Heavy Quark Production

Part I: Theoretical Perspective

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OUTLINE

- Statement of the problem
- Status report:

  Comparison of Data & Theory
- How do we make heavy quarks
- Case studies:
- Mass-Independent Evolution

  Why is it valid?
- Conclusions
- Lunch

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Statement of the Problem

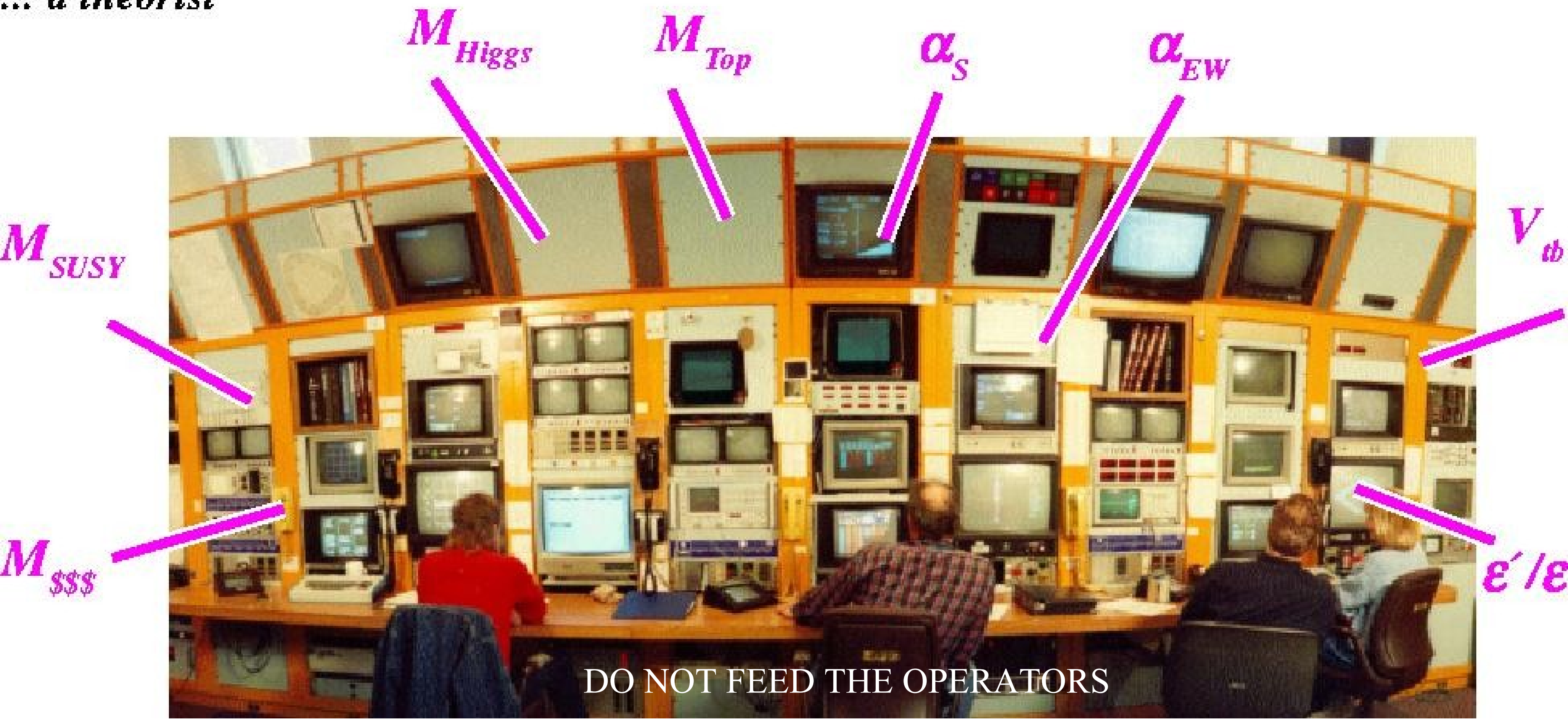
What is the ideal way to learn about quark masses and their effects on a physical process?

As a theorists, I simply run my calculation over the full range of mass values from 0 to ∞, and study the behavior.

Wouldn't it be great if the experiments could do the same???

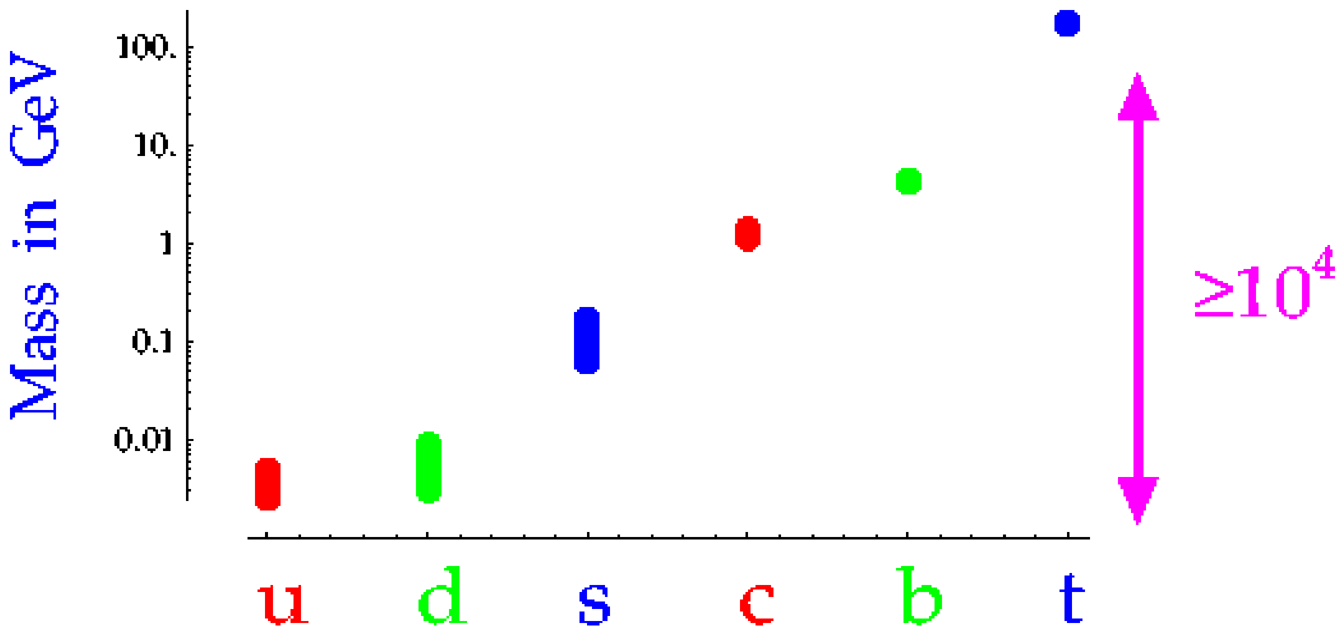
What's really in the Fermilab control room ...

... a theorist



Unfortunately, in real life, we can't vary parameters continuously

Quark Masses Span Wide Dynamical Range ~ 10^4



U	0.003
D	0.005
S	100
C	1.3
B	4.5
T	173

We can't vary the quark mass continuously, but these ``notches" on our control panel give us a lot of flexibility

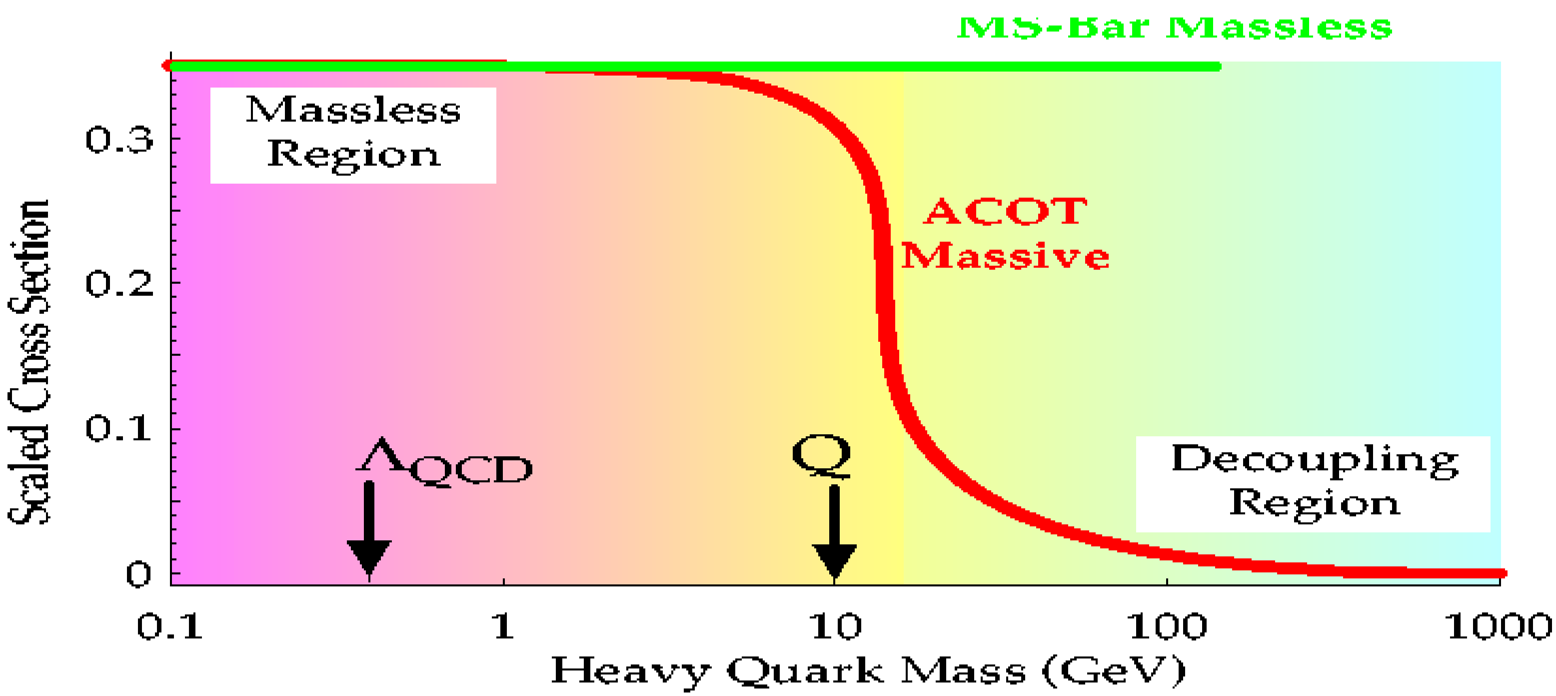
Theorists would much prefer that quark masses only come in 2 varieties:

m = 0: Massless case.

Mass plays no dynamic role
Well understood.

m = ∞: Infinite case.

Mass Decouples.
We can forget about this object



Single-Scale Problem in Perturbation Theory:

$$\sigma = \sum_{N=1}^{\infty} (\alpha_s L)^N = \sum_{N=1}^{\infty} \left\{ \alpha_s(\mu) \log\left(\frac{E^2}{\mu^2}\right) \right\}^N$$

... where E is any relevant scale of the problem: Q, P_T, E_T,...

Use RGE to solve this. $\frac{d\sigma}{d \log \mu^2} = \ldots$

Multi-Scale Problem in Perturbation Theory:

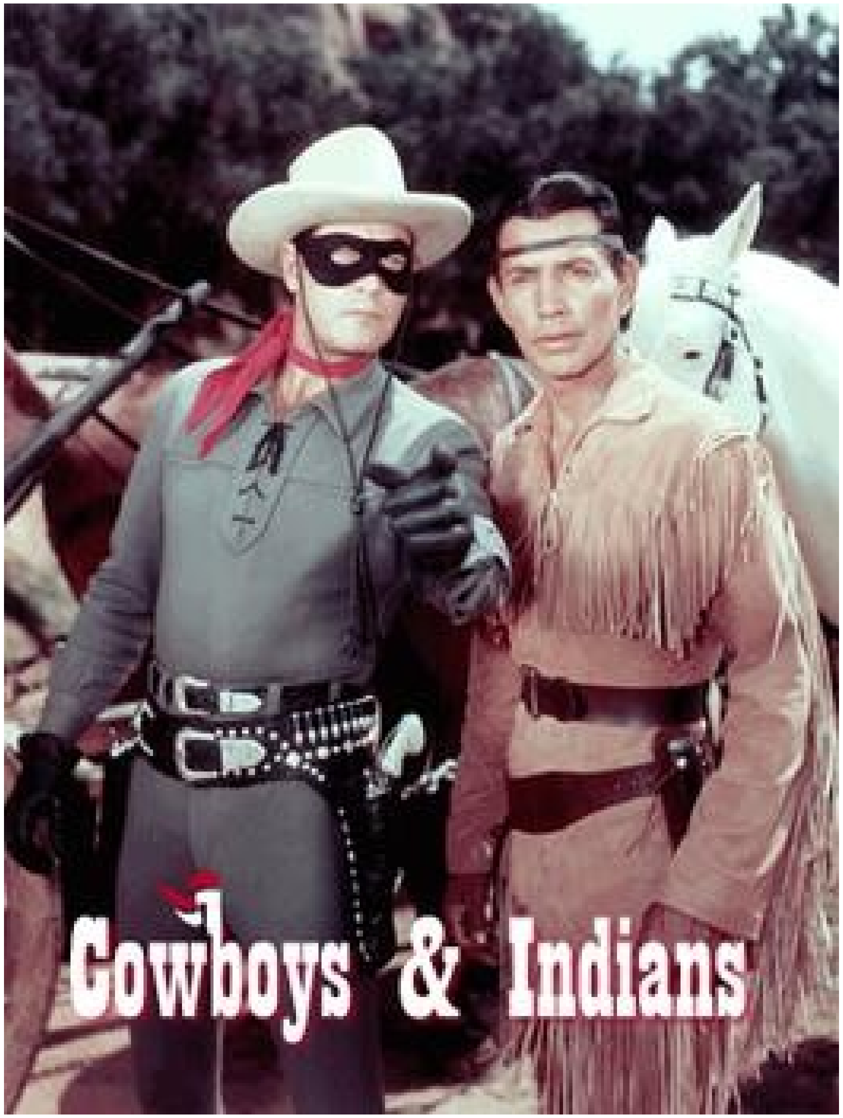
What do we do if we have 2 scales???

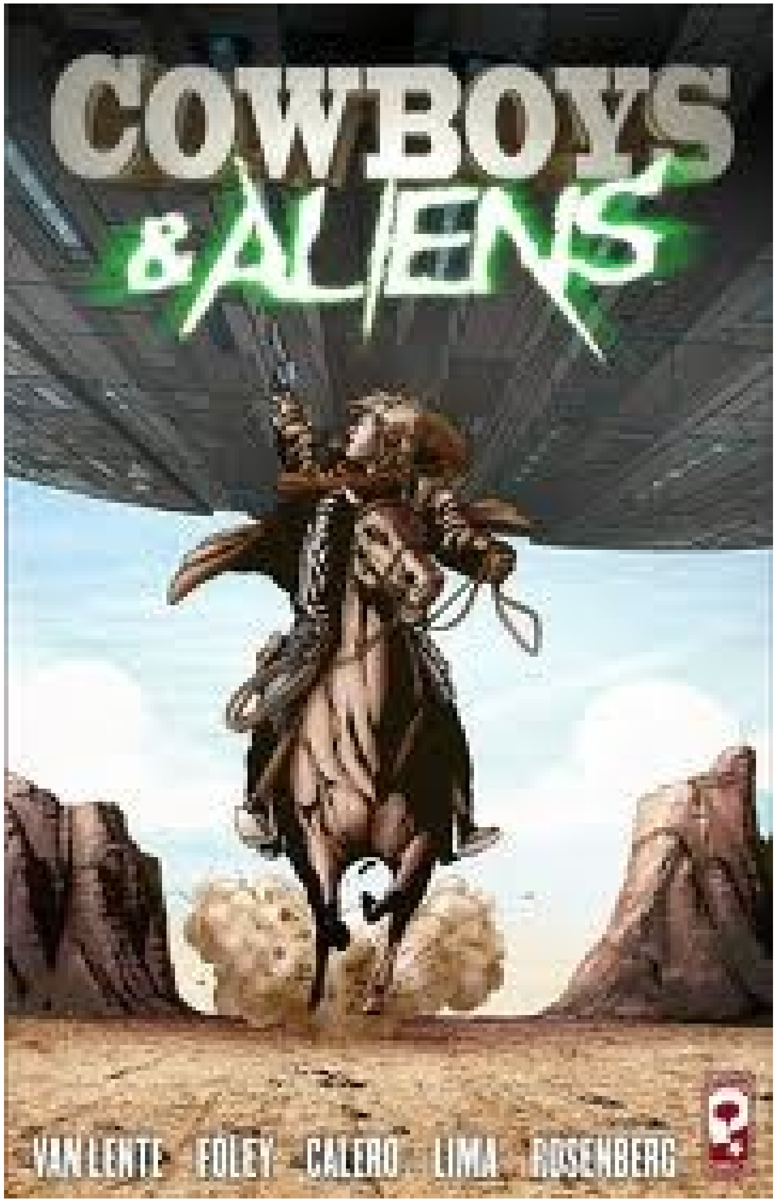
$$\log\left(\frac{E^2}{\mu^2}\right) \qquad \log\left(\frac{M_H^2}{\mu^2}\right)$$

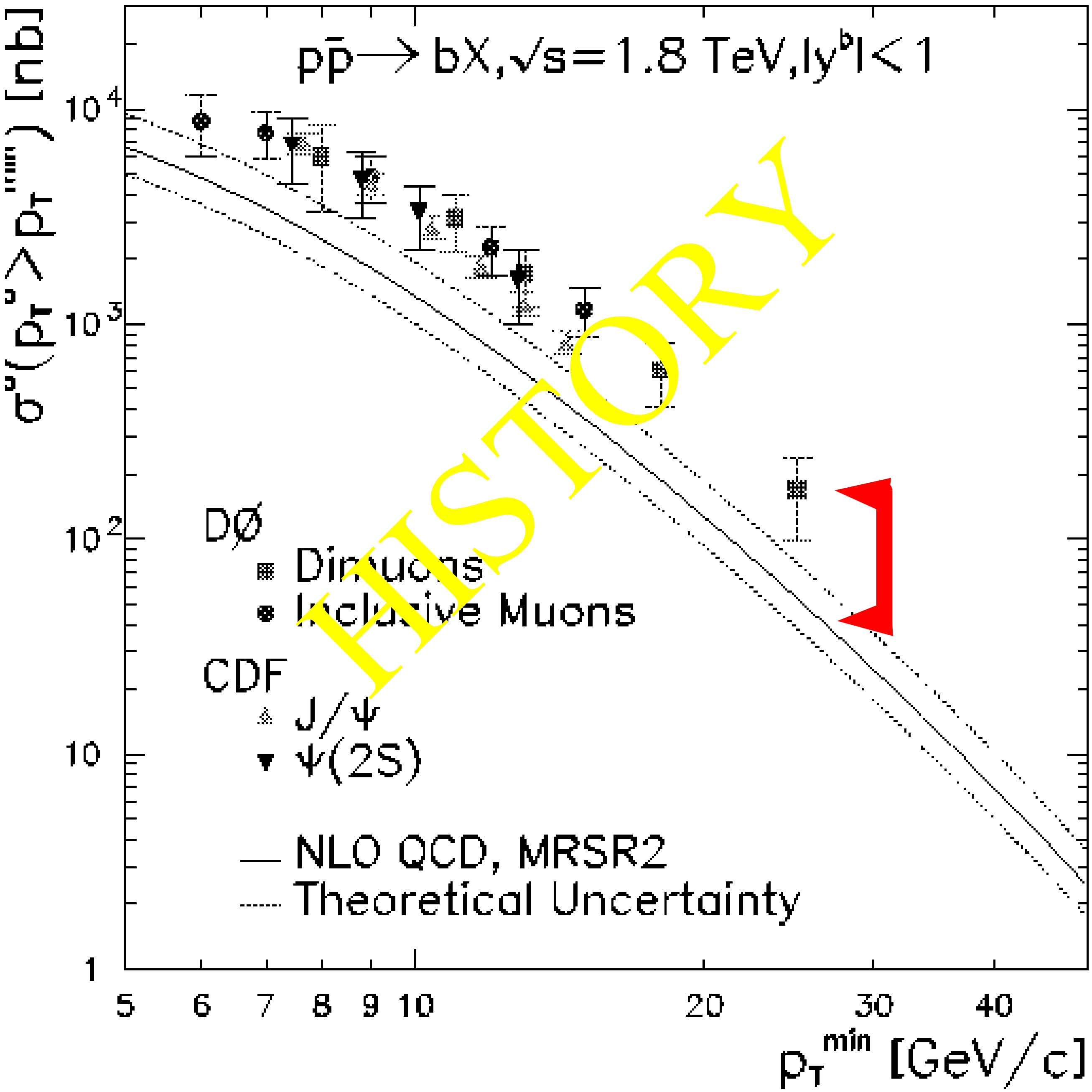
... life gets interesting.

