMINED for Q > Q_0 by the RG evolution equations.

Find the parameter values ***$\{a_{i0} \ldots a_{i5}\}$*** such that theory and data agree most closely.

Propagation of Experimental "Errors"

Consider the measurement of an observable.

In the simplest statistical analysis we have

$$D_i = T_i(a) + \varepsilon_i \qquad (i = 1 \ldots N_{dp})$$

and $\langle \varepsilon_i^2 \rangle = \sigma_i^2$.

Data point; Theory with unknown constants to be determined; experimental error.

The experimental collaboration would publish

$\{ D_i \, , \sigma_i \; ; \; i = 1 \ldots N_{dp} \}$

Define a "measure of agreement"

$$\chi^2(a) = \sum_{i=1}^{N_{dp}} \left(\frac{D_i - T_i(a)}{\sigma_i} \right)^2$$

Minimize $\chi^2 \Rightarrow$ central fit $\{a_0\}$

The variation of χ^2 around the minimum

$\Rightarrow$ the uncertainty

Propagation of Experimental "Errors"

In a small neighborhood of $\{a_0\}$

$$\chi^2(a) = \chi^2(a_0) + \sum_{k,\ell=1}^{D} \underbrace{\frac{1}{2}\frac{\partial^2 \chi^2}{\partial a_k \partial a_\ell}}_{\text{the Hessian matrix}} (a_k - a_{0k})(a_\ell - a_{0\ell})$$

Now, make a prediction based on the theory, for some other observable Q

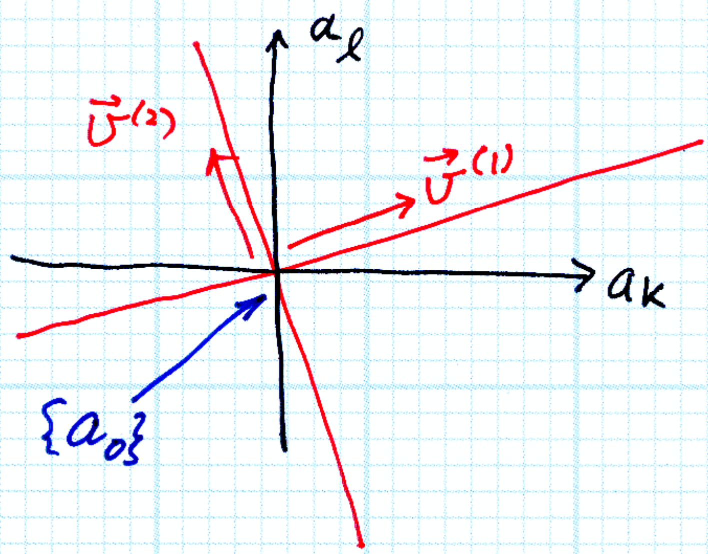
$$Q(a) = Q(a_0) + \sum_{k=1}^{D} \frac{\partial Q}{\partial a_k}(a_k - a_{k0}) + \ldots$$

$$Q_{\text{prediction}} = Q(a_0) \pm \delta Q$$

$$(\delta Q)^2 = \sum_{k,\ell} \underbrace{(a_k - a_{0k})(a_\ell - a_{0\ell})}_{\text{allowed variations of the parameters}} \frac{\partial Q}{\partial a_k}\frac{\partial Q}{\partial a_\ell}$$

Ludwig Otto Hesse

Parameter Variations in the Eigenvector Basis



$$\sum_{\ell=1}^{D} H_{k\ell}\, v_\ell^{(m)} = \lambda^{(m)} v_k^{(m)}$$

$$m = 1\ 2\ 3 \ldots D$$

The **distance** along the eigenvector $\vec{v}^{(k)}$ is allowed to vary by an amount of order $1/\sqrt{\lambda^{(k)}}$.

From the CT10 Global Analysis, we have $26 \times 2 = 52$ alternative sets of PDF's, and one central set.

Results of the Global Analysis

$\{ f_i^{(0)}(x, Q^2) \}$ ← central fit

$\{ f_i^{(K)}(x, Q^2) \}$ ← alternative fits, or PDF varieties $K = 1\ 2\ 3 \ldots 52$

$K = 1, 2$ Eigenvector 1 { + direction, − direction }

$K = 3, 4$ Eigenvector 2 { + direction, − direction }

⋮

$K = 2\ell-1, 2\ell$ Eigenvector ℓ { + direction, − direction }

⋮

Accessible at the CTEQ web site, or the Durham Parton Distribution Generator

Uncertainties

Ideally, $(\delta Q)^2 = \sum_{\ell=1}^{26} \left[Q(a_{2\ell-1}) - Q(a_0) \right]^2$

and $(\delta Q)^2 = \sum_{\ell=1}^{26} \left[Q(a_{2\ell}) - Q(a_0) \right]^2$

and $(\delta Q)^2 = \sum_{\ell=1}^{26} \left[\frac{Q(a_{2\ell}) - Q(a_{2\ell-1})}{2} \right]^2$

This is the "Master Formula" for symmetric errors.

But the behavior in the neighborhood of the minimum is not quadratic. So, instead,

$$(\delta^+ Q)^2 = \sum_{\ell=1}^{26} \left[\max\left(Q_\ell^{(+)} - Q_0 \,,\, Q_\ell^{(-)} - Q_0 \,,\, 0 \right) \right]^2$$

$$(\delta^- Q)^2 = \sum_{\ell=1}^{26} \left[\max\left(Q_0 - Q_\ell^{(+)} ,\, Q_0 - Q_\ell^{(-)} ,\, 0 \right) \right]^2$$

// Q_0 = best fit ; $Q_\ell^{(+)} = Q[a_{2\ell-1}]$, $Q_\ell^{(-)} = Q[a_{2\ell}]$ //

asymmetric errors

Systematic Errors

$$D_i = T_i(a) + \sum_{\nu=1}^{N_{sy}} \sigma_{\nu i}\, r_\nu + \sigma_{0i}$$

($\sigma_{\nu i}$): correlated errors; published by the expt.