Let's start with a historical perspective

as things change over time



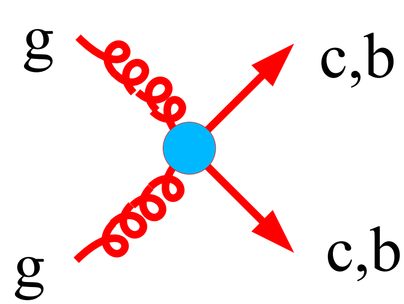


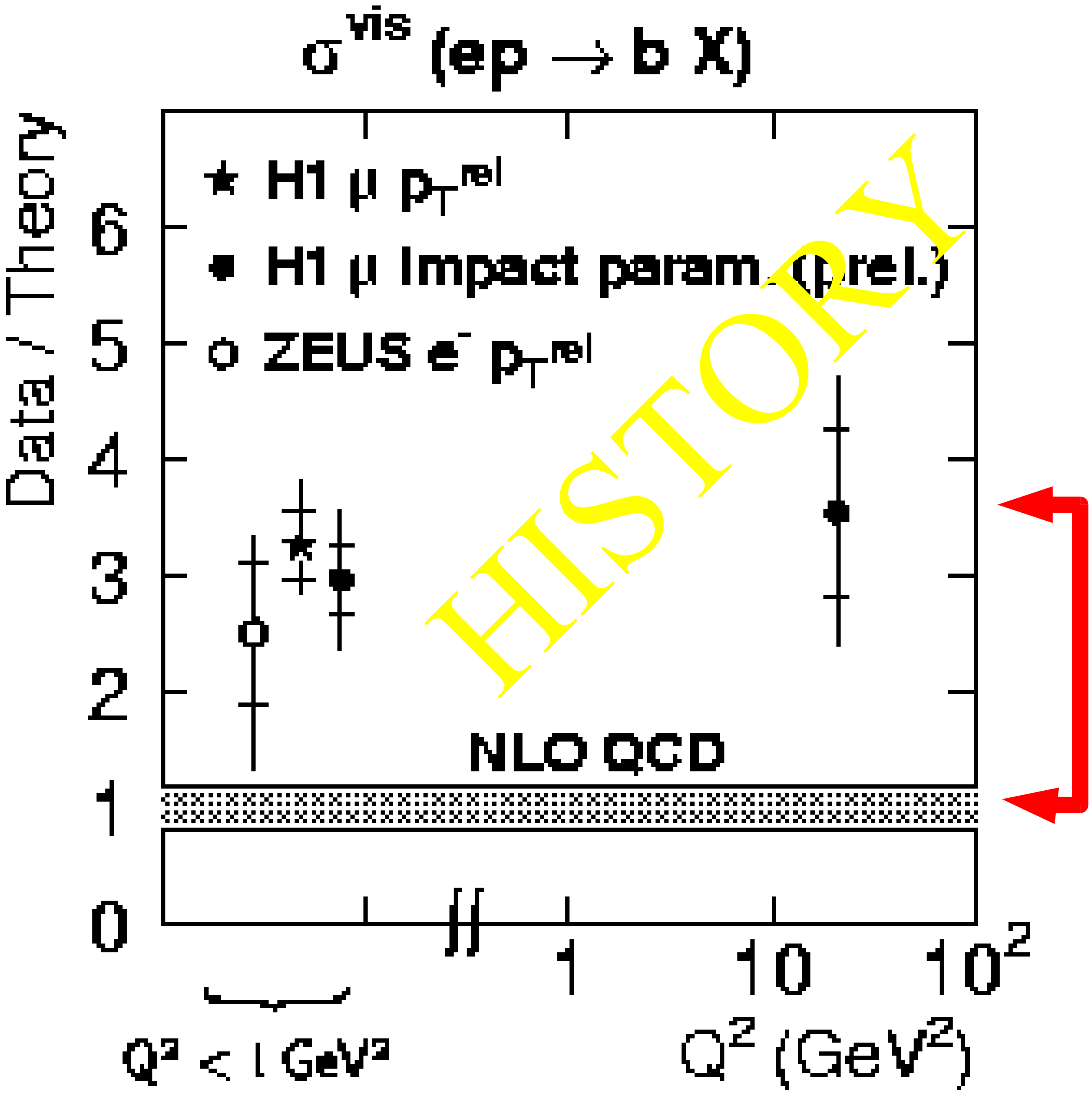


Comparison of Run I data with NLO Theory

Data is high by factor of 2x or 3x

... even given μ variation



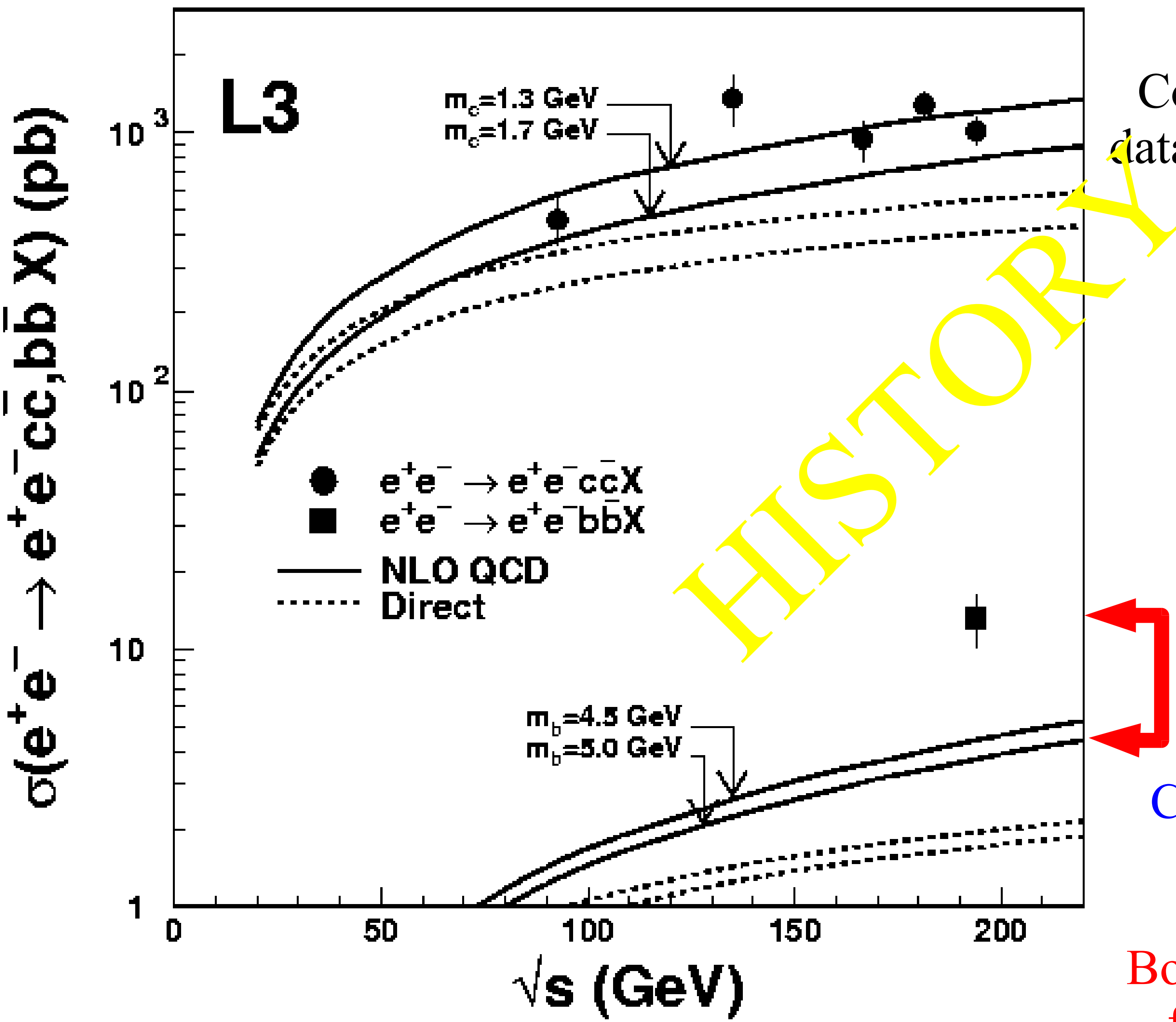


Comparison of HERA data with NLO Theory

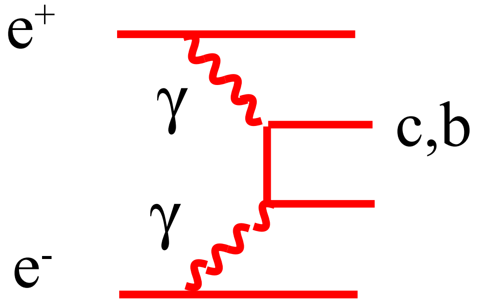
Data is high by factor of 2x or 3x

... even given μ variation

Charm and Bottom Production at LEP



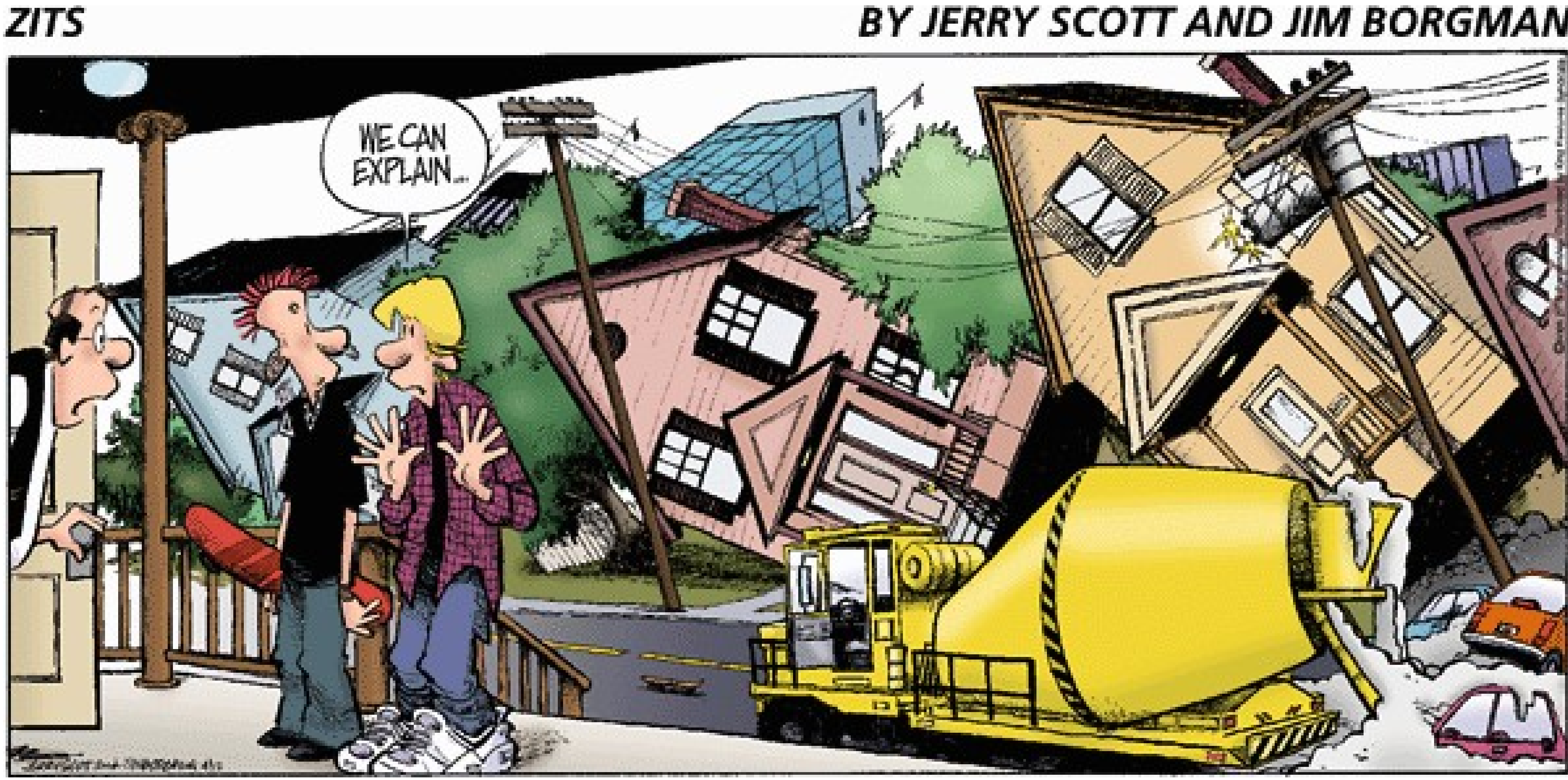
Comparison of LEP data with NLO Theory



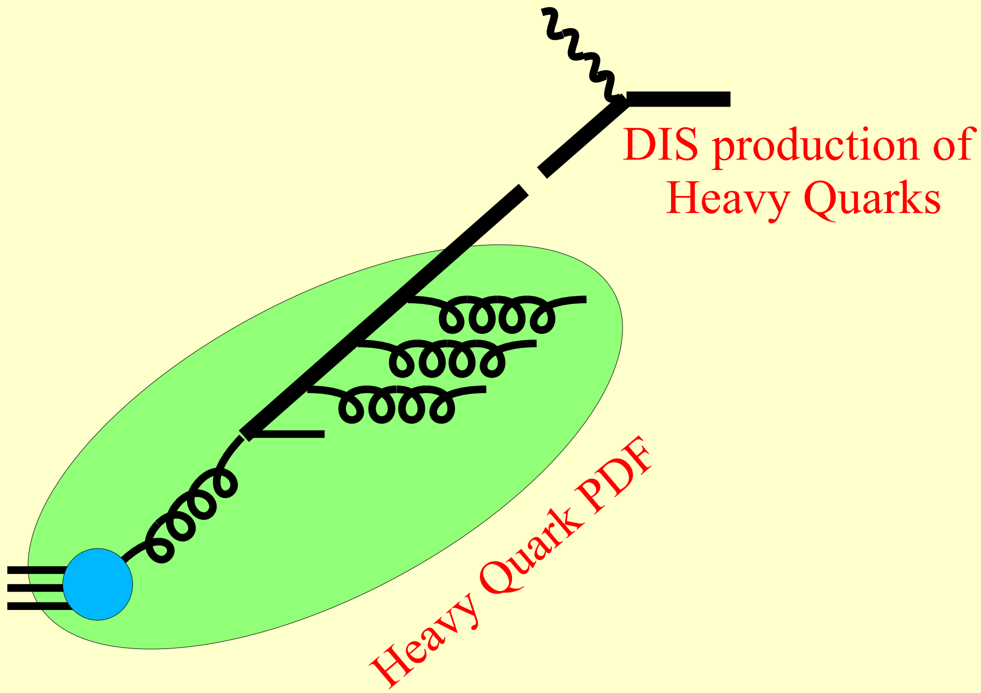
Charm is reasonable

Bottom data is high by factor of 2x or 3x

Historically, calculation of processes with heavy quarks has been a challenge



Part of the answer ...



Heavy Quarks PDF's

Essential for disparate mass scales

Heavy Quarks: How do we deal with disparate scales???

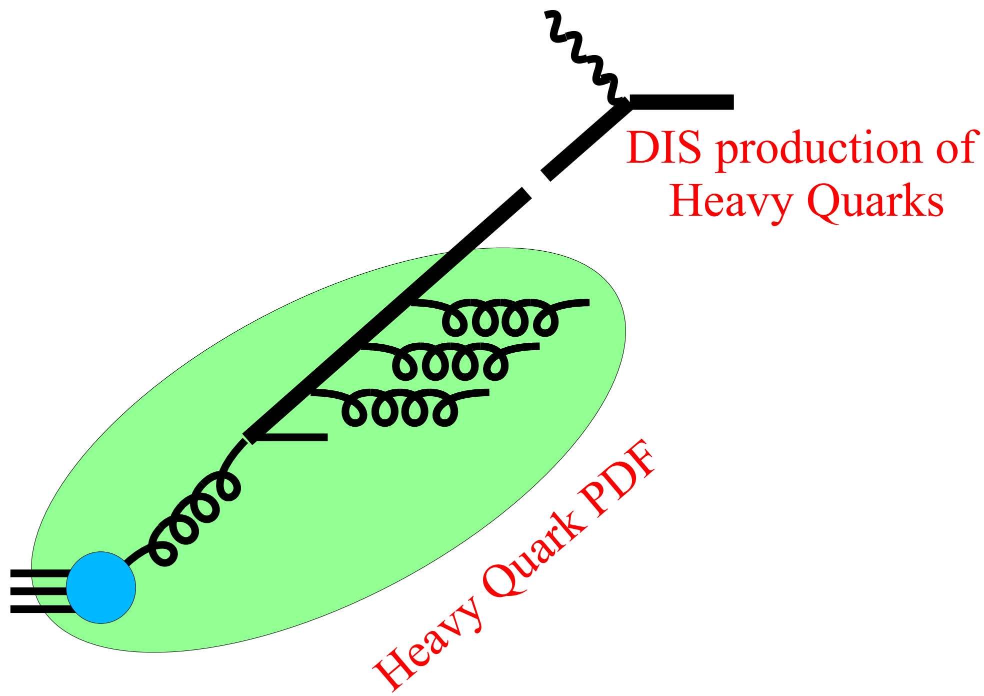
Problem: Heavy Quark introduces new scale: $\log\left(\frac{Q^2}{\mu^2}\right) \quad \log\left(\frac{M_H^2}{\mu^2}\right)$

... life gets interesting.

Solution: Resum $Log(M_H)$ in the Heavy Quark PDF's:

...include charm and bottom in the PDFs

DGLAP equation
Resums iterative splittings
inside the proton

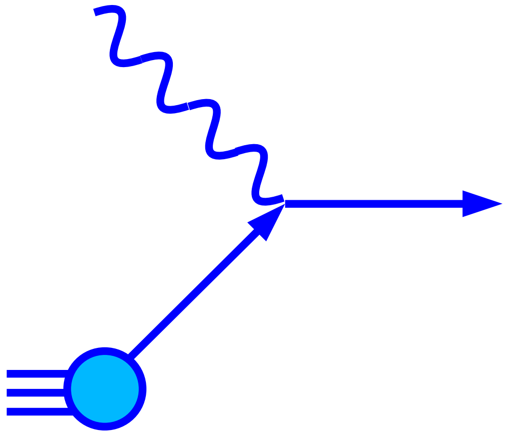


Result: We can describe the full kinematic range from low to high

this is the essence of the ACOT renormalization scheme

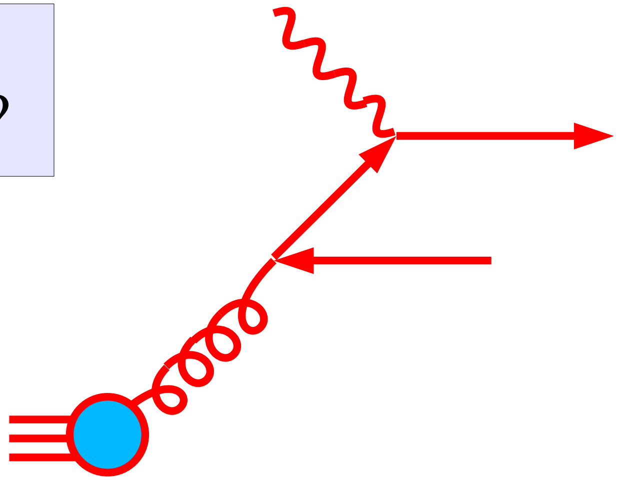
ACOT, PRD 50, 3102

Which is the correct production mechanism?



Heavy Excitation (HE)

Quark	Channel
s	YES
t	NO
c	???
b	???

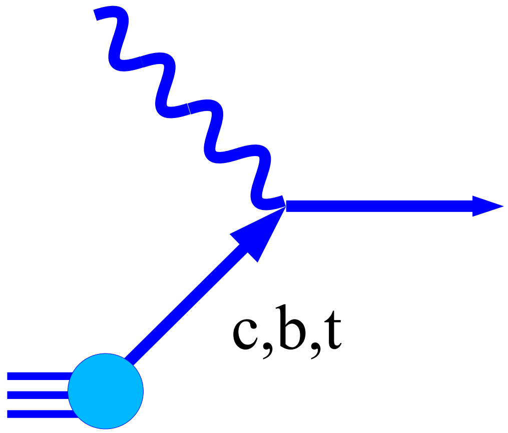


Heavy Creation (HC)

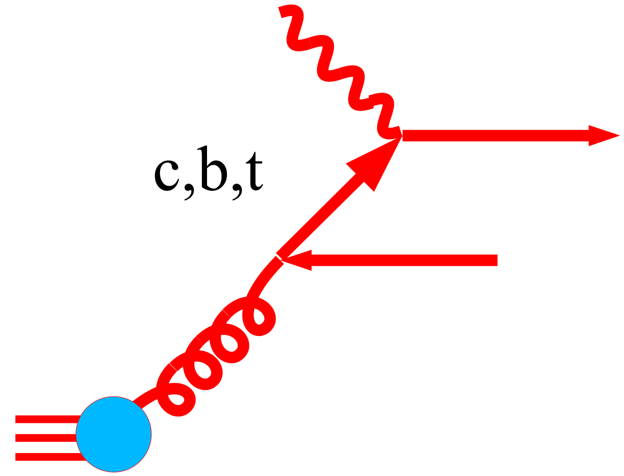
Quark	Channel
s	NO
t	YES
c	???
b	???

If you can't beat 'em, join 'em.

How to Join without ``Double Counting"???

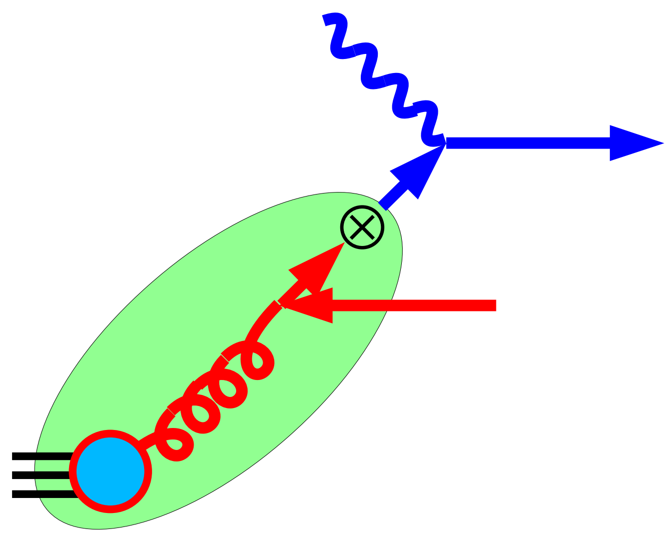


Heavy Excitation (HE)

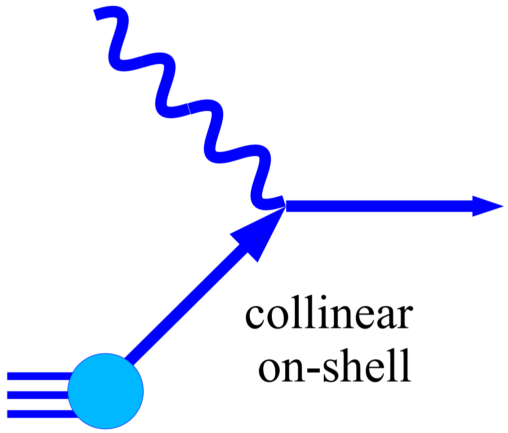


Heavy Creation (HC)

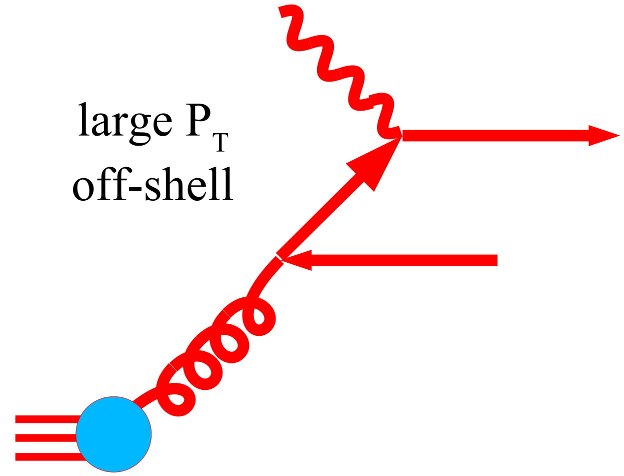
Wait a minute!
Since the heavy quark originally came from a gluon splitting, these diagrams are ***Double Counting***



How to Join without ``Double Counting"???

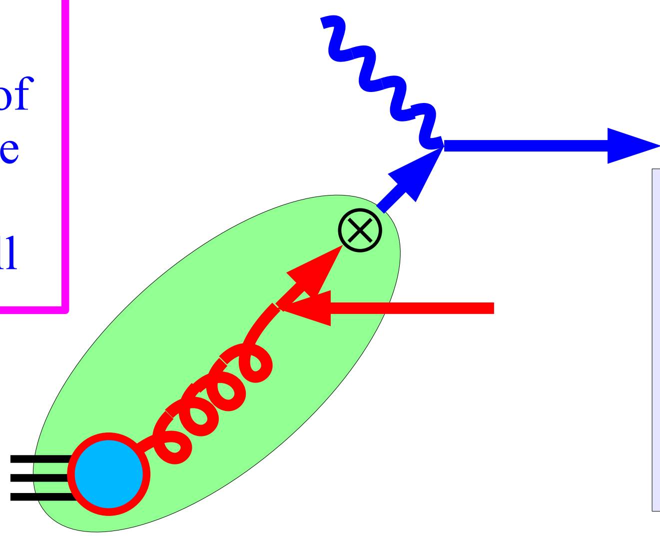


Heavy Excitation (HE)



Heavy Creation (HC)

SUB removes the overlapping regions of phase space where the t-channel quark is collinear and on shell



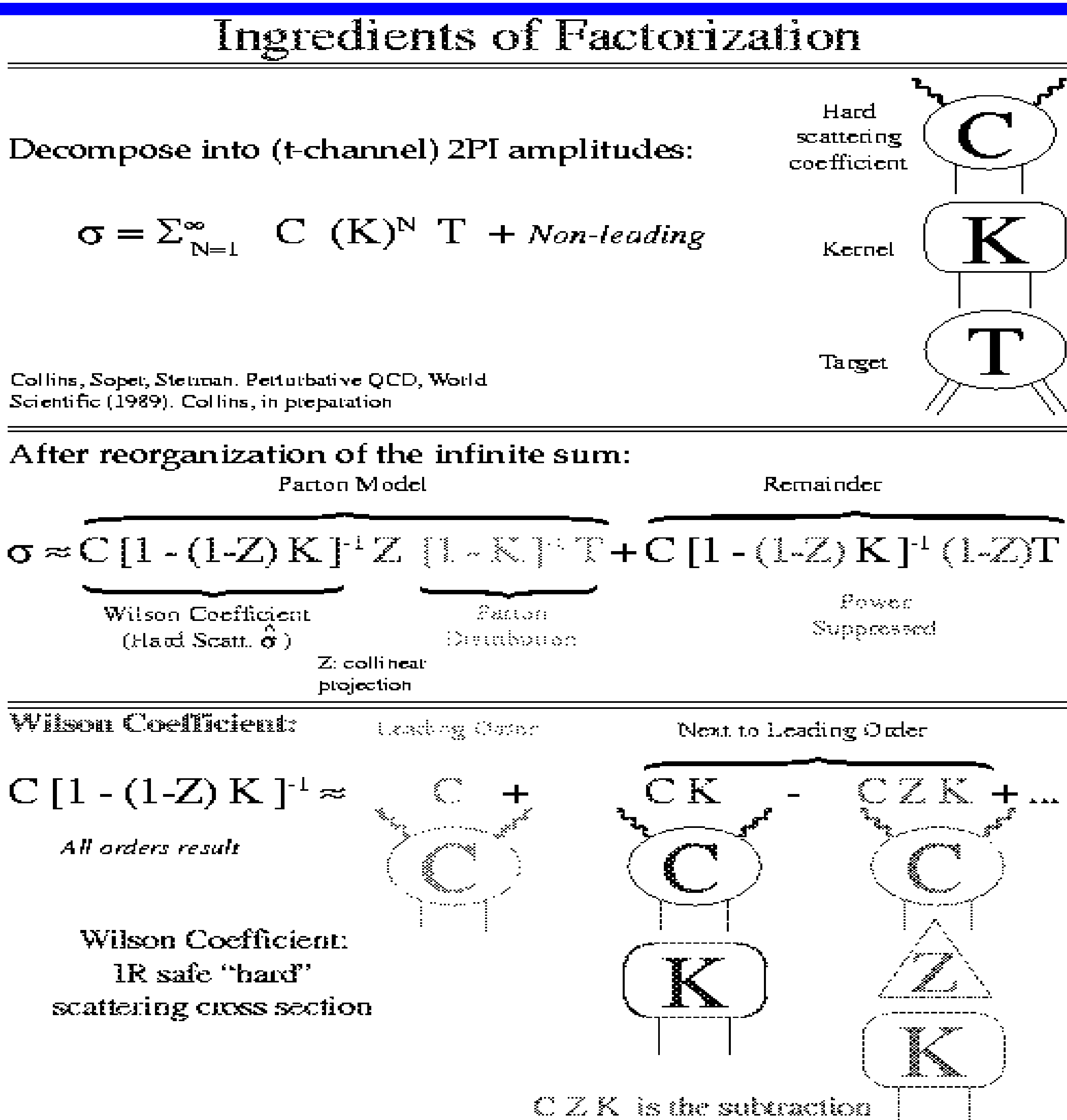
Subtraction (SUB)

TOT = HE + (HC – SUB)

Formally, NLO (HC – SUB)

How do we actually derive???

There is a rigorous factorization proof ...



A formal proof was constructed by numerous groups.

This proof was explicitly extended to the case of massive quarks

(Collins,1998)

Though Experiment:
To keep things simple, let's consider scattering off a parton target.

Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

Note: not m^2/Q^2

At Zeroth Order:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 \;\; + O(\Lambda^2/Q^2)$$

f^0 f^1

for parton target

Use: $f^0 = \delta$ and $d^0 = \delta$ for a parton target.

Therefore:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 = \delta \otimes \omega^0 \otimes \delta = \omega^0$$

$$\sigma^0 = \omega^0$$

Warning: *This trivial result leads to many misconceptions at higher orders*

Application of Factorization Formula at Next to Leading Order NLO)

Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

<u>At First Order:</u>

$$\sigma^1 = f^1 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^1 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^1$$

$$\sigma^1 = \quad f^1 \otimes \sigma^0 + \omega^1 + \sigma^0 \otimes d^1$$

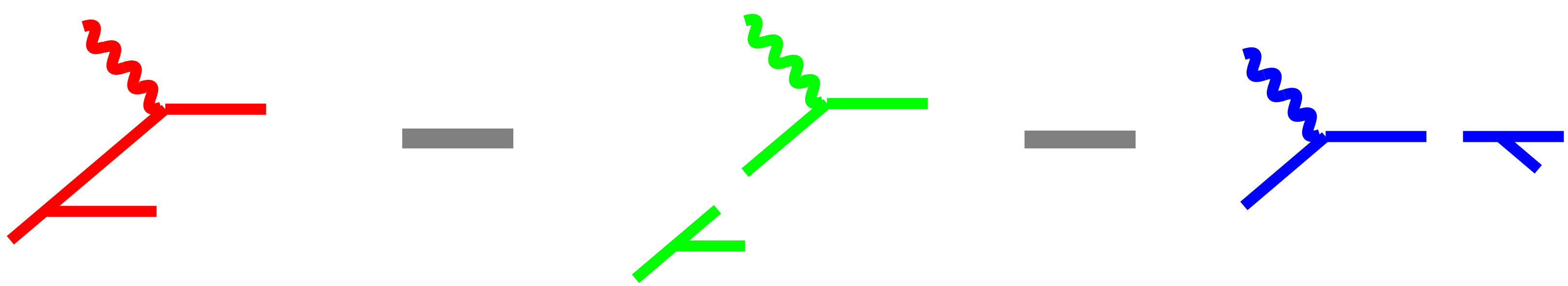
f^0 $\qquad$ f^1

We used: $f^0 = \delta$ and $d^0 = \delta$ for a <u>parton</u> target.

Therefore:

$$\omega^1 = \sigma^1 - f^1 \otimes \sigma^0 - \sigma^0 \otimes d^1$$

Not just σ



σ^1 $\qquad$ $f^1 \otimes \sigma^0$ $\qquad$ $\sigma^0 \otimes d^1$

Use the Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

At Second Order *(NNLO):*

Therefore:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + \ldots$$
$$+ f^1 \otimes \omega^1 \otimes d^0 + \ldots$$

$$\omega^2 = ???$$

Compute ω^2 at second order.
Make a diagrammatic representation of each term.

HOMEWORK PROBLEM: CONVOLUTIONS

Part 1) Show these 3 definitions are equivalent; work out the limits of integration.

$$f \otimes g = \int f(x) g(z/x) \frac{dx}{x} = \int f(z/y) g(y) \frac{dy}{y}$$
$$= \int f(x) g(y) \delta(z - x * y) dx dy$$

Part 2) Show convolutions are the ``natural" way to multiply probabilities.

If f represents the heads/tails probability distribution for a single coin flip, show that the distribution of 2 coins is f⊕f and 3 coins is f⊕f⊕f.

$$f \oplus g = \int f(x) g(y) \delta(z - (x + y)) dx\, dy$$
$$f(x) = \frac{1}{2} (\delta(1 - x) + \delta(1 + x))$$

BONUS: How many processes can you think of that don't factorize?

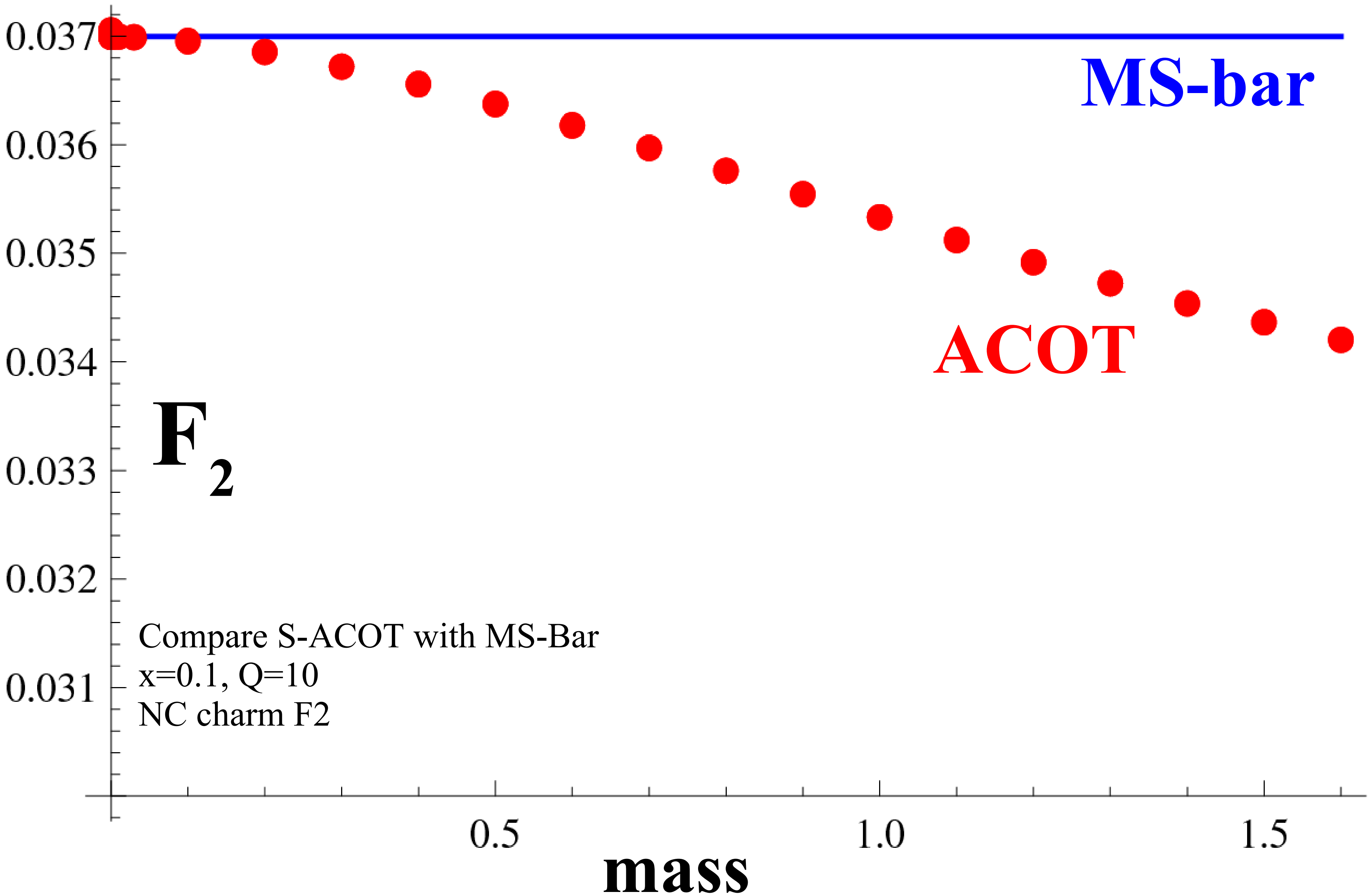
ACOT is a minimal massive extension of the MS-bar scheme

In particular, $m \to 0$ limit of ACOT yields MS-Bar

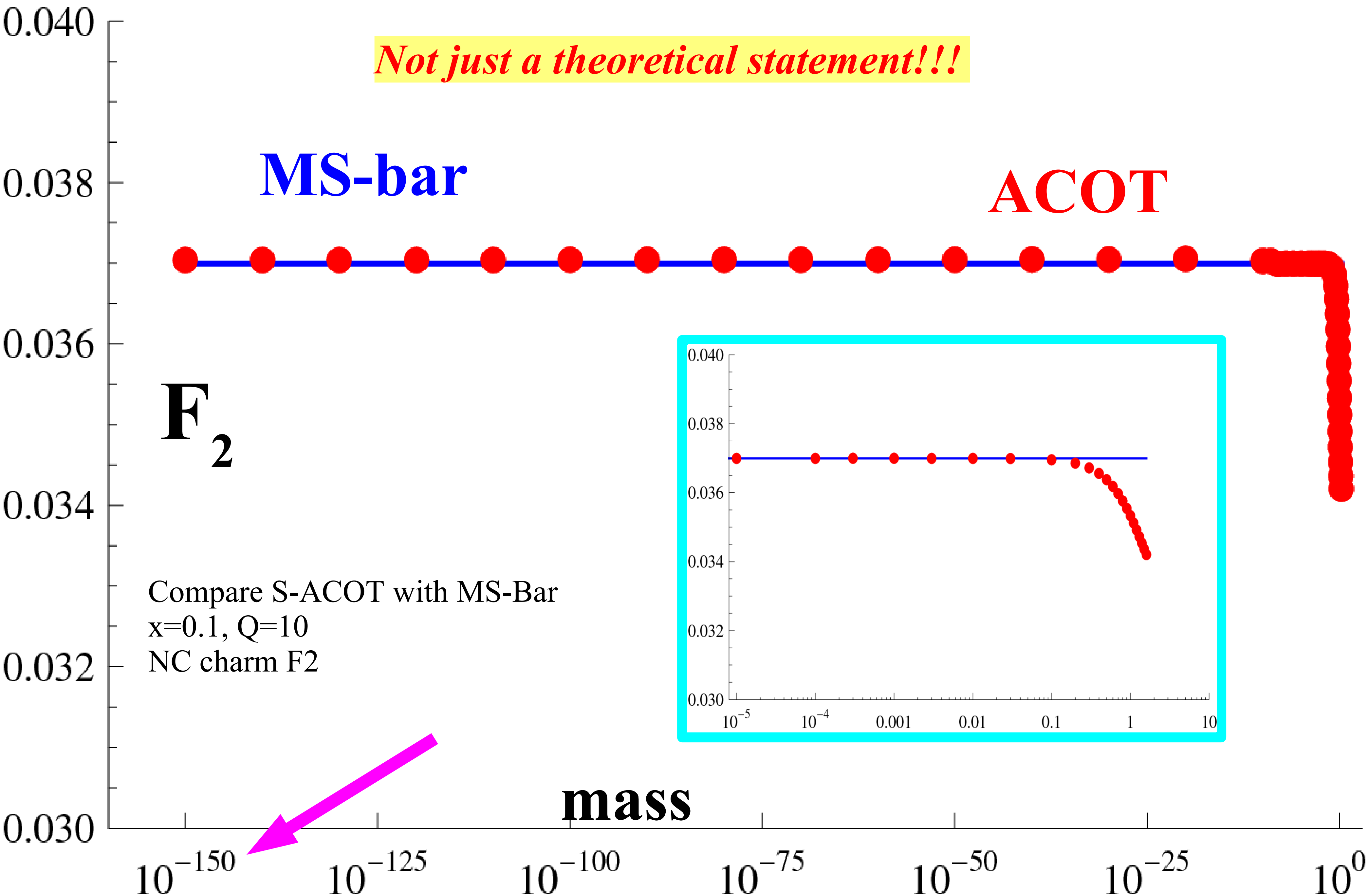
no finite renormalization

**Aivazis, Collins, Olness, Tung*

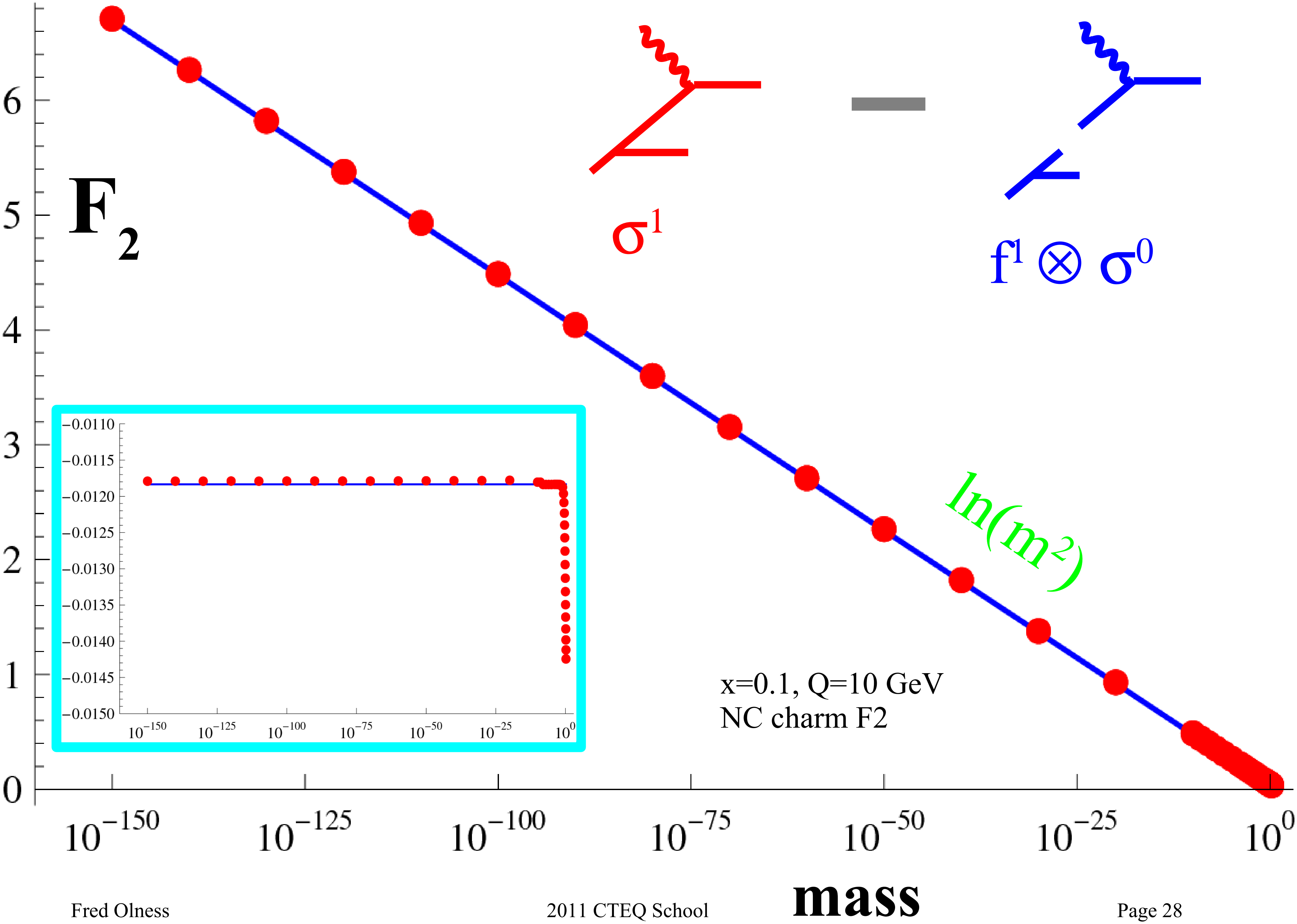
ACOT m→ 0 limit yields MS-Bar: *No finite renormalization*