(σ_{0i}): uncorrelated error

$$\chi^2(a;\{r_\nu\}) = \sum_{i=1}^{N} \left\{ \frac{D_i - T_i(a) - \sum_{\nu=1}^{N_{sy}} \sigma_{\nu i}\, r_\nu}{\sigma_{0i}} \right\}^2 + \sum_{\nu=1}^{N_{sy}} r_\nu^2$$

Minimize w.r.t. both $\{a_k\}$ and $\{r_\nu\}$

We are fitting $T_i(a)$ to "optimally shifted data".

Democracy among Experiments

$$\chi^2_{global}(a) = \sum_{\text{experiments}} \left\{ \chi^2_{\text{expt.}}(a) + \mathbf{P}_{\text{expt.}} \right\}$$

... from the previous page, including the correlated shifts $\{\mathbf{r}_\nu\}_{\mathbf{expt}}$.

$\mathbf{P}$ is a penalty that prevents ***its*** experiment from deviating too much (90% C.L.) from the theory.

- Minimize χ^2.
- Diagonalize the Hessian Matrix.
- Final Result: $\boldsymbol{f_i^{(0)}(x,Q^2)}$ and 52 variations $\boldsymbol{\{f_i^{(k)}(x,Q^2)\}}$

...the ***eigenvector-basis PDF variations.***

Experimental Data Used in the CT10 Analysis

Democracy among Experiments

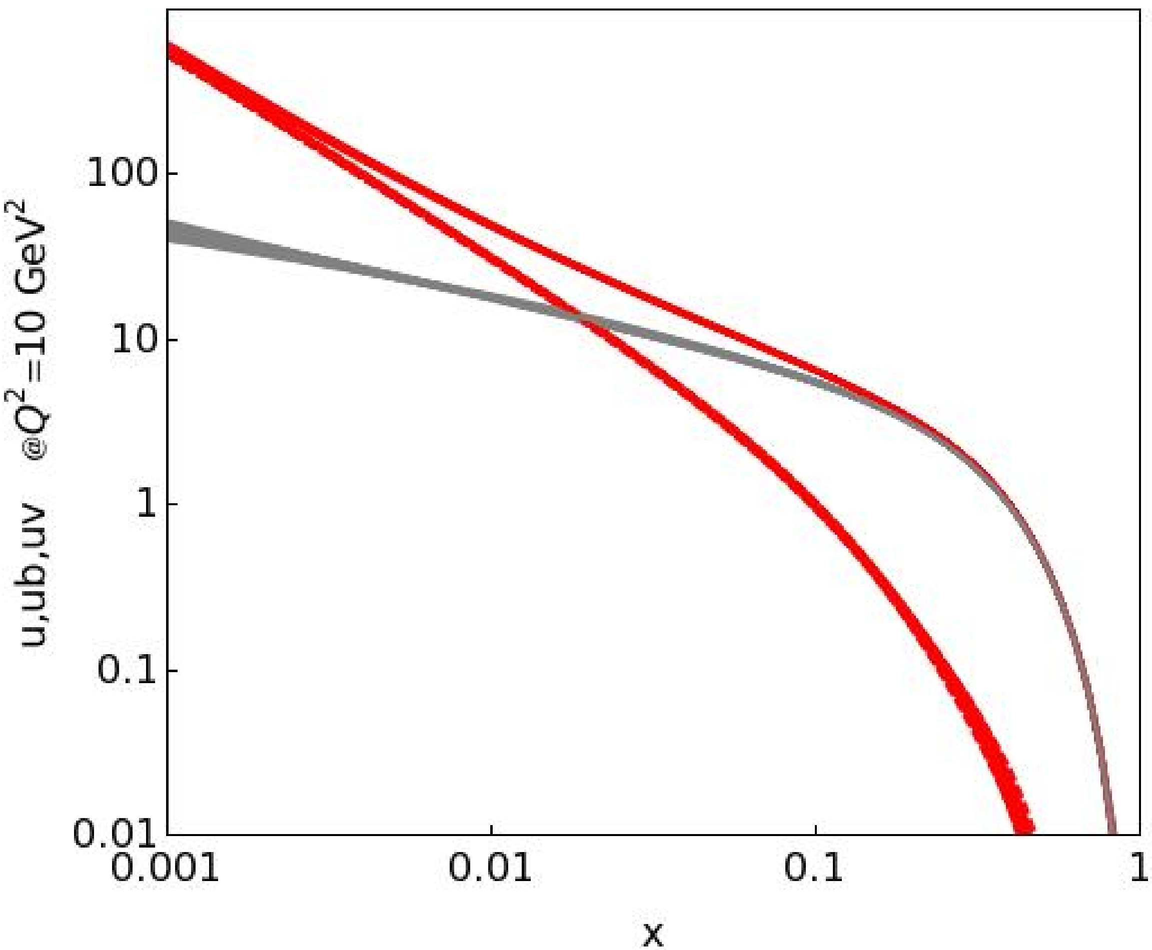
code#	*EXPT*	*Chi2/N*	*EXPT*
159	Hera-I	1.18	Combined HERA1 NC+CC DIS (2009)
101	BcdF2pCor	1.12	BCDMS F2 proton (CERN-EP 89-06)
102	BcdF2dCor	1.05	BCDMS F2 deuteron (CERN-EP 89-170)
103	Nmcf2pCor	1.7	NMC F2 (Nucl Phys B483, 3, (1997)
104	NmcRatCor	1.01	NMC F2d/F2p (Nucl Phys B483, 3, (1997)
108	cdhswf2	0.73	P Berge et al Z Phys C49 187 (1991)
109	cdhswf3	0.69	P Berge et al Z Phys C49 187 (1991)
110	ccfrf2.mi	1.04	CCFR F2 (PMI): Phys.Rev.Lett.86:2742-2745 (2001) Yang&
111	ccfrf3.md	0.37	CCFR xF3: Phys. Rev. Lett. 79: 1213 (1997) Shaevitz&Seli
201	e605	0.78	E605 dimuon yield PRD, s*dsig/drtaudy (nbarnGeV**2/r
203	e866f	0.45	E866 final: hep-ex/0103030 -> pd / 2pp
225	cdfLasy	0.79	W production: decay lepton asymmetry CDF Run-1
505	cdf1jtCorB	1.64	Run 1b 1800 GEV central jet xsecs to be used with the Cl
515	d0jetR1B	0.74	D0 inclusive jet xsecs (nb/GeV); Run IB; PRL86,1707(200
140	HN+67F2c	1.28	H1 96/97 data on F2c - e+p; hep-ex/0108039 Ref: Phys. I
143	HN+90X0c	1.55	H1 99/00 rsigmac for c-cbar, e+p; hep-ex/0507081,0411
145	HN+90X0b	0.78	H1 99/00 NC rsigmab for b-bbar, e+p; hep-ex/0507081,(
156	ZN+67F2c	0.9	ZEUS 96/97 data on F2c - e+p; hep-ex/9908012
157	ZN+80F2c	0.77	ZEUS 98/00 F2c from e+ p ; hep-ex/0308068
124	NuTvNuChXN	0.89	NuTev Neutrino Dimuon Reduced xSec--corrected for NL
125	NuTvNbChXN	0.83	NuTev Neutrino Dimuon Reduced xSec--corrected for NL
126	CcfrNuChXN	1.25	Ccfr Neutrino Dimuon Reduced xSec--corrected for NLO
127	CcfrNbChXN	0.77	Ccfr Neutrino Dimuon Reduced xSec--corrected for NLO
504	cdf2jtCor2	1.56	(run II: cor.err; ptmin & ptmax)
514	d02jtCor2	1.14	(run II: cor.err; ptmin & ptmax)
204	e866ppxf	1.24	DY pp: Q^3 dSig/dQ dxf
260	ZyD02a	0.57	Z rapidity dist. (D0 TeV II-a)
261	ZyTeV2	1.74	Z rapidity dist. (CDF TeV II)
227	cdfLasy2	1.45	W production: decay lepton asymmetry CDF Run-2