ACOT m→ 0 limit yields MS-Bar: *No finite renormalization*



ACOT m→ 0 limit yields MS-Bar: *No finite renormalization*

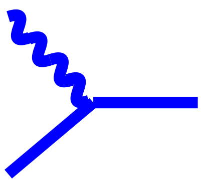


Combined Result:

$$\omega^{0}+\omega^{1}=\sigma^{0}+\sigma^{1}-\left\{f^{1}\otimes\sigma^{0}+\sigma^{0}\otimes d^{1}\right\}$$

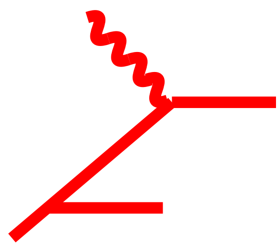
TOT

HE



Heavy Excitation

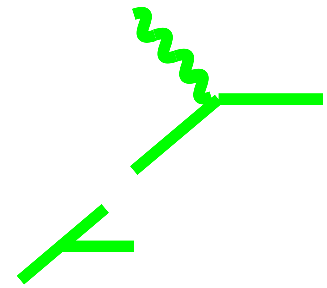
HC



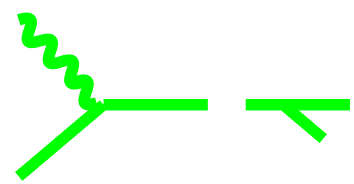
Heavy Excitation

SUB

Subtraction



$f^{1}\otimes\sigma^{0}$

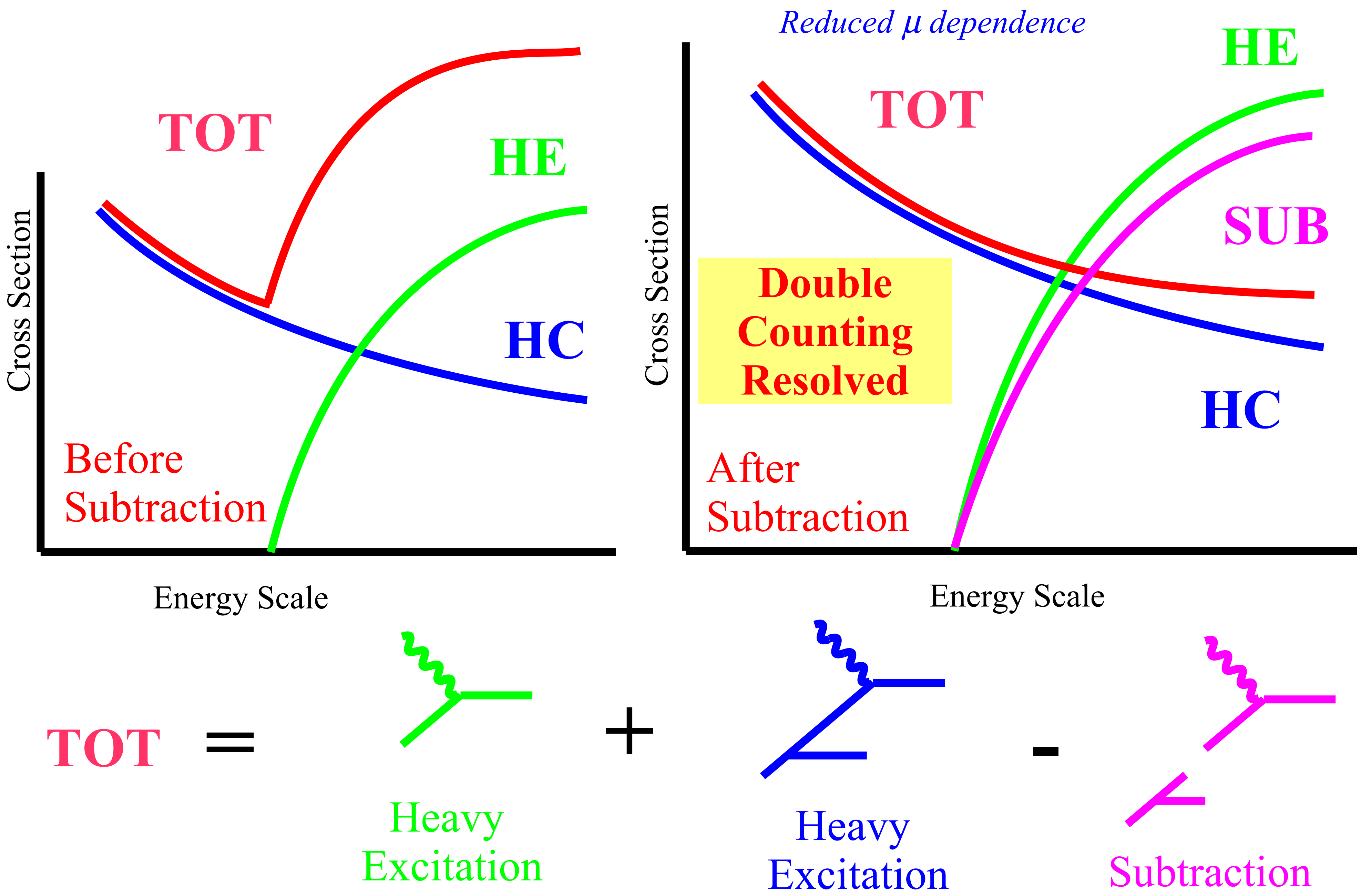


$\sigma^{0}\otimes d^{1}$

TOT = HE + HC – SUB

Double Counting Resolved

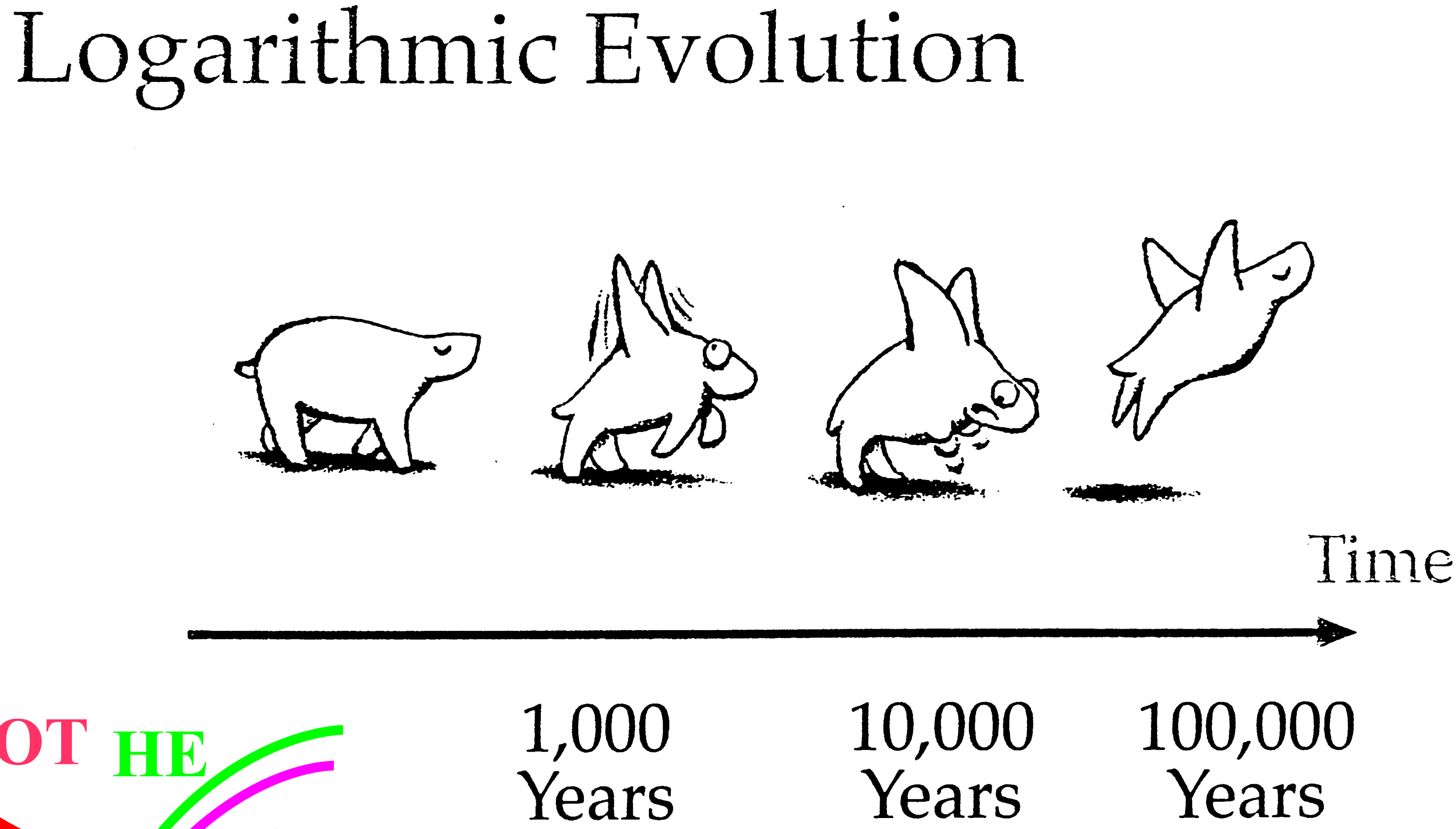
Interaction of the separate contributions vs. energy scale

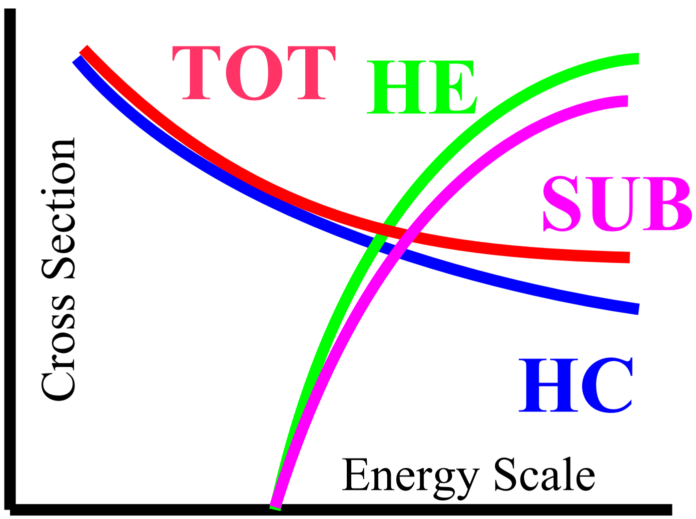


When do we need to worry about Heavy Quark PDFs???

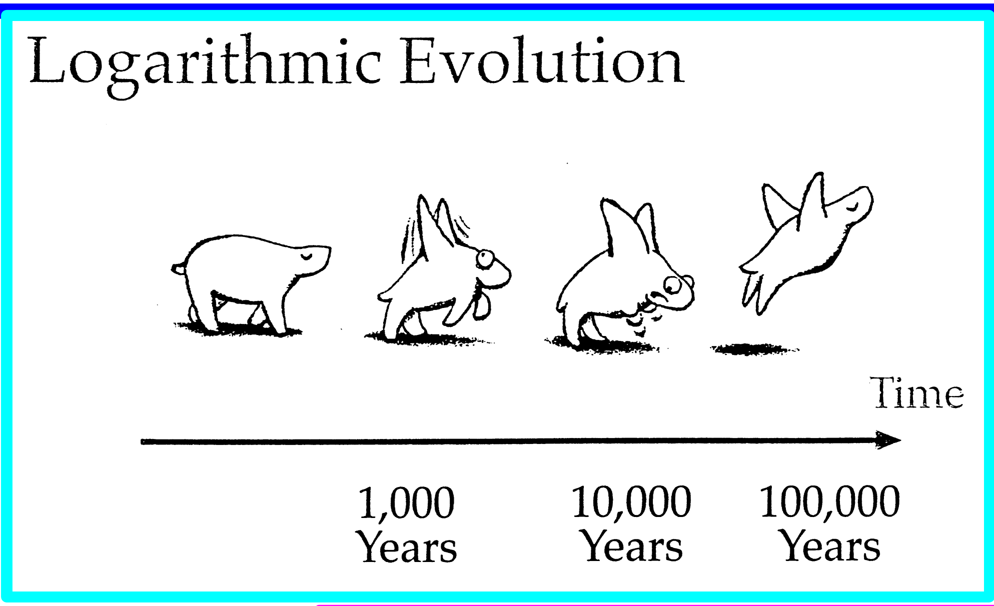
Why do $c(x,\mu)$ and $b(x,\mu)$ increase so quickly???

``Standard'' Evolution

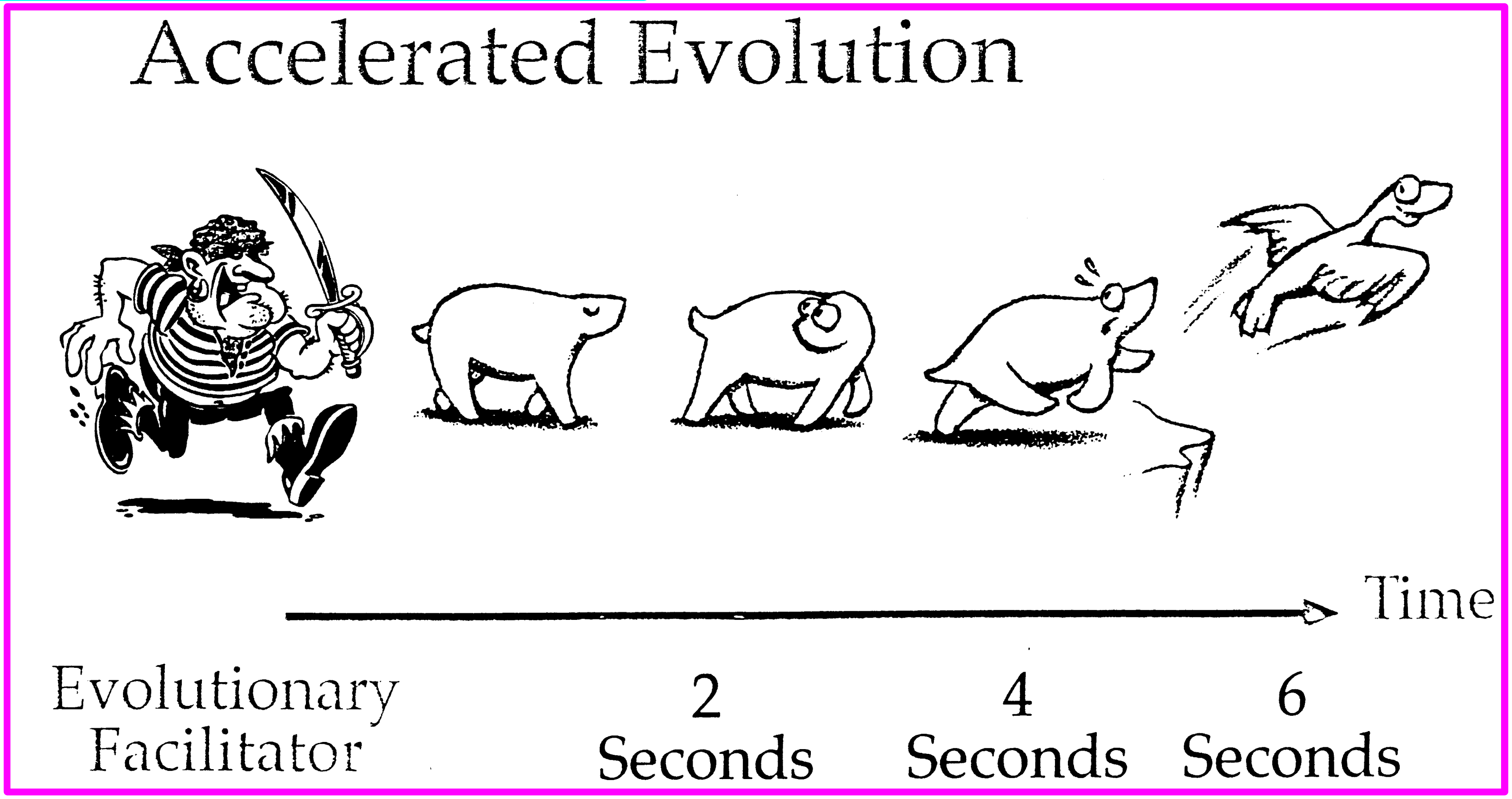




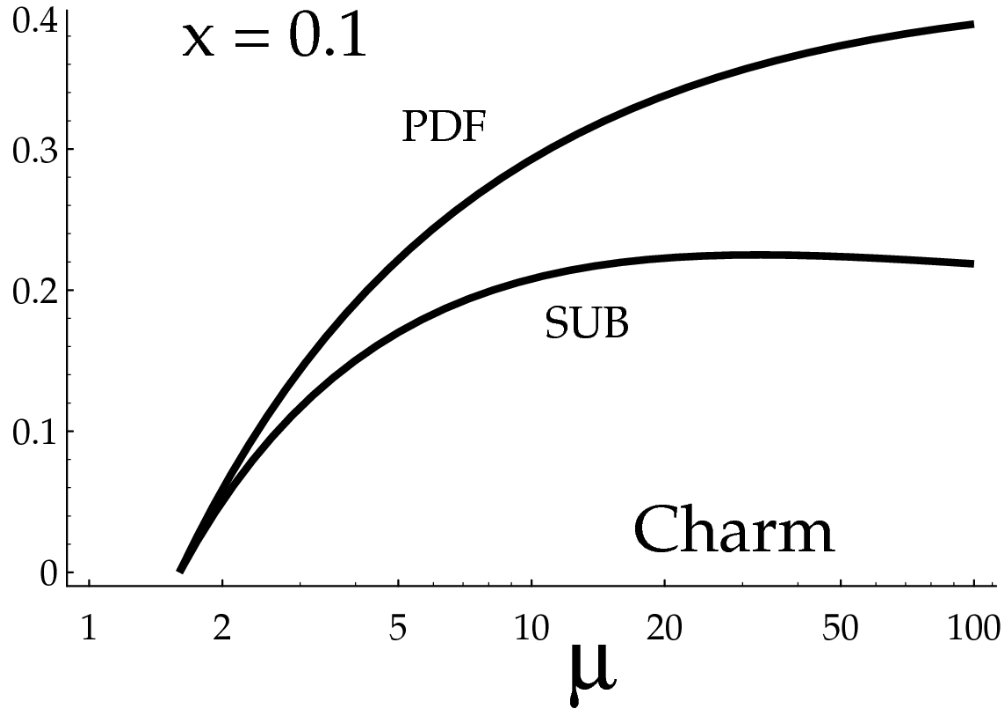
Why does $f_b(x,\mu)$ increase so quickly???

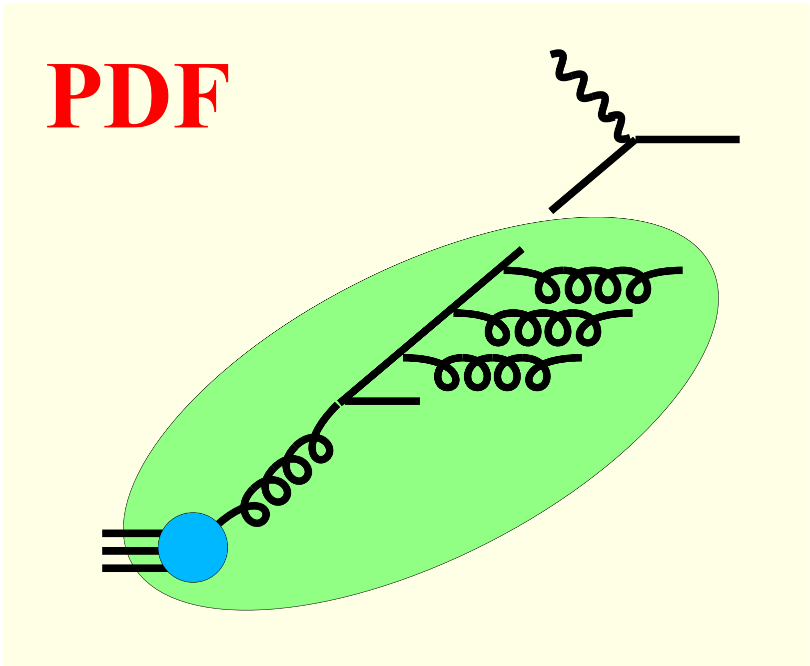


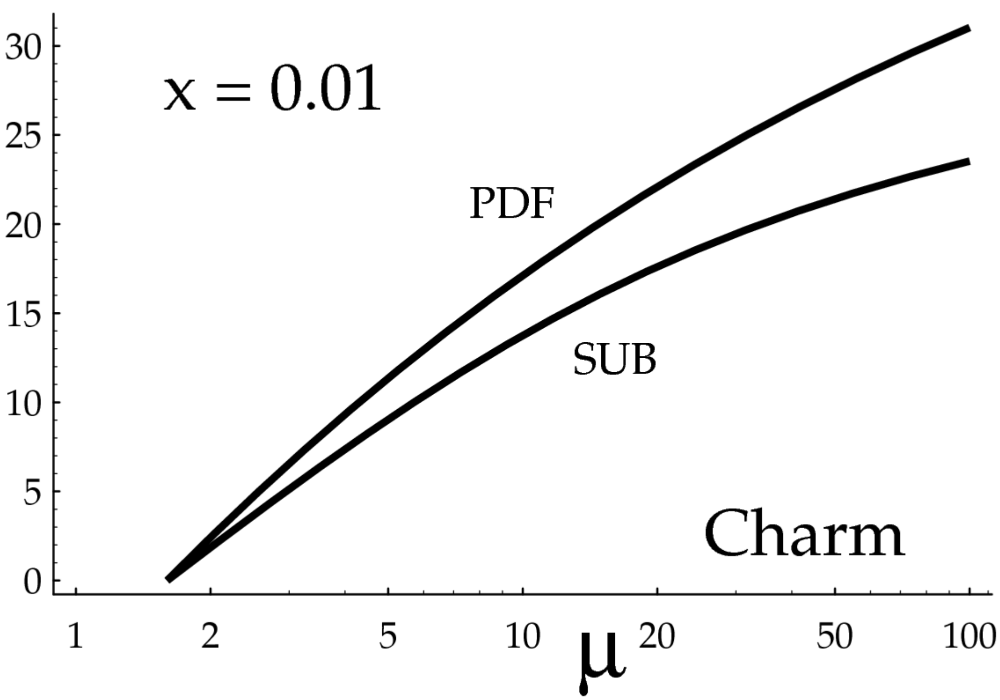
On occasion,
certain factors can accelerate
evolution

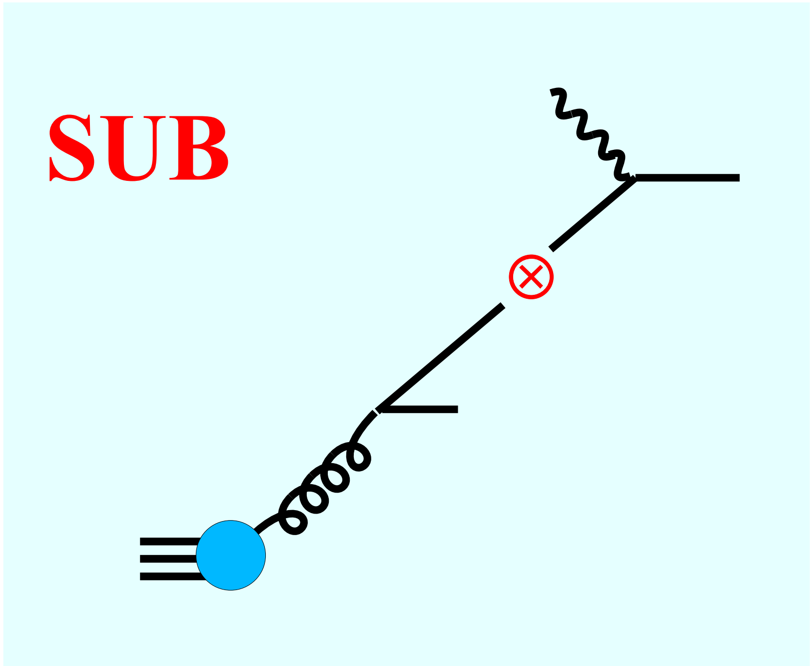


When do we need to consider heavy quark PDF evolution ???



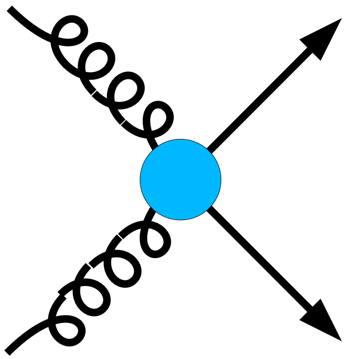
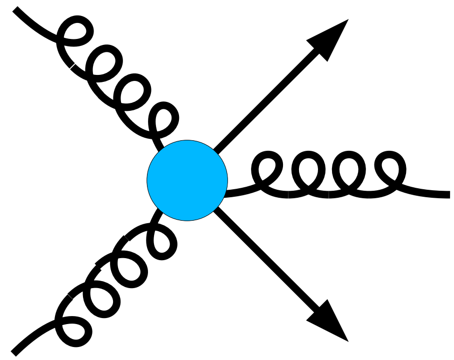
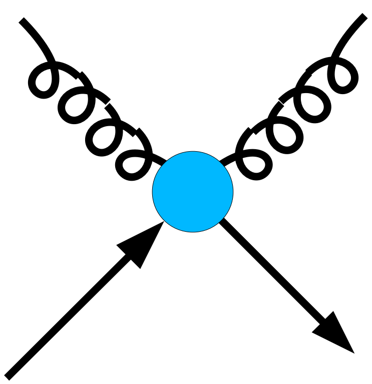
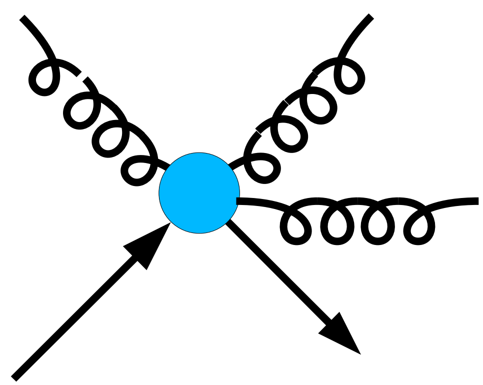






B-Hadroproduction: A Case Study

The Basic Contributions to Heavy Flavor Production

	Leading Order	Next to LO
Heavy Creation		
Heavy Excitation		

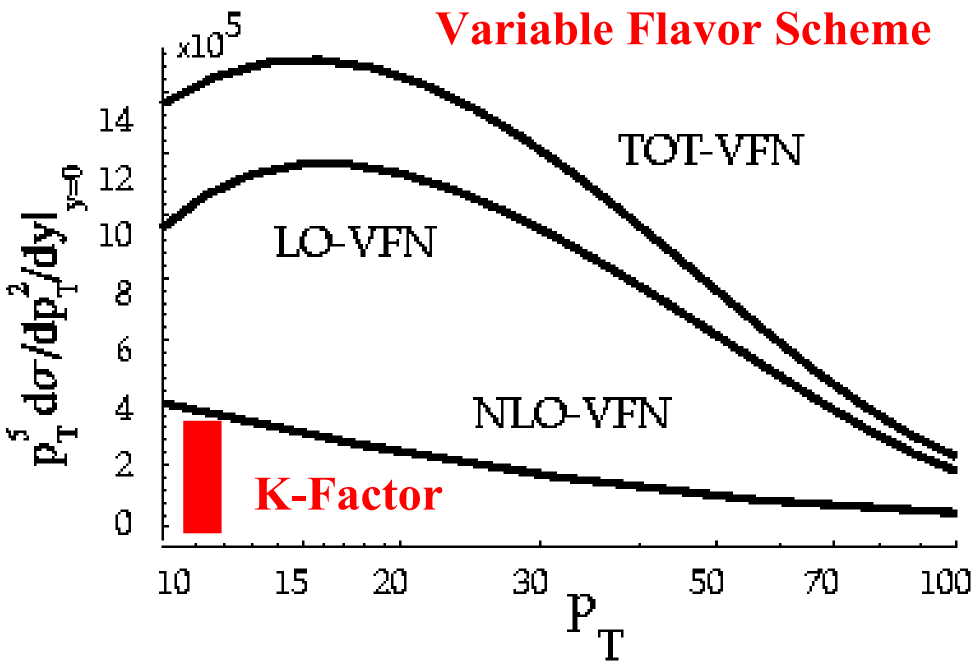
Fixed-Flavor Scheme

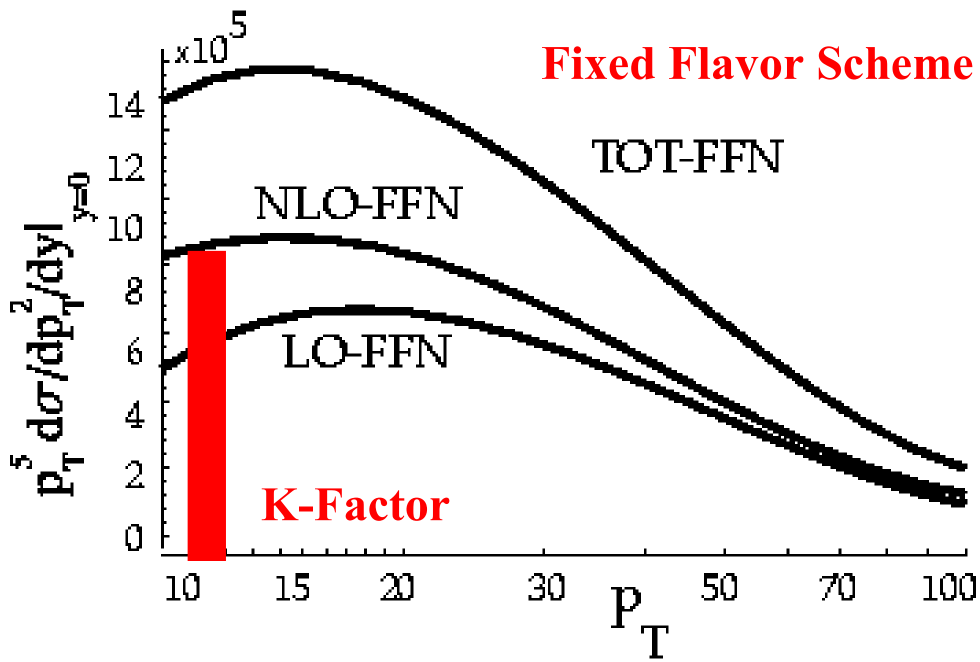
Variable-Flavor Scheme

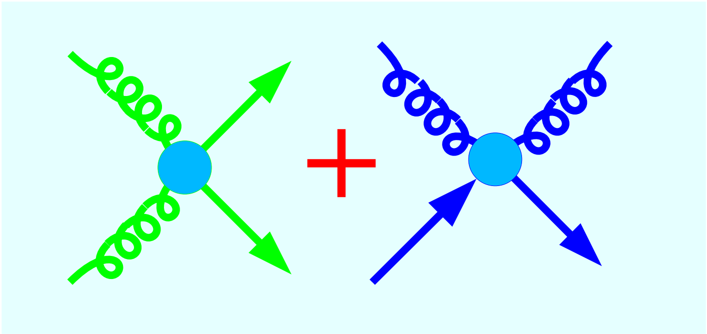
<u>Surprise</u>: NLO / LO ~ 2
But, theory still below data

Nason, Dawson, Ellis
Beenakker, Kuijf, Van Neerven, Smith

Compare Fixed & Variable Flavor Scheme







- To NLO, different schemes are comparable.
- K-Factor very different.
- Suggestion: VFS may provide more efficient organization of perturbation series than FFS.
- Recall: Choice of expansion point x_0 in Taylor series.

The Moral

You don't have to choose which expansion point you use;
by using the Heavy Quark PDF,
QCD will compensate

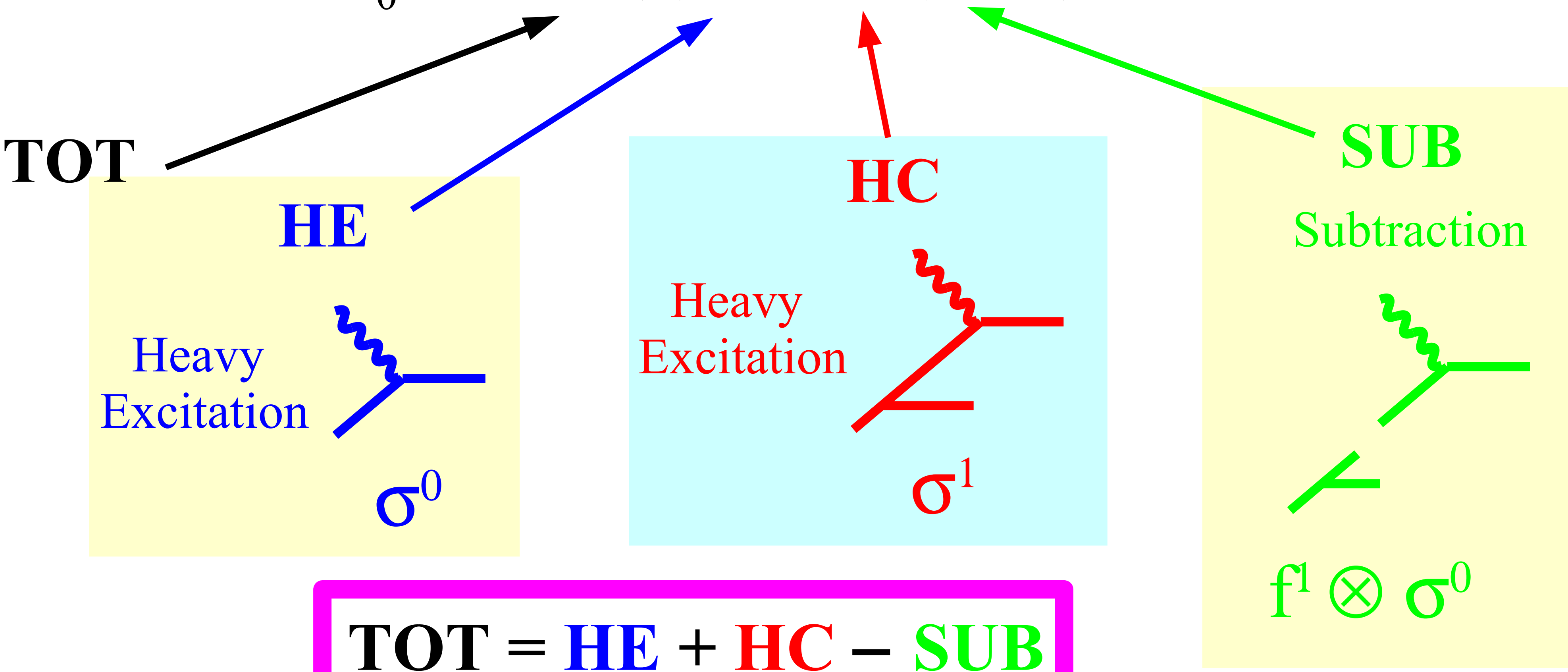
In practice ...

Using the heavy quark PDF's we can accommodate quark masses of any values: e.g., 10^{-150} to 10^{+150}

Expand $f(x)=x$ in Taylor Series about x_0.

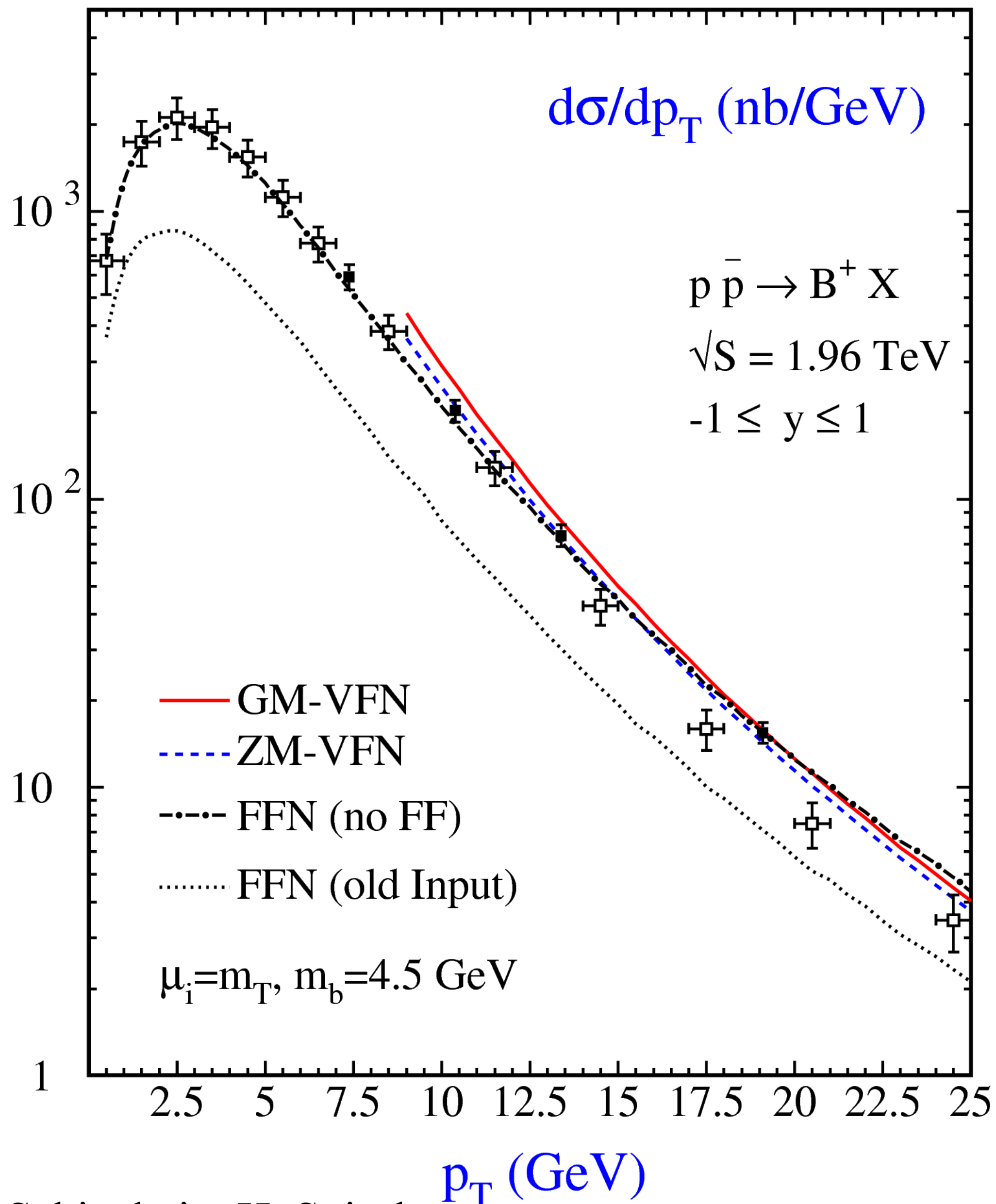
For $x_0=0$: $f(\varepsilon) = 0 + (\varepsilon - 0) + \ldots = \varepsilon$

For $x_0=1$: $f(\varepsilon) = 1 + (\varepsilon - 1) + \ldots = \varepsilon$



TOT = HE + HC – SUB

Hadroproduction of Beauty at Tevatron: Current Understanding



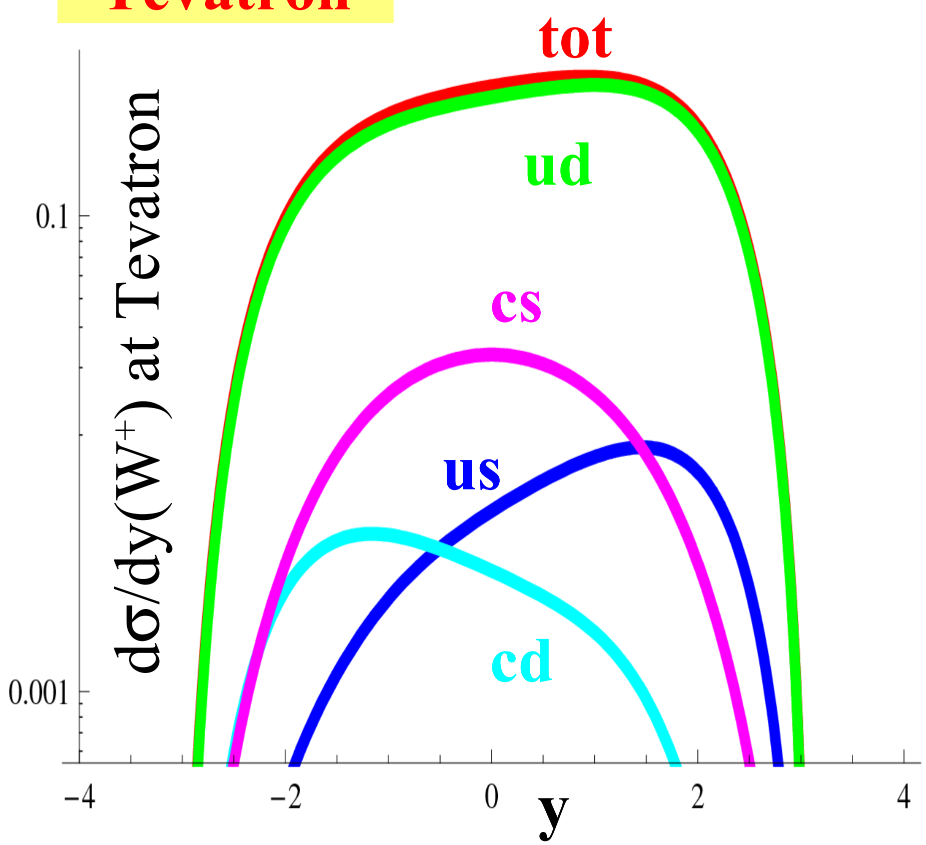
B.A. Kniehl, G. Kramer, I. Schienbein, H. Spiesberger
PhysRevD.77.014011

W/Z PRODUCTION

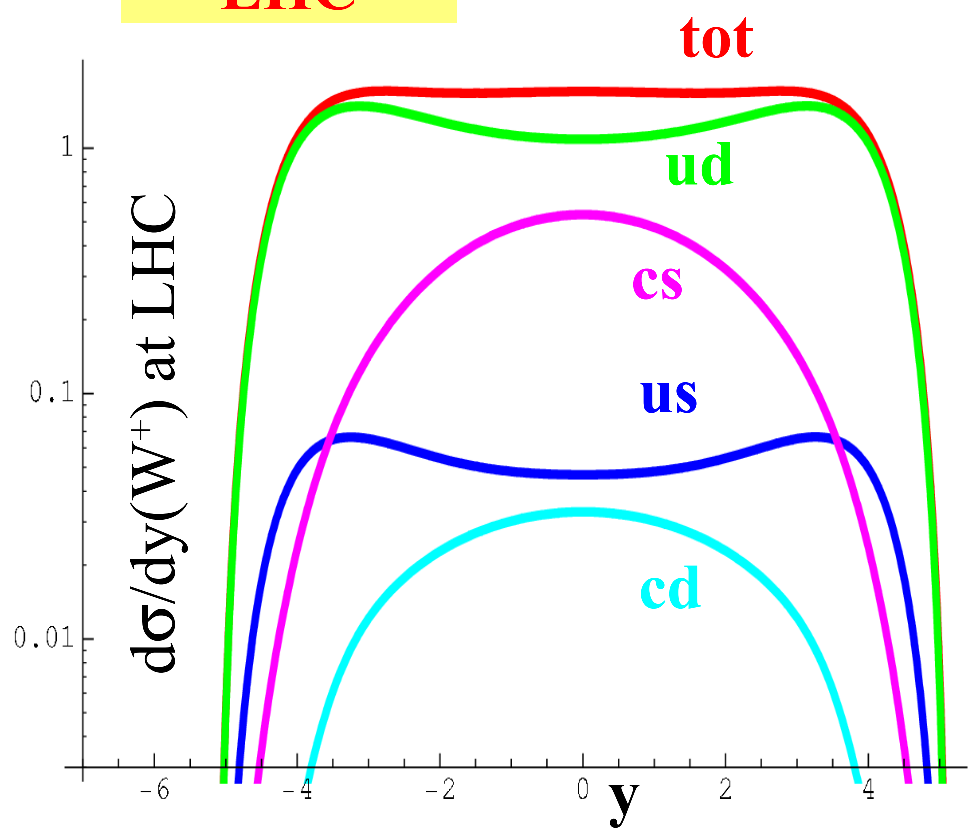
Heavy Quark contribution to W/Z Production