next ……

Next …

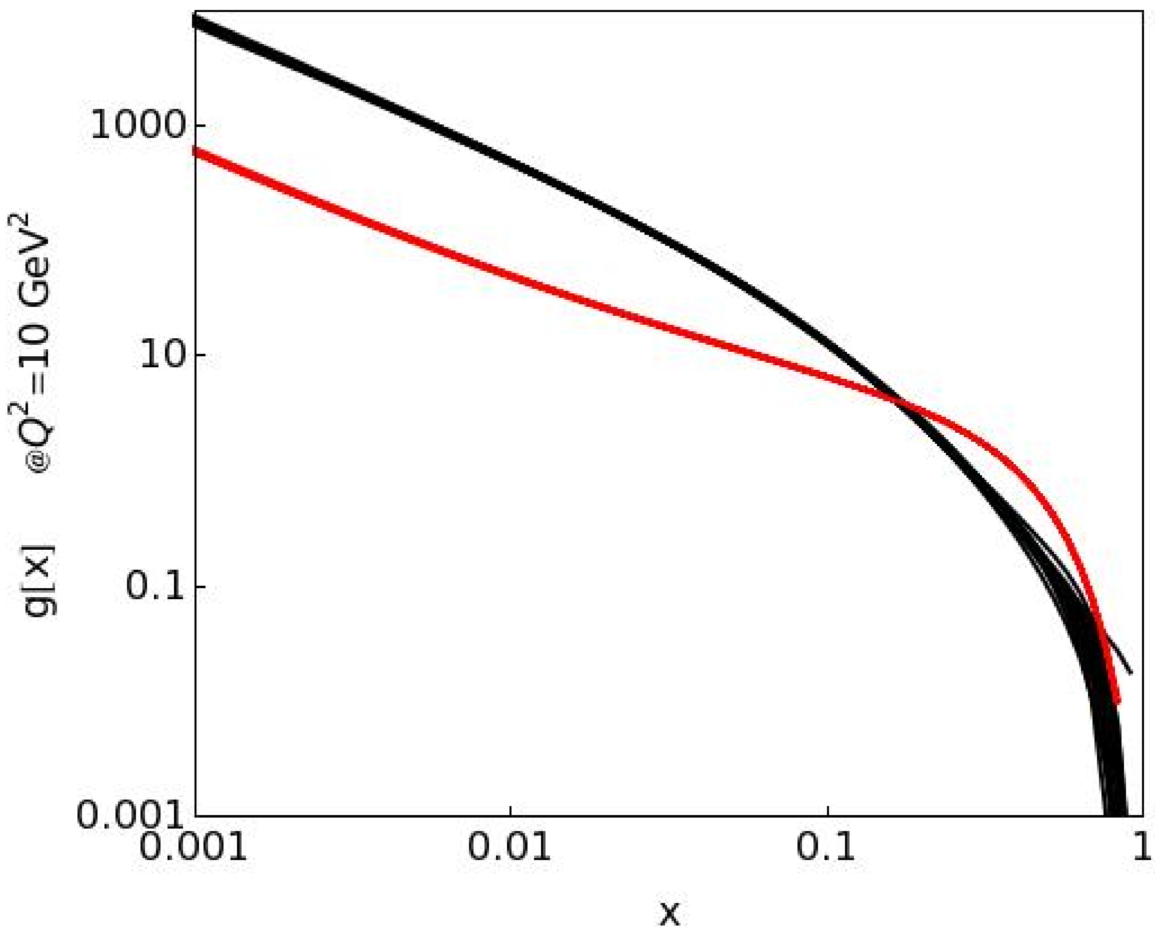
Uncertainties of Parton Distribution Functions

- PDFs u, $\bar{u}$ and $u_{valence}$ versus ***x*** at $\boldsymbol{Q^2}$ = 10 GeV^2