Heavy quark PDFs essential ingredient

Tevatron

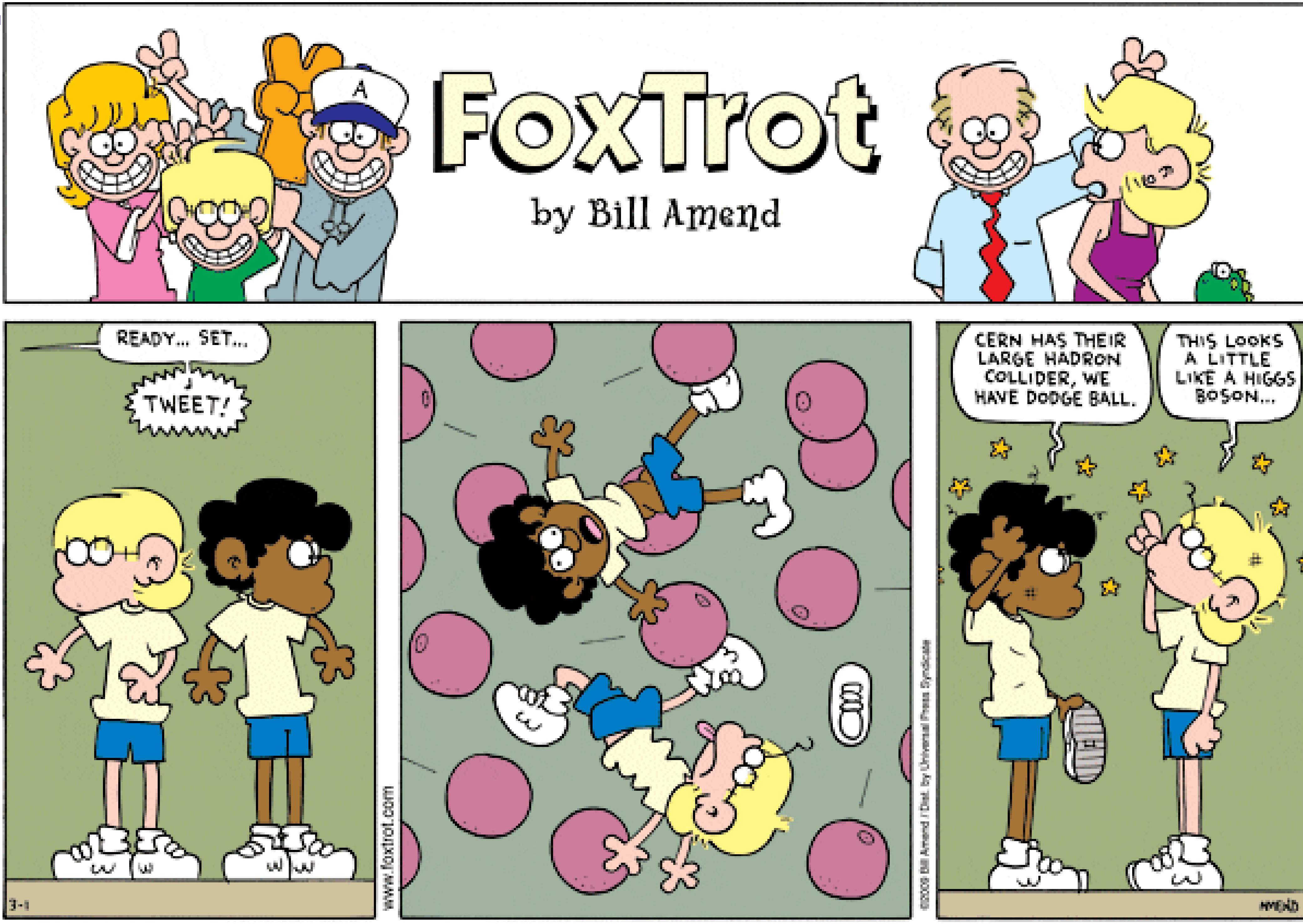


LHC

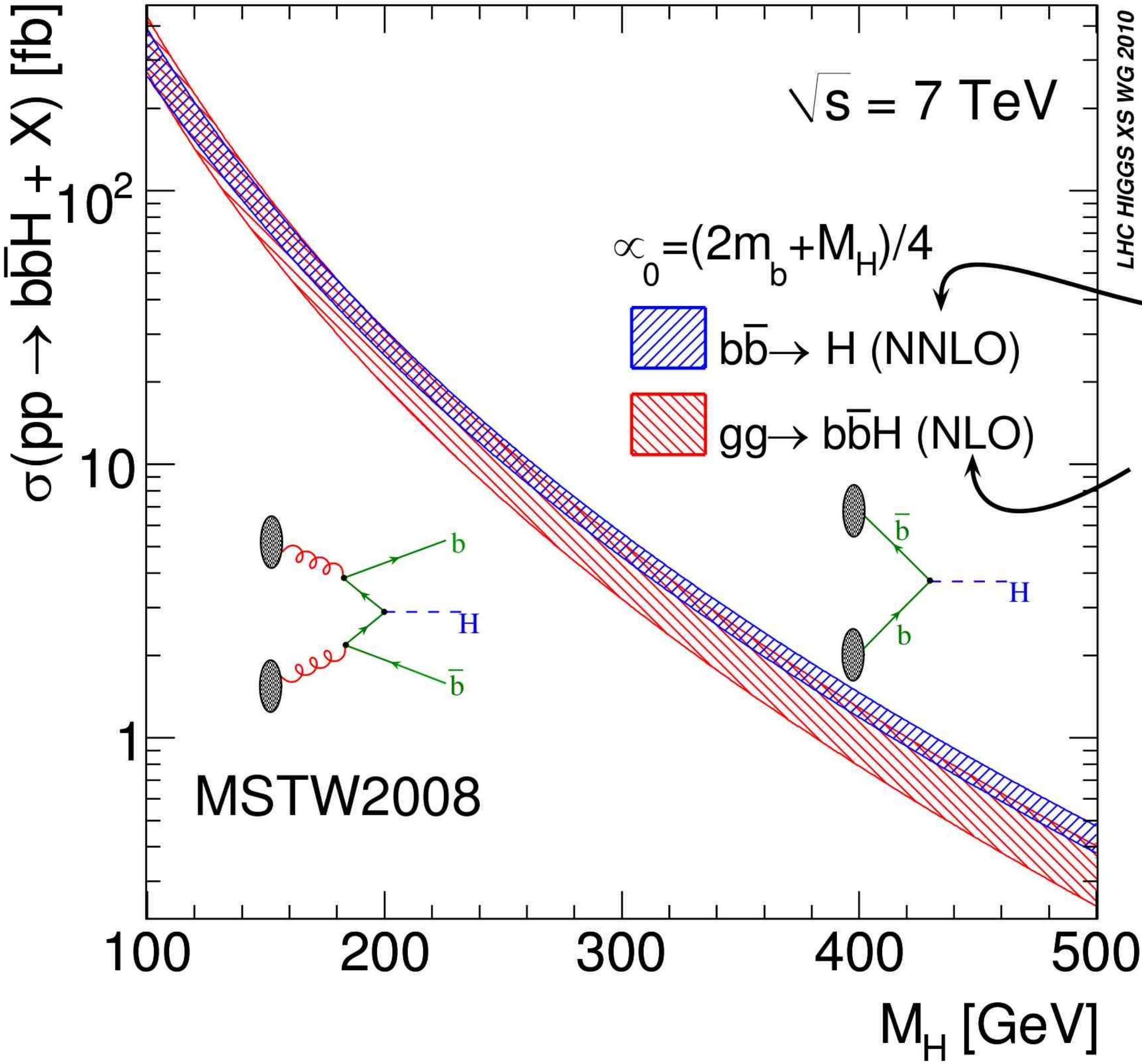


Heavy Quark components play an increasingly important role at the LHC

HIGGS PRODUCTION



Higgs Production via Heavy Quark PDFs



[RH, Kilgore '03]

[Dittmaier, Krämer, Spira '04]

[Dawson, Jackson, Reina, Wackeroth '04]

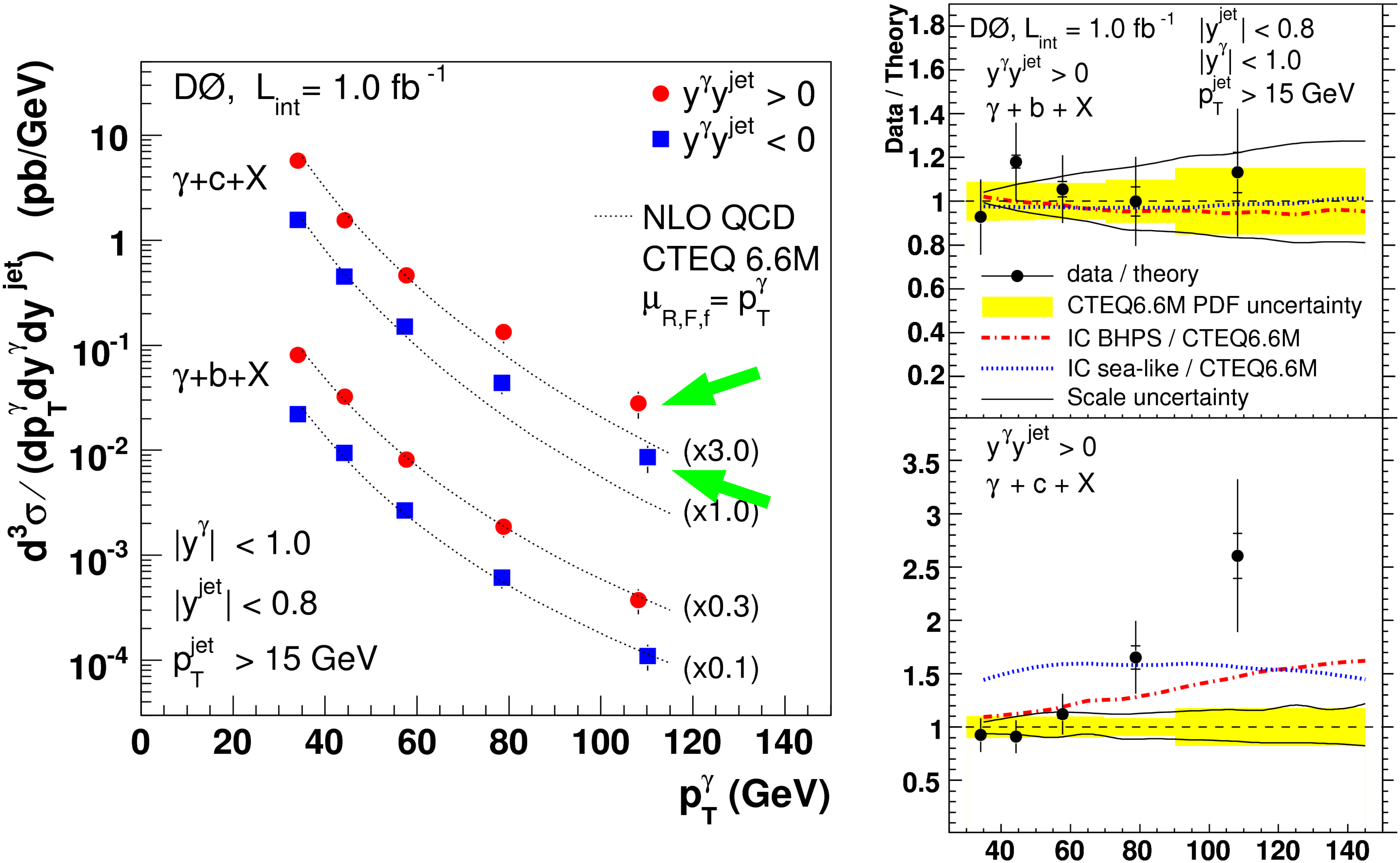
electro-weak:

[Dittmaier, Krämer, Mück, Schlüter '06]

R. Harlander (BU Wuppertal) Higgs theory at NNLO June 2011 36

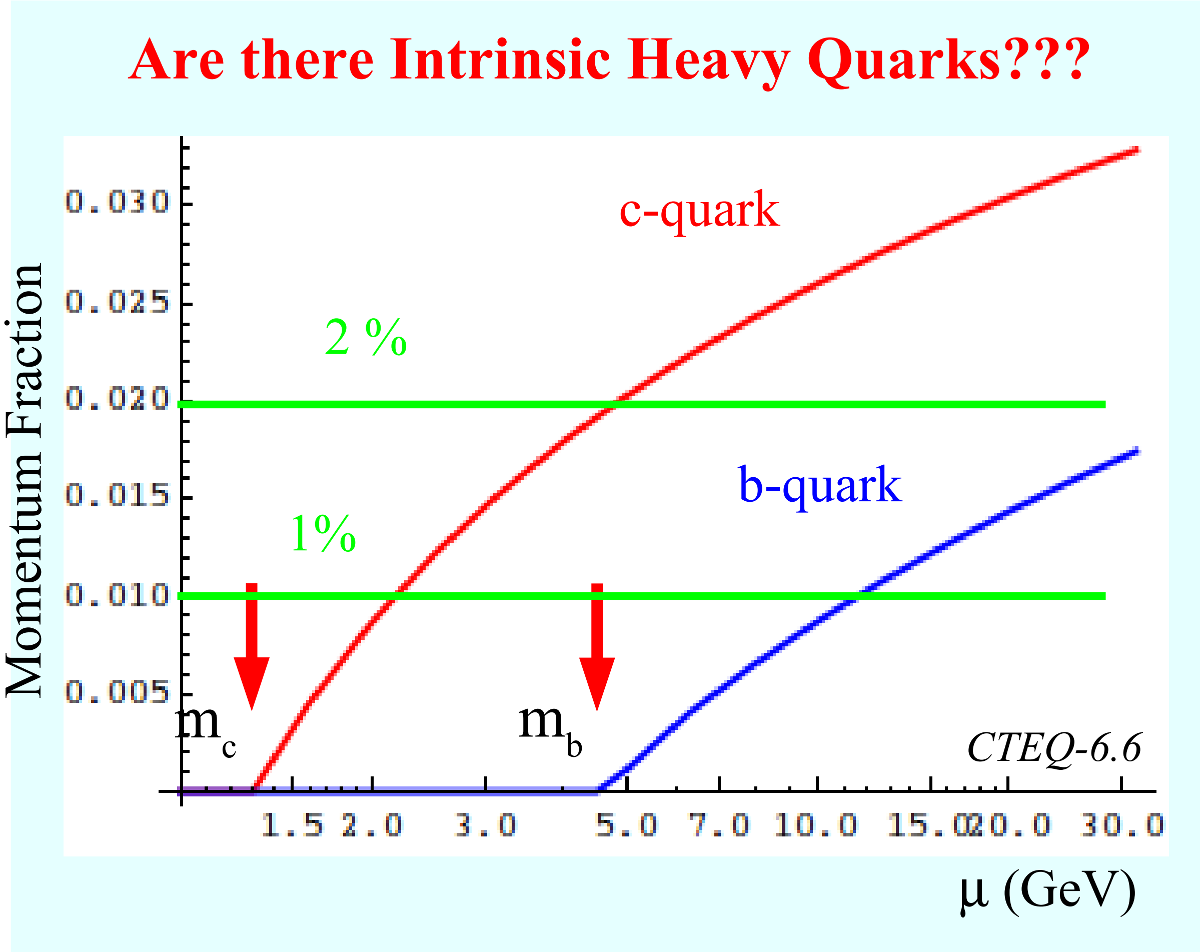
INTRINSIC CHARM & BOTTOM

Heavy Quarks at the Tevatron: γ+c and γ+b



D. Duggan (D0) arXiv:0906.0136

Are there Intrinsic Heavy Quarks??? Do they matter???

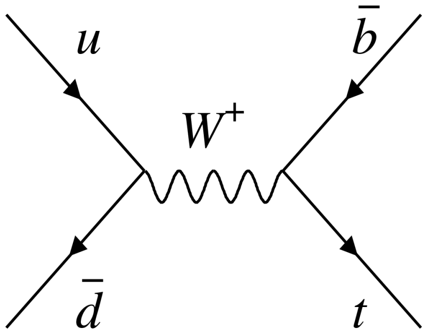


* Most sensitive near threshold
* What happens if we allow the evolution to determine charm?

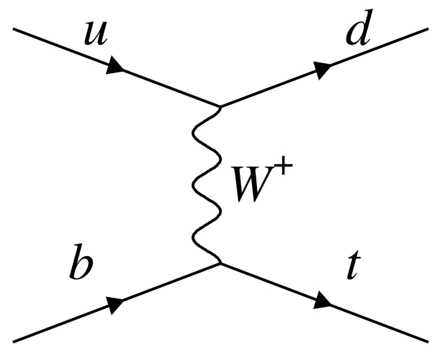
Zero:	No intrinsic charm
Positive:	Intrinsic charm
Negative:	Inconsistent

SINGLE TOP

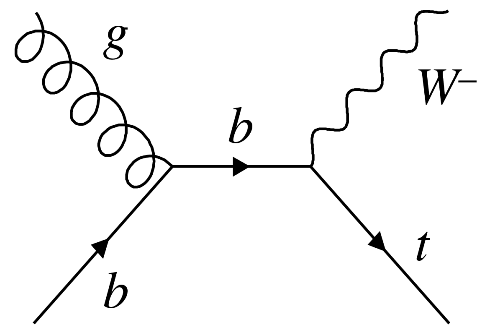
Single Top



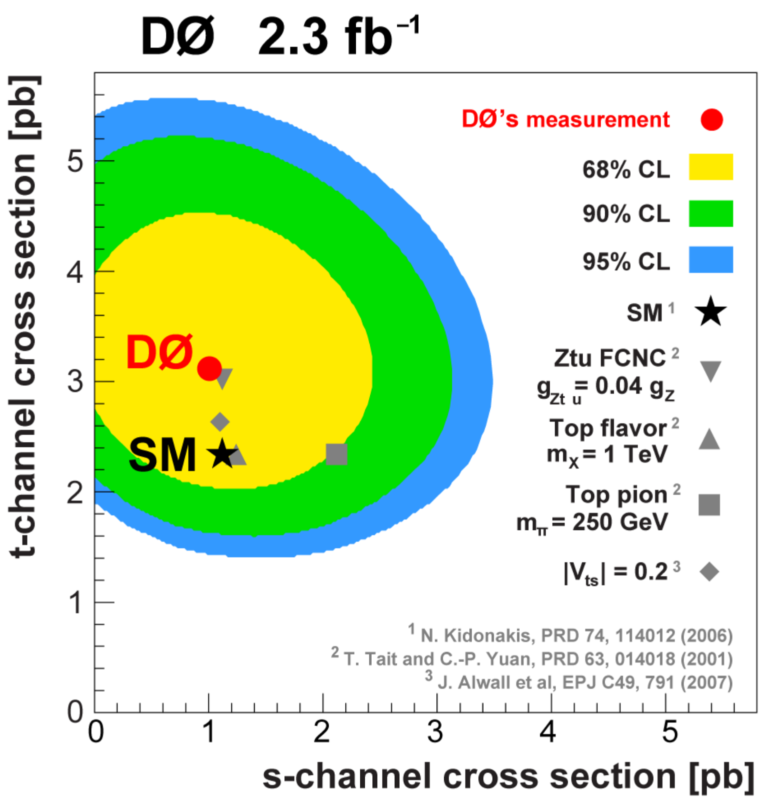
s-channel

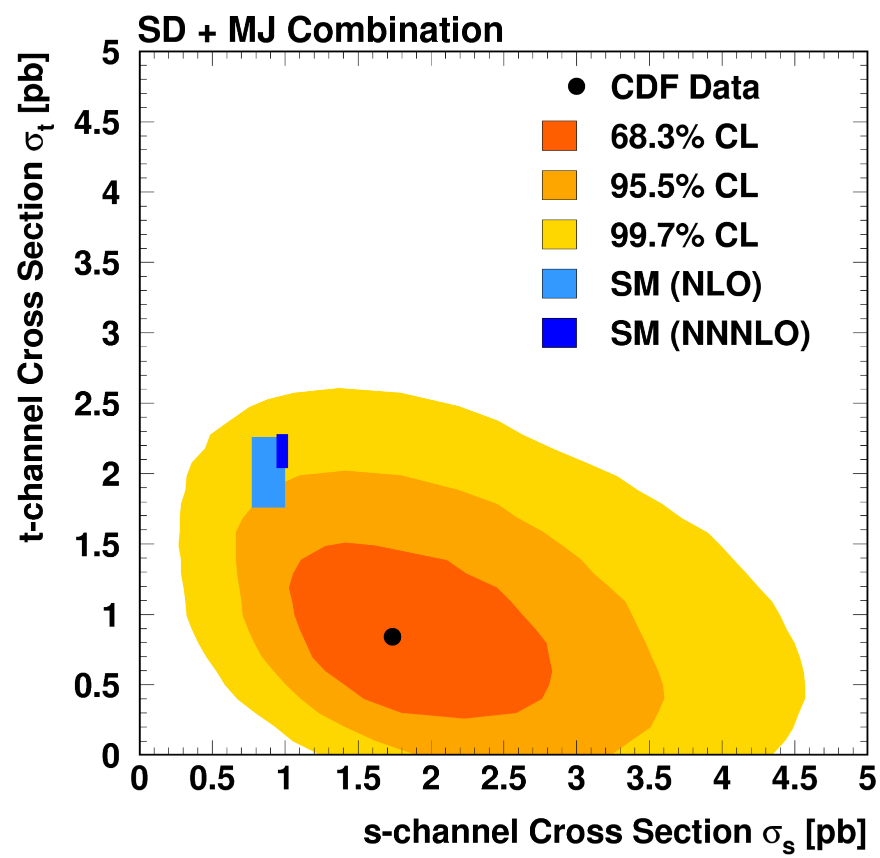


t-channel



associated W

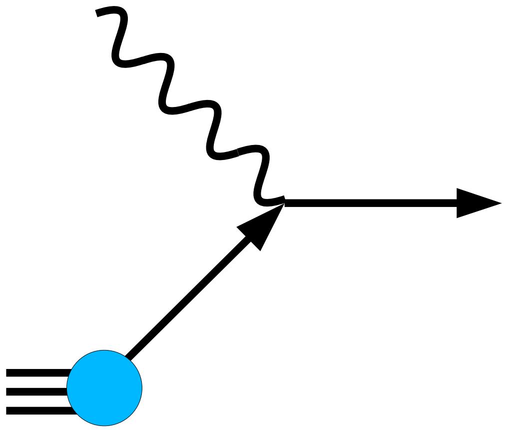


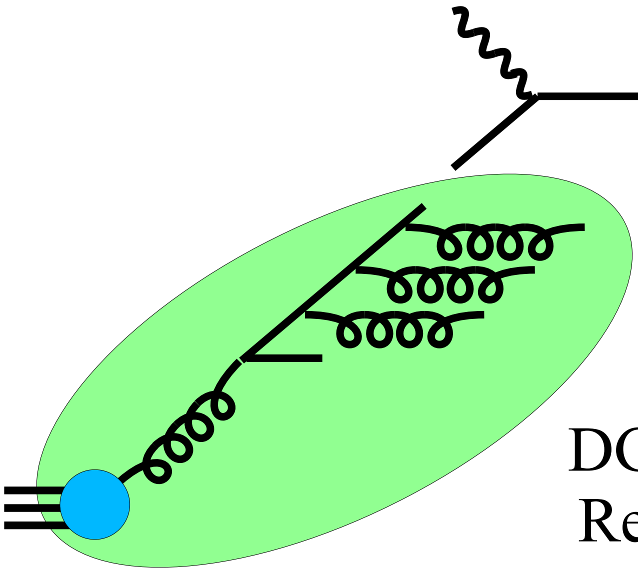


Mass-Independent Evolution.

Why is it valid?

DGLAP Equation and the Heavy Quark PDF





DGLAP equation
Resums iterative
splittings inside
the proton

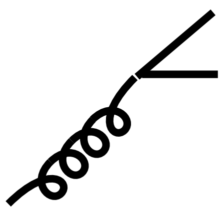
$$HE = \int f(P \to a) \otimes \sigma(a \to c)$$

DGLAP Equation

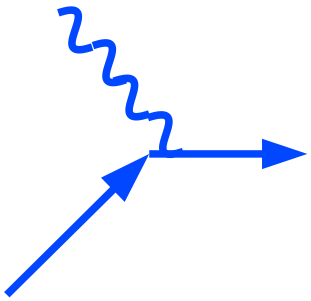
$$\frac{df_i}{d\log\mu^2} = \frac{\alpha_s}{2\pi} \, {}^1P_{j\to i} \otimes f_j + \ldots$$

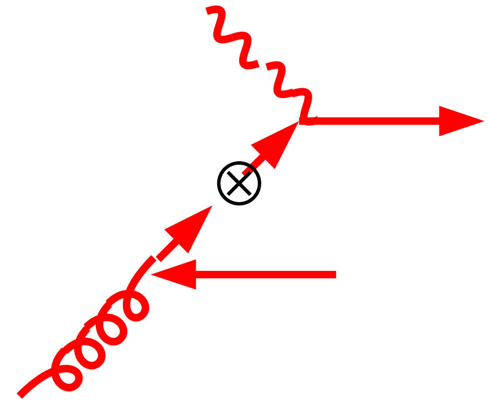
Splitting Function

$${}^1P_{g\to q} = \frac{1}{2}\left[x^2 + (1-x)^2\right] + \left(\frac{M_H^2}{\mu^2}\right)\left[x(1-x)\right]$$



Effect of Heavy Quark Mass in the Calculation





$$HE = \int f(P \to a) \otimes \sigma(a \to c)$$

$$\approx f(P \to g) \otimes {}^{1}P(g \to a)$$

valid near threshold ($M_H \sim Q$)

$$SUB = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

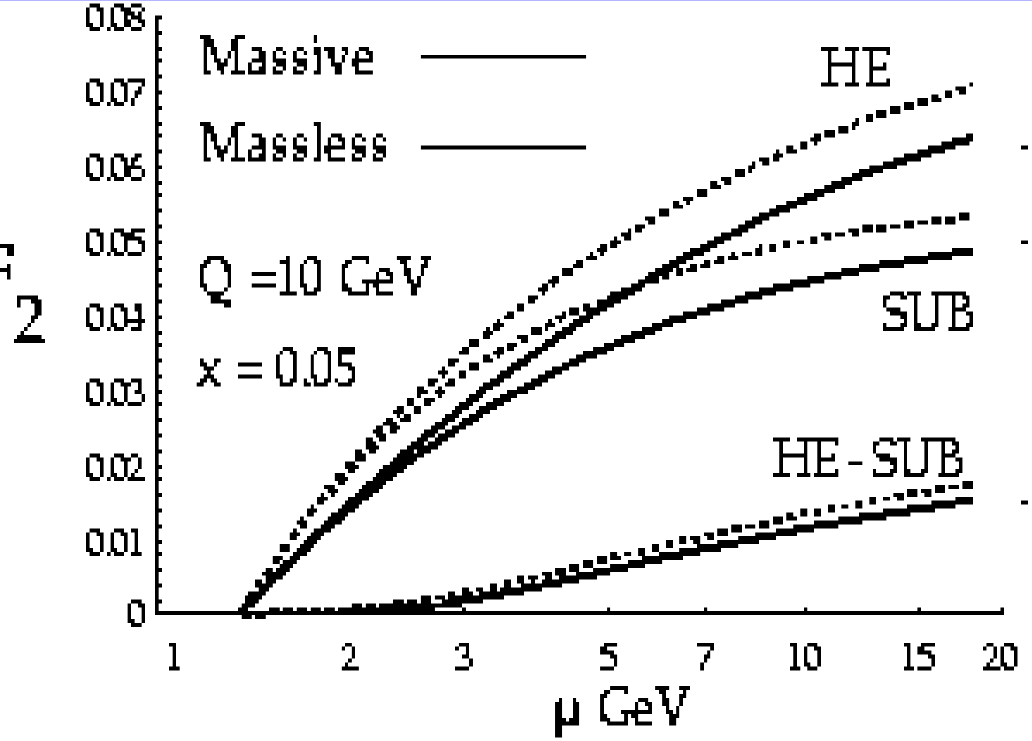
${}^{1}P$ splittings must match

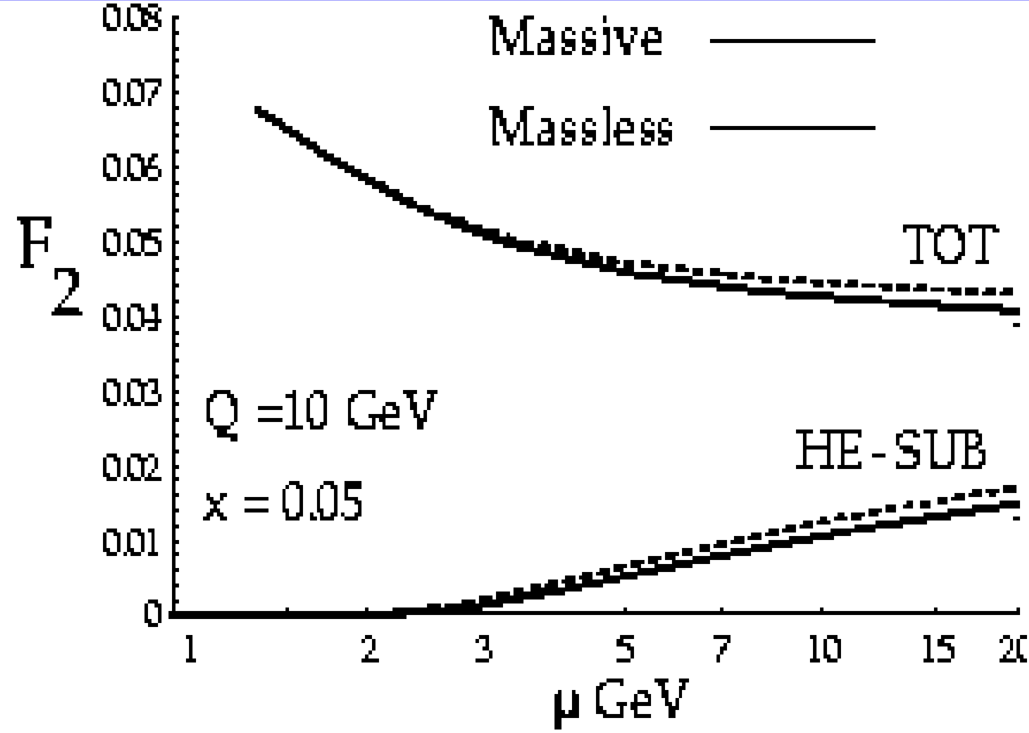
In Summary:

Near threshold($M_H \sim Q$), mass effects cancel between HE and SUB

Above threshold($M_H \ll Q$), mass effects can be ignored

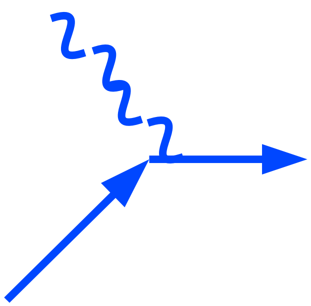
Effect of Heavy Quark Mass in the Calculation is Trivial

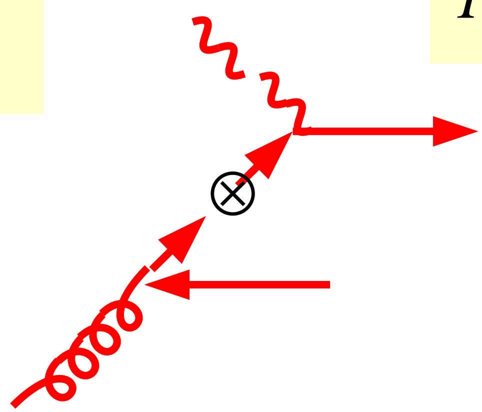


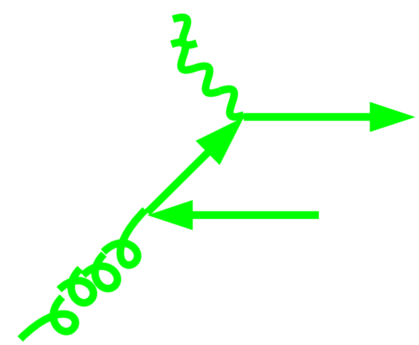


$$HC = \int f(P \to g) \otimes \sigma(g \to c)$$

$$HE = \int f(P \to a) \otimes \sigma(a \to c)$$

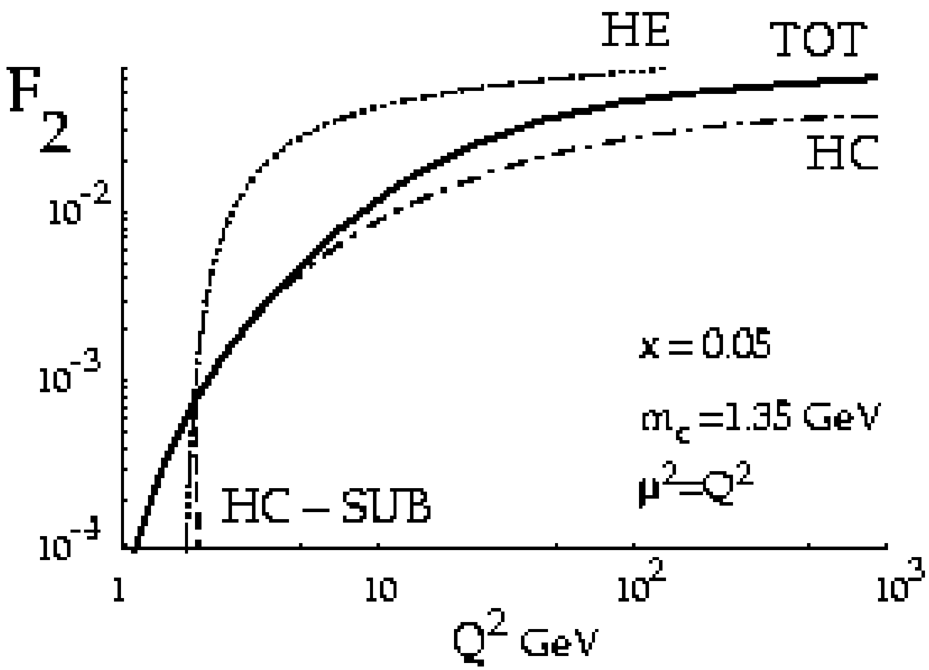


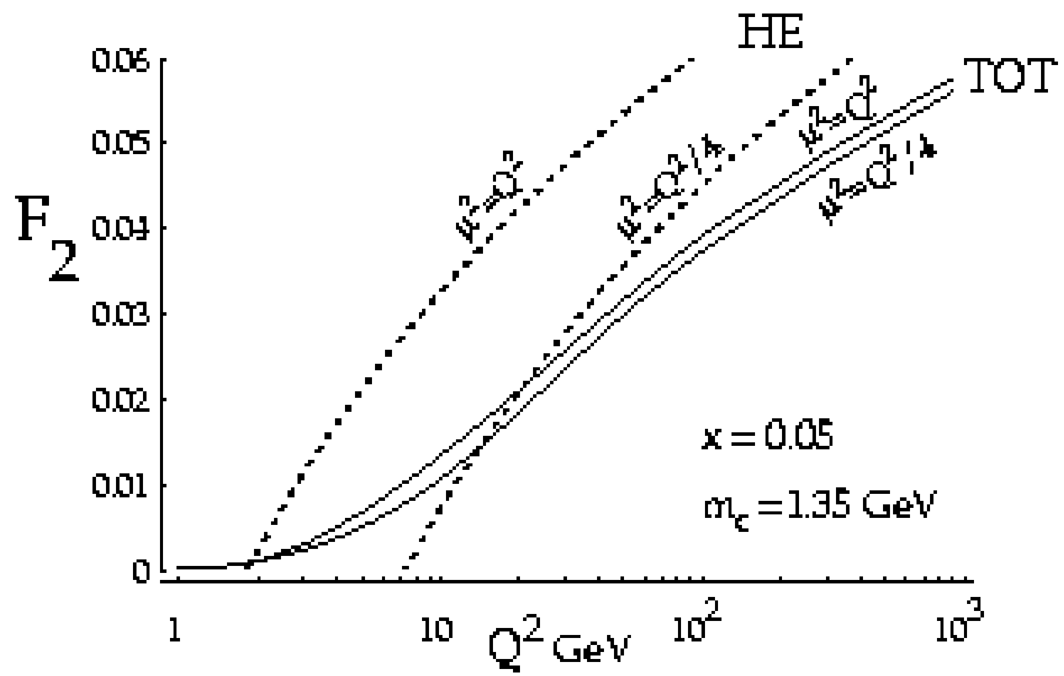




$$SUB = \int f(P \to g) \otimes {}^{1}P(g \to a) \otimes \sigma(a \to c)$$

Variation of σ vs. renormalization scale μ





LO = HE result is very sensitive to the choice of scale (i.e., $\mu^2=Q^2$ or $Q^2/4$)
TOT result (higher order) is stable w.r.t. the choice of scale

An accurate calculation must be stable as the renormalization scale varies

Outlook & Conclusions

The race is not always to the swift, nor the battle to the strong -
but that's the way to bet.

Runyon's Law

An interesting subject because:

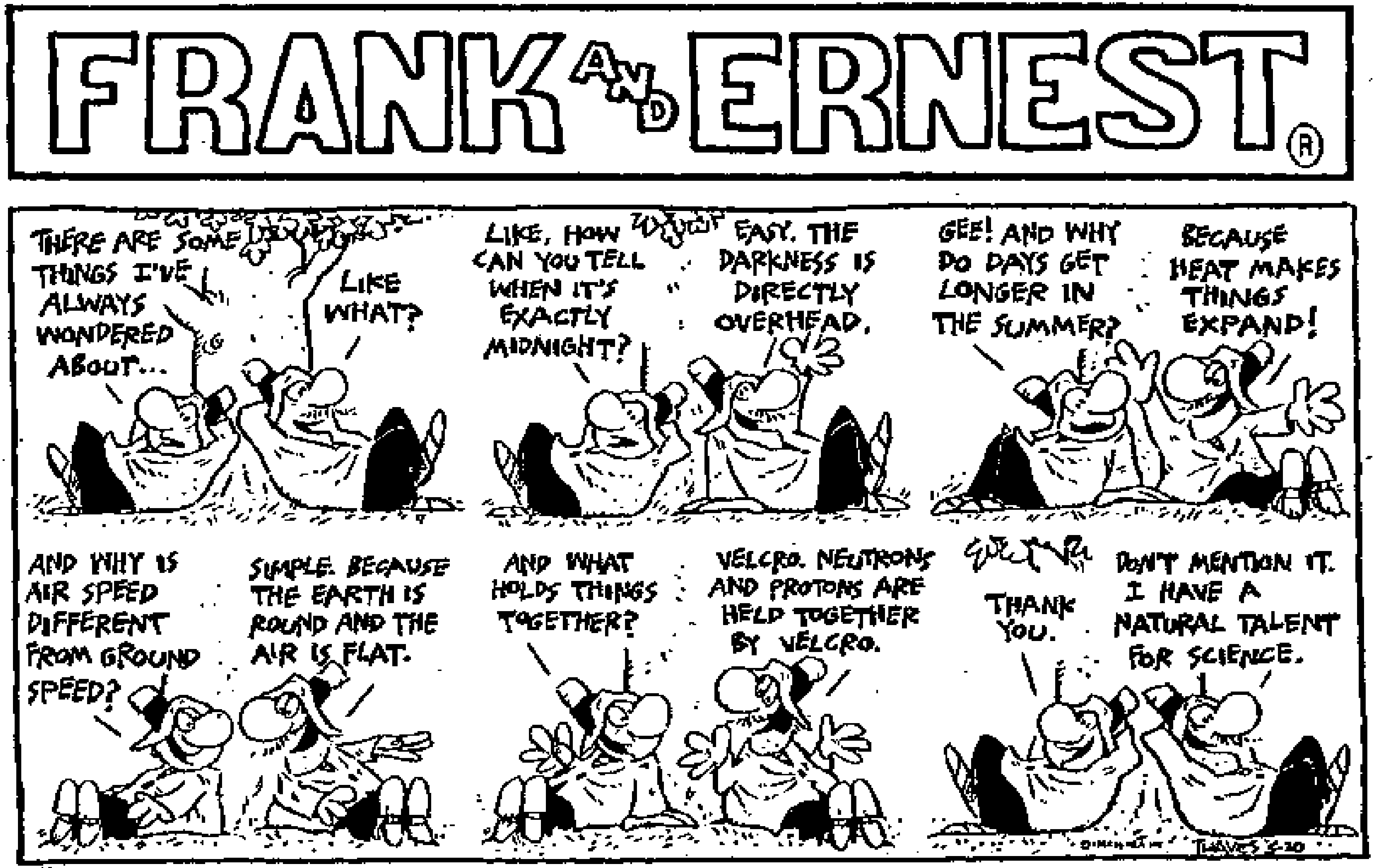
- Lots of data at present; more in near future
- Theoretical issues of multi-scale problem

A fascinating subject because:

- Theory & data not fully consistent
- This should be a region we can compute

Fred Olness olness@smu.edu

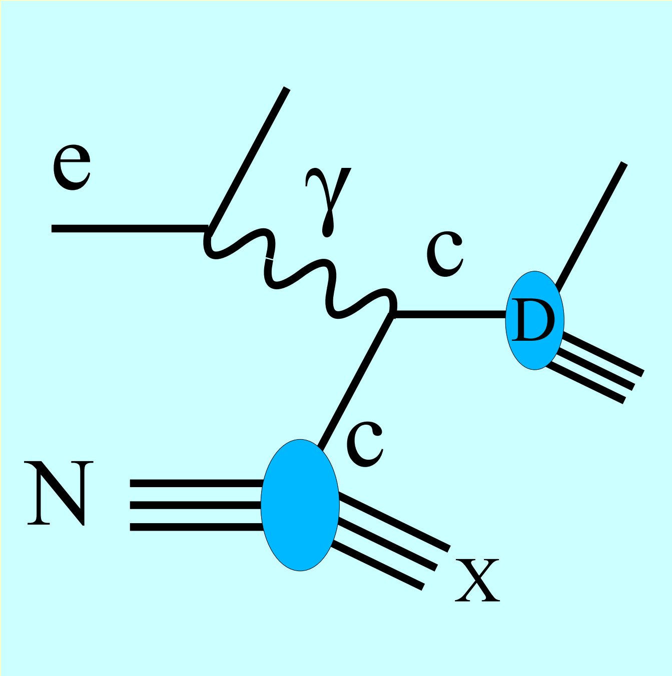
Keep an open mind!!!



Page 2

LEFTOVERS

Heavy Quarks



Dynamics & Kinematics

Effect of Kinematic Mass Re-Scaling

ACOT (Aivazis, Collins, Olness, Tung) A general framework for including the heavy quark components.

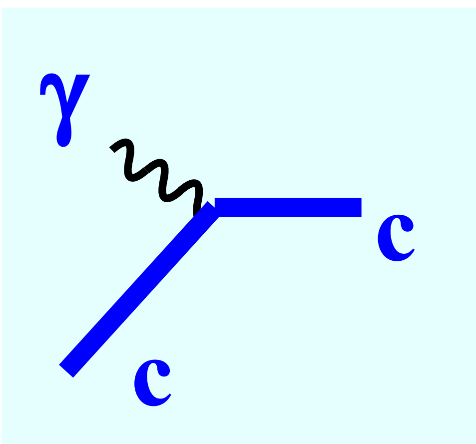
Phys.Rev.D50:3102-3118,1994.

S-ACOT (Simplified-ACOT) ACOT with the initial-state heavy quark masses set to zero.