53 alternate sets of PDFs

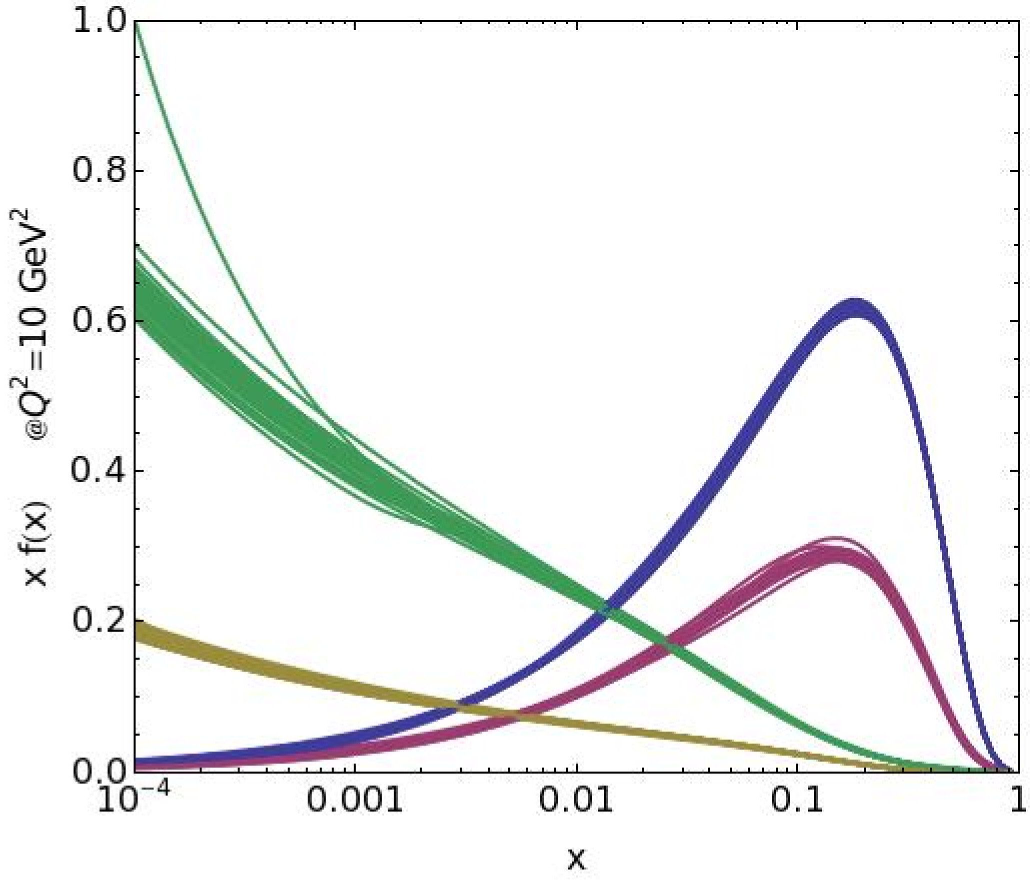
- PDFs u and g , versus ***x*** at $\boldsymbol{Q^2}$ = 10 GeV^2



53 alternate sets of PDFs

The u quark is well constrained by the HERA data;
The gluon is not so well known, so it has a wider band.

- PDFs $u_{valence}$, $d_{valence}$, .05*sea and .05*gluon versus ***x*** at $\boldsymbol{Q^2}$ = 10 GeV^2

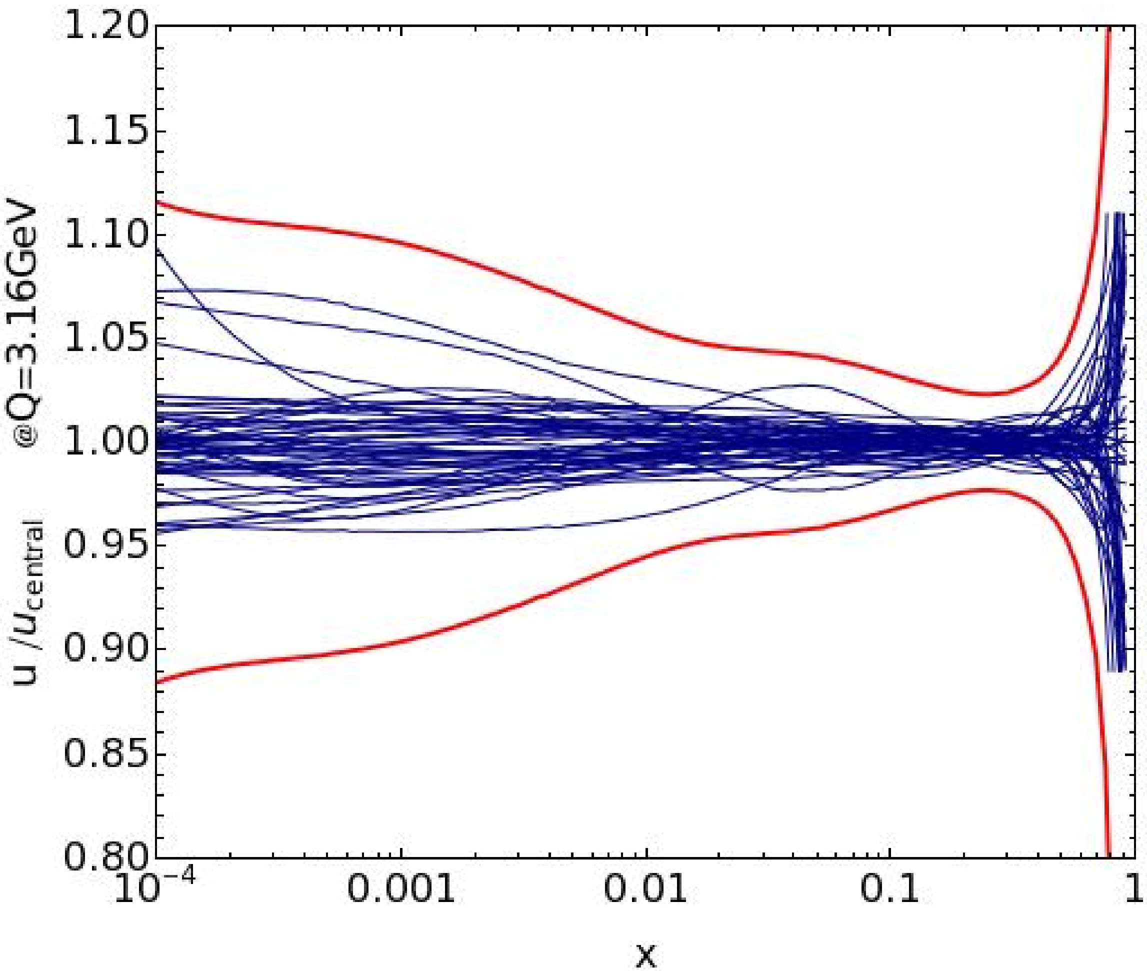


53 alternate sets of PDFs

The u quark is better constrained than the d quark.