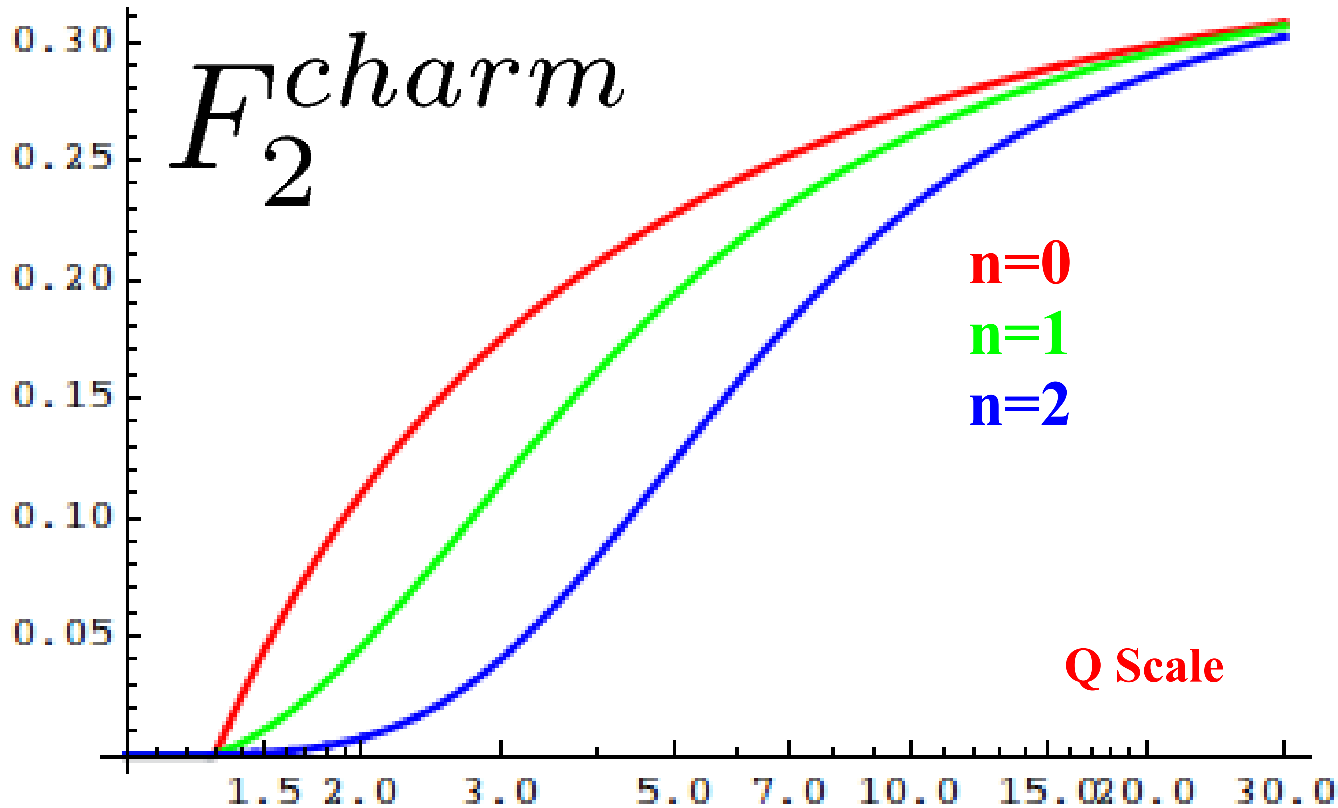
Phys.Rev.D62:096007,2000.

ACOT-χ & **S-ACOT-χ:** As above with a generalized slow-rescaling

Phys.Rev.D62:096007,2000.

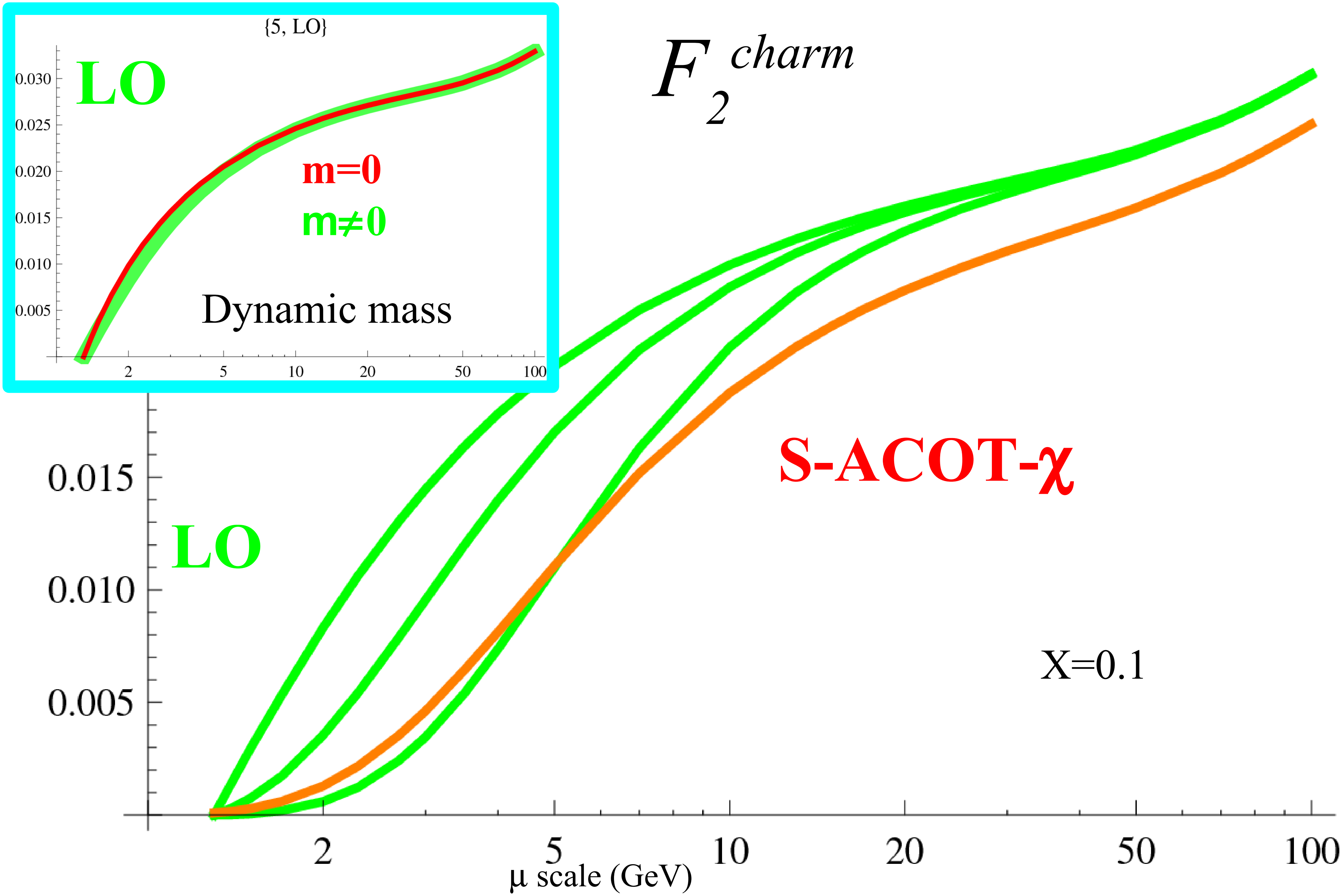


$$\chi = x\left[1 + \frac{(\mathbf{n} m_c)^2}{Q^2}\right]$$



Kinematic Masses are more important than Dynamical Masses ***(in general)***

F_2 Charm in the threshold region



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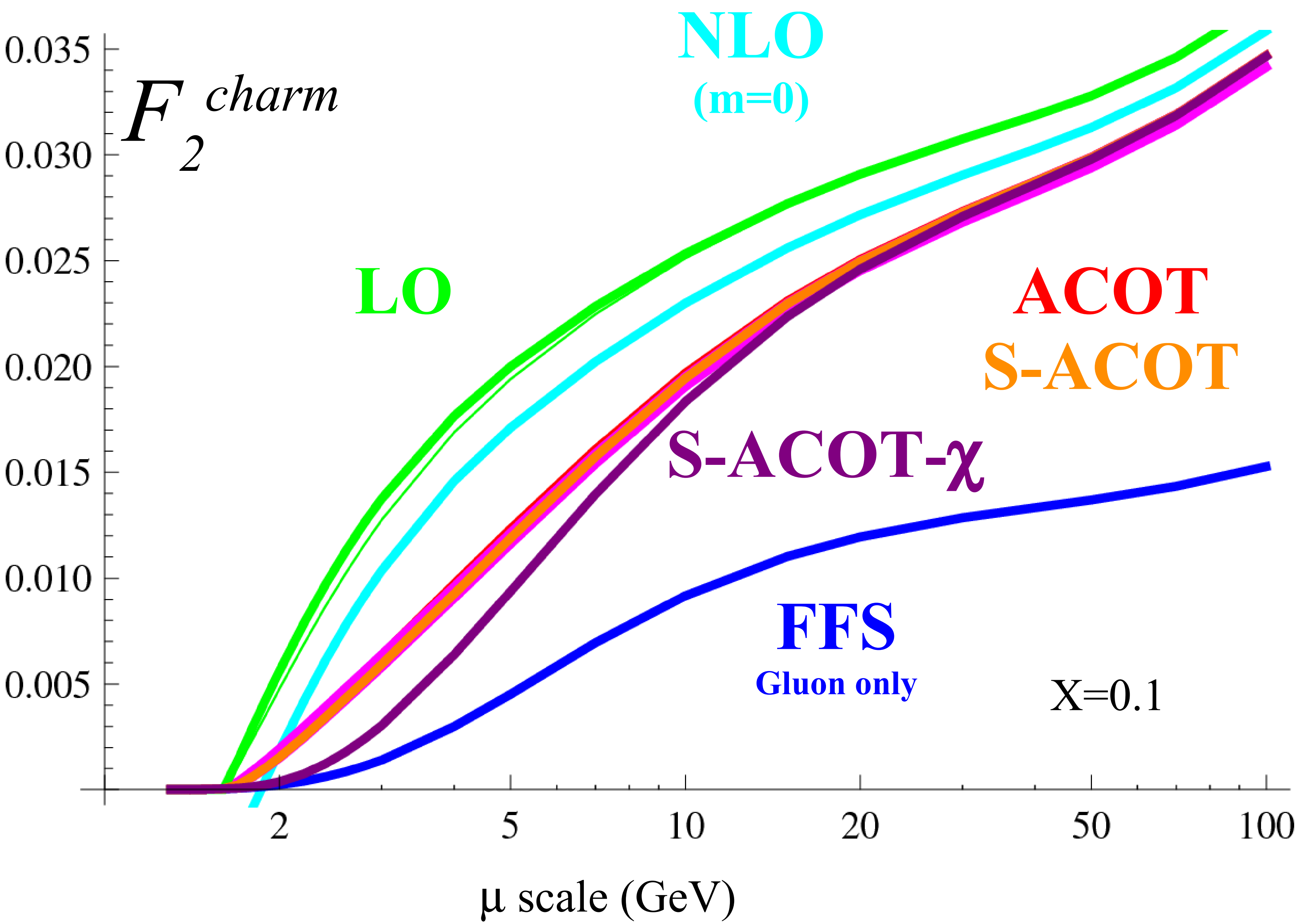
F_2 Charm in the threshold region



A man with one watch knows what time it is; a man with two is never sure.

Les Houches 2009

Comparative Studies

Physics at TeV Colliders

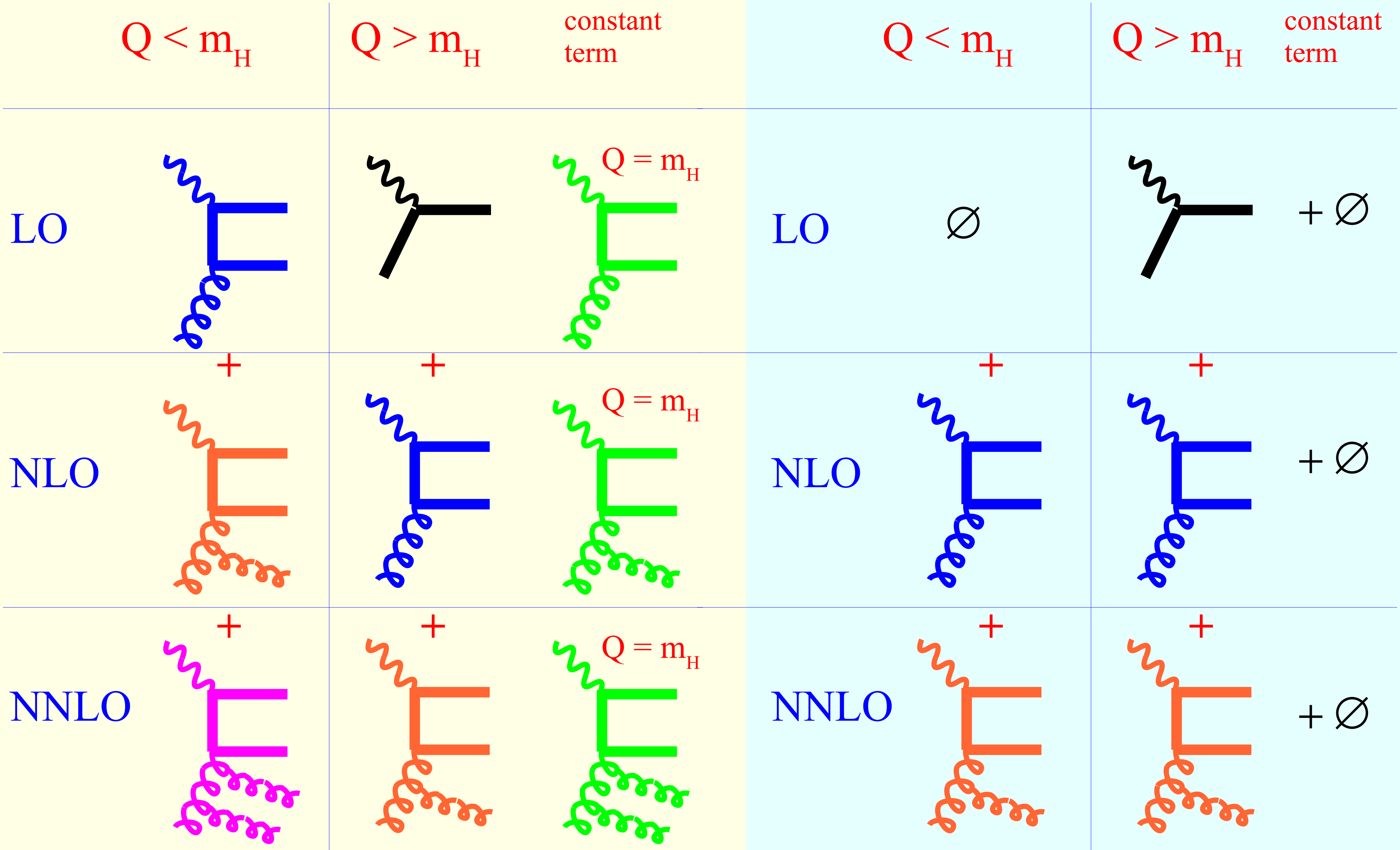
Les Houches 8-26 June 2009

LES HOUCHES

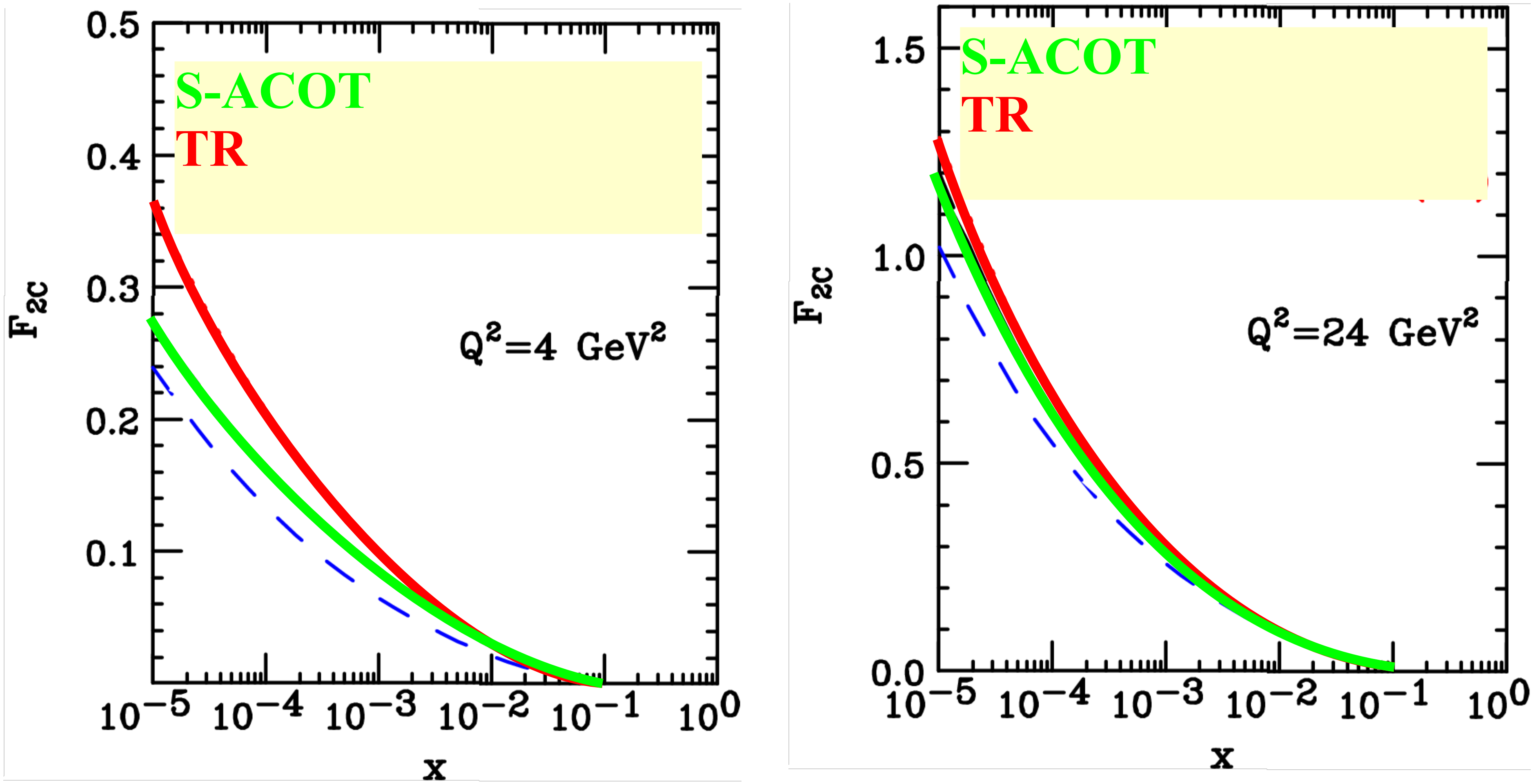
centre de physique

Schematic Summary of ACOT & TR Schemes

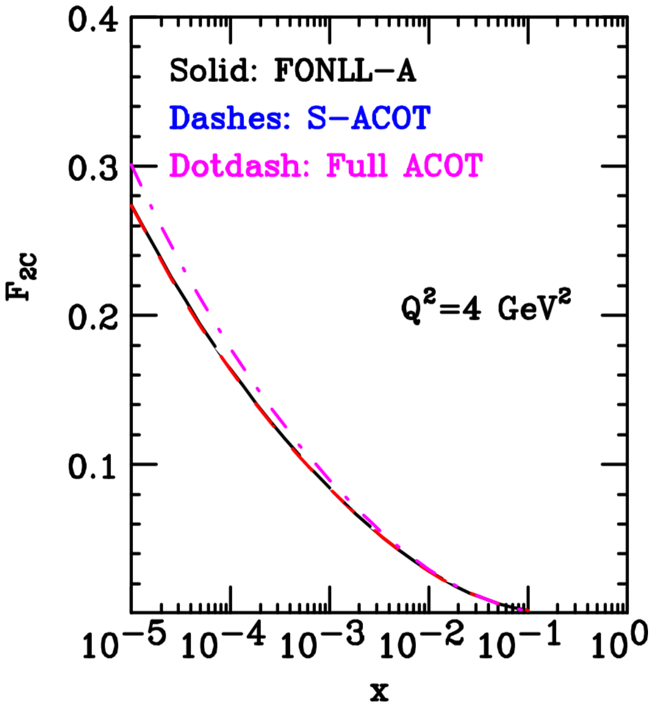
TR type schemes ACOT type schemes

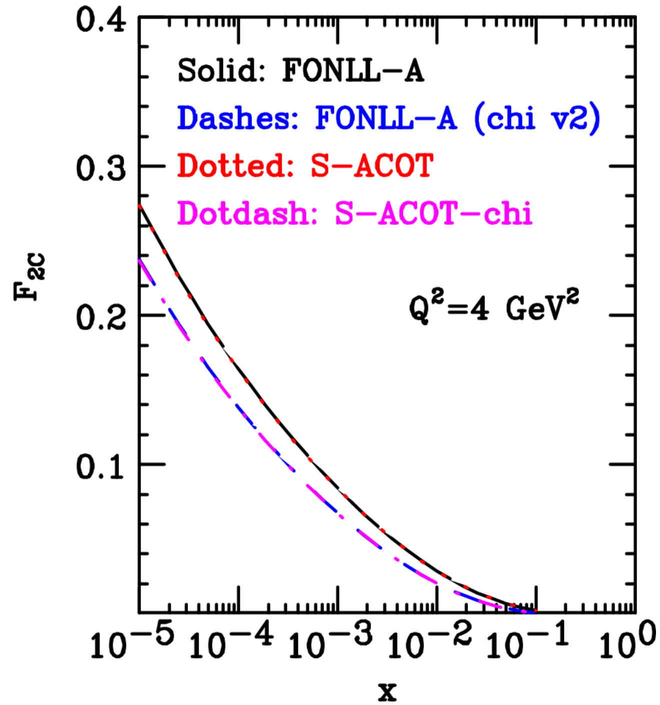