PDF Bounds – u-quark, symmetric

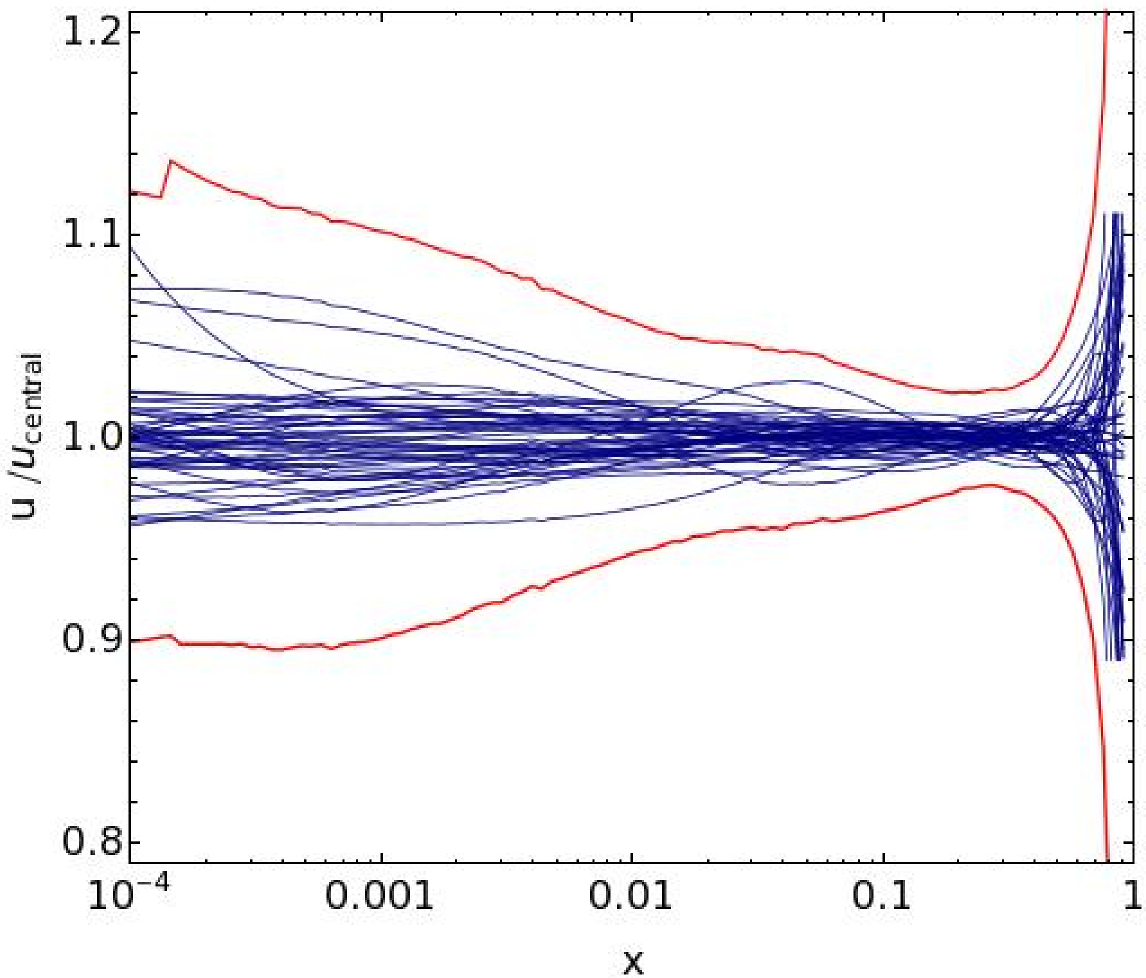
RATIO PLOT



Red curves: Upper and lower boundaries of the uncertainly band, according to the symmetric Master Formula

PDF Bounds – u-quark, asymmetric

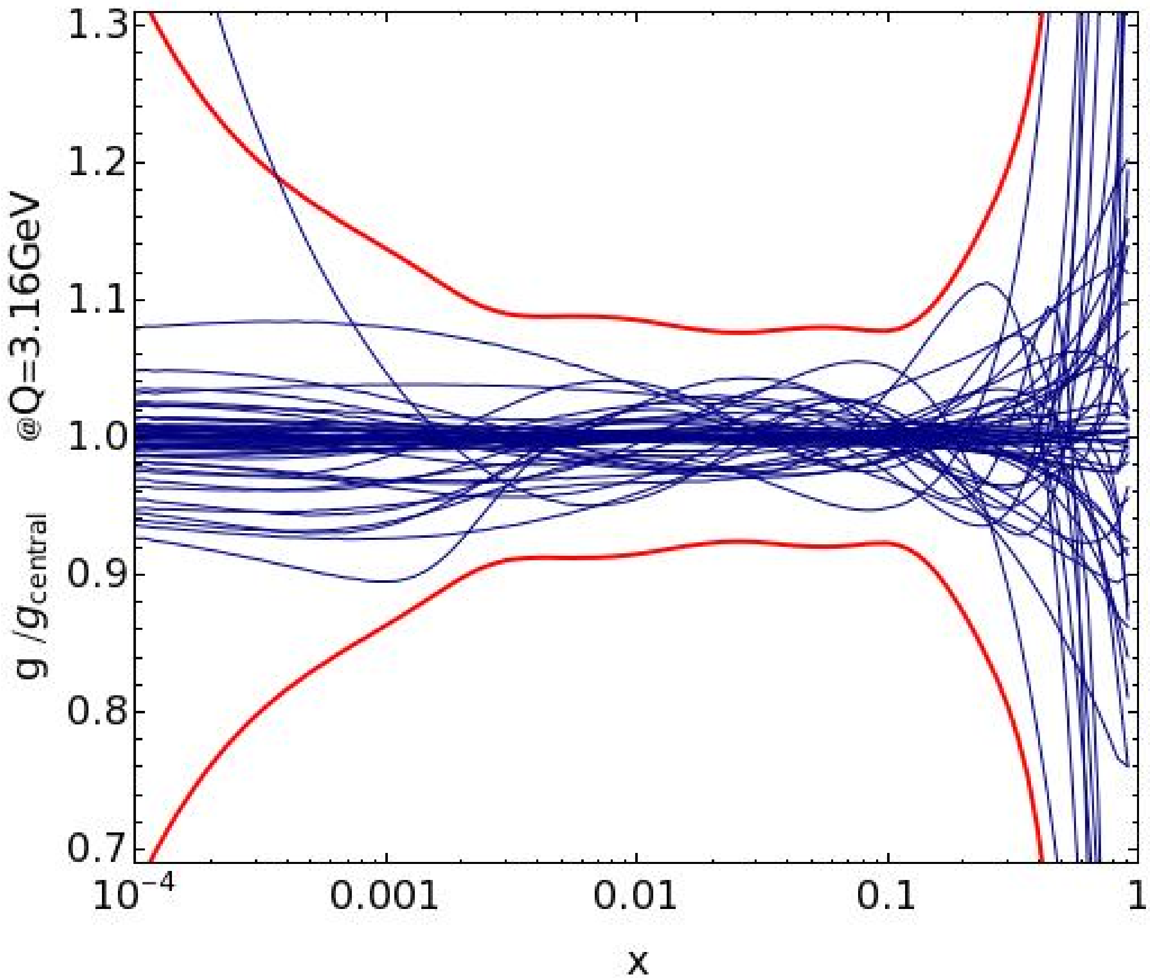
RATIO PLOT



Red curves: Upper and lower boundaries of the uncertainly band, according to the Asymmetric Version Master Formula

PDF Bounds – gluon, symmetric

RATIO PLOT



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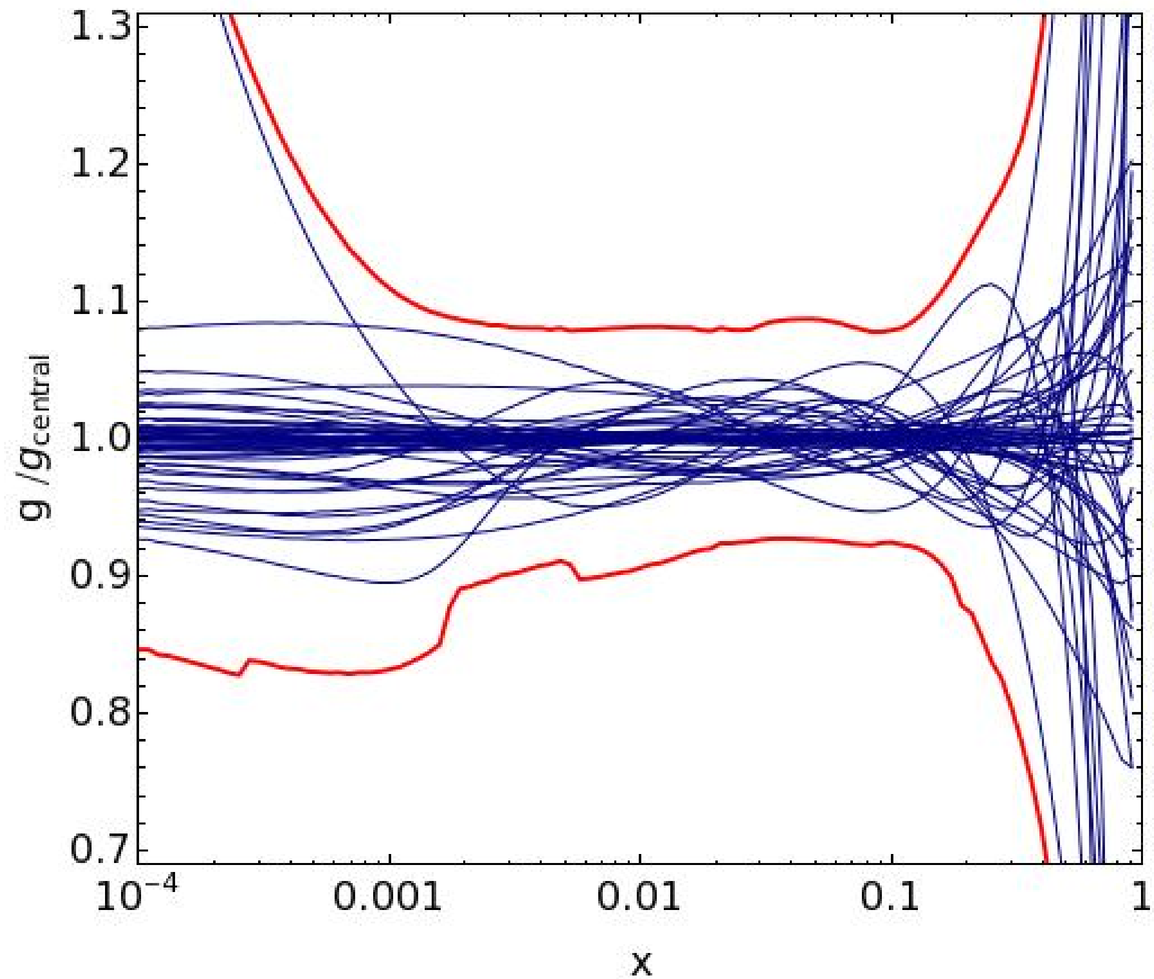
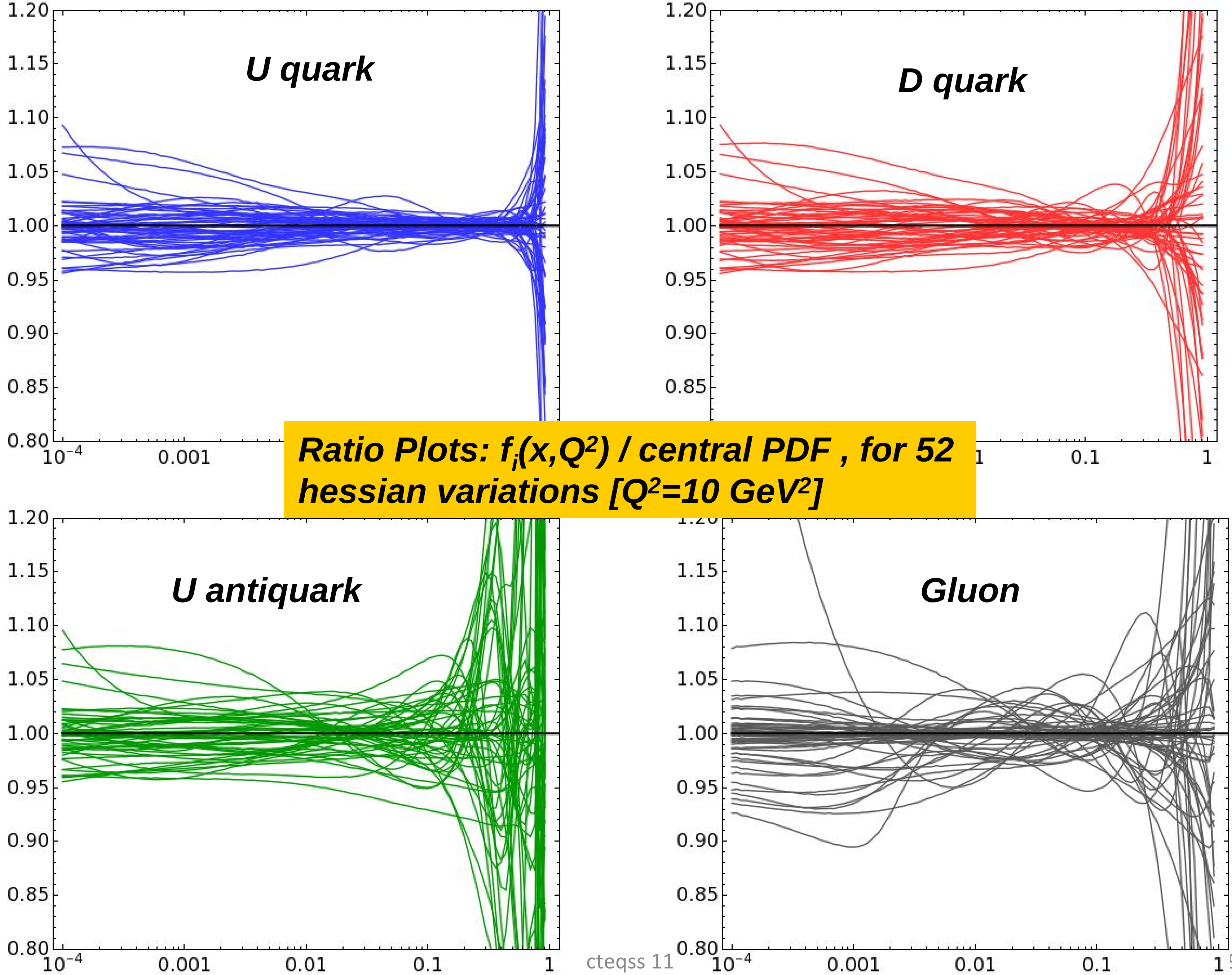
PDF Bounds – gluon, asymmetric

RATIO PLOT



Red curves: Upper and lower boundaries of the uncertainly band, according to the Asymmetric Version Master Formula



Ratio Plots: $f_i(x,Q^2)$ / central PDF , for 52 hessian variations [Q^2=10 GeV^2]

Implications for LHC predictions

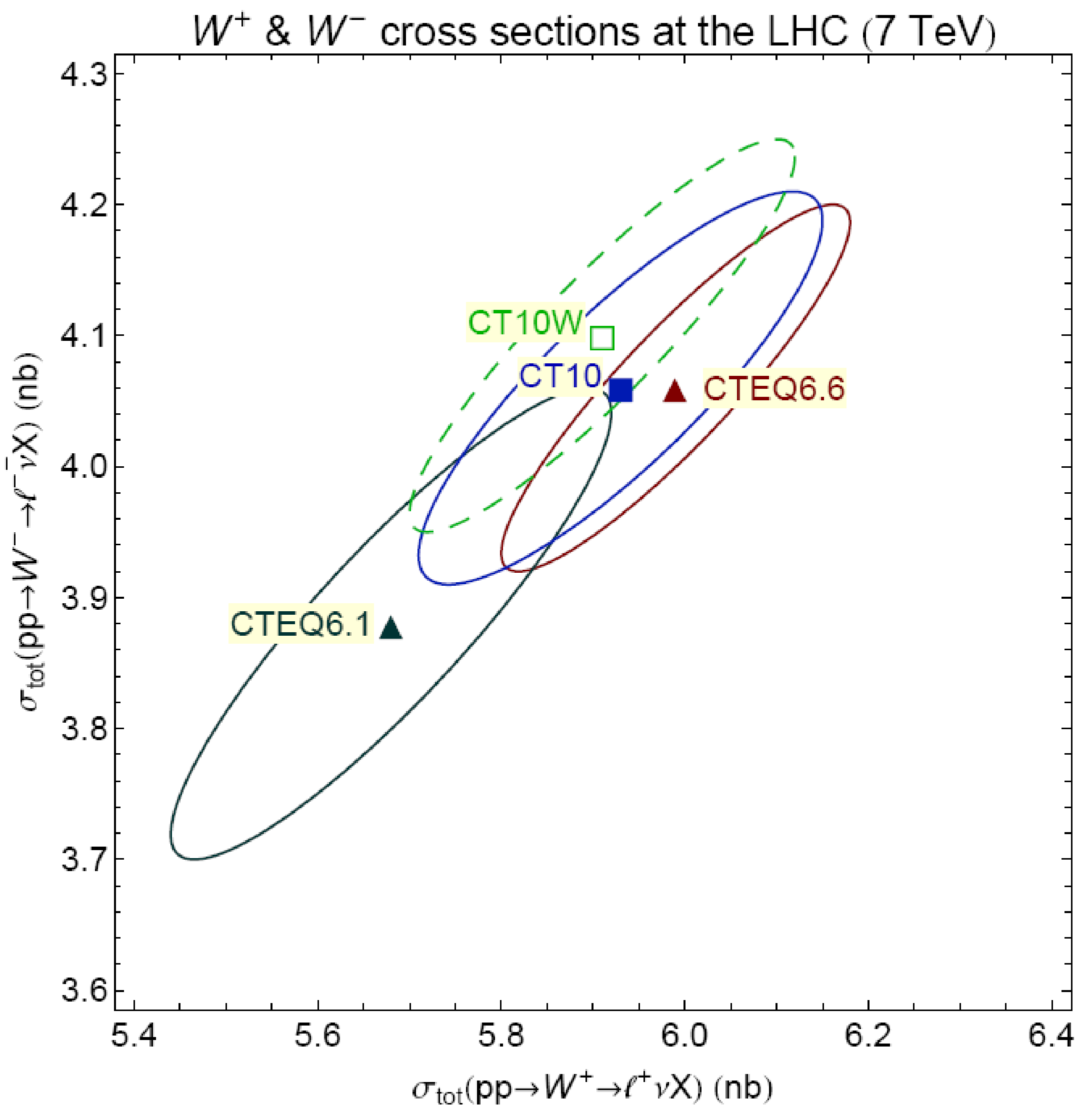
The search for New Physics will probably require high precision comparisons between standard model predictions and experimental measurements.

We seek discrepancies between standard model theory and data.

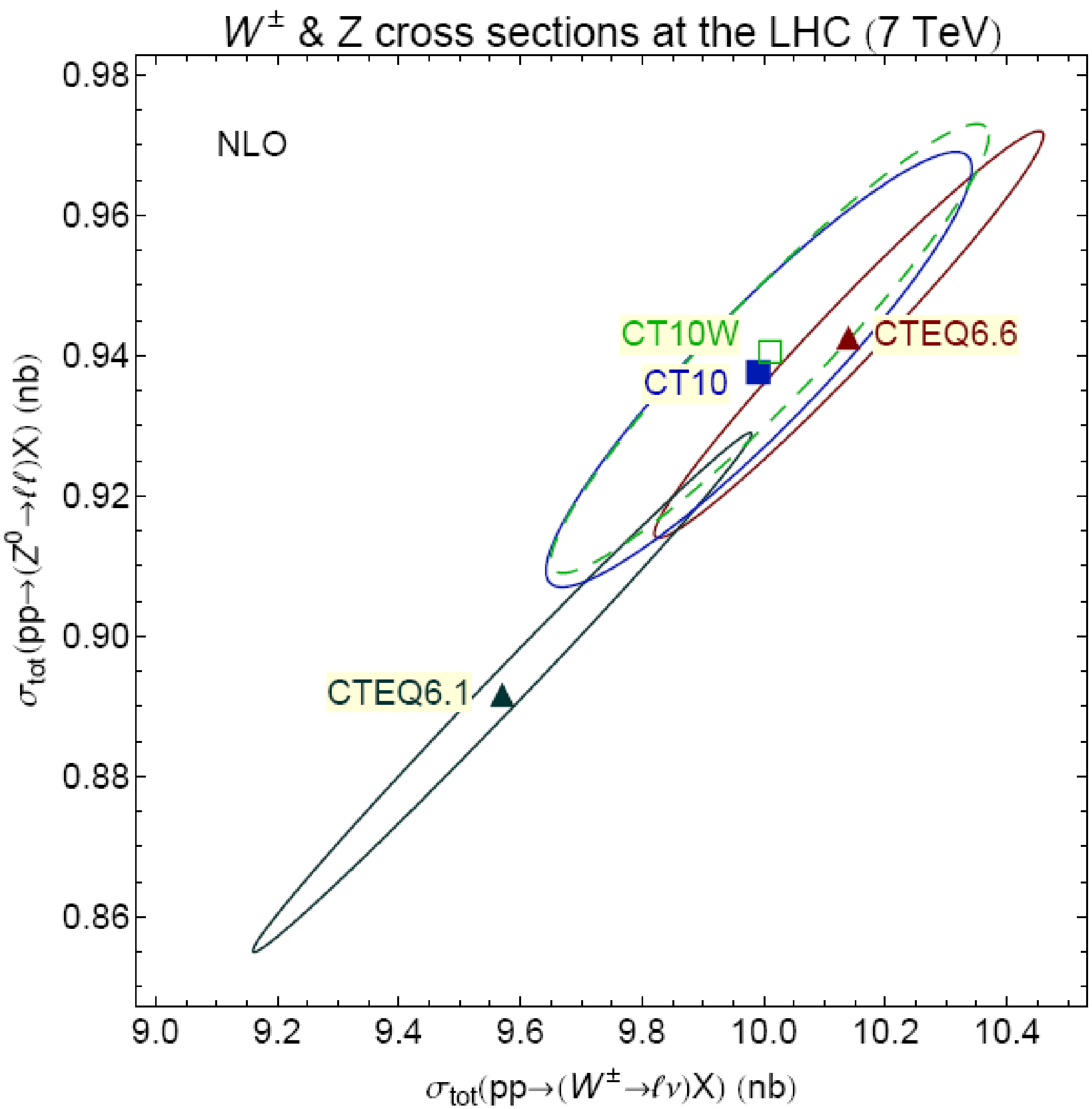
The CT10 PDFs – central fit and eigenvector basis variations – will be used in the theory predictions.

Please understand the importance of the Master Formula!

$(\delta Q)^2 = \text{SUM } 0.25 \ [Q^{(+)} - Q^{(-)}]^2$ ***summed over the 26 eigenvector directions***



… from the CT10 paper



… from the CT10 paper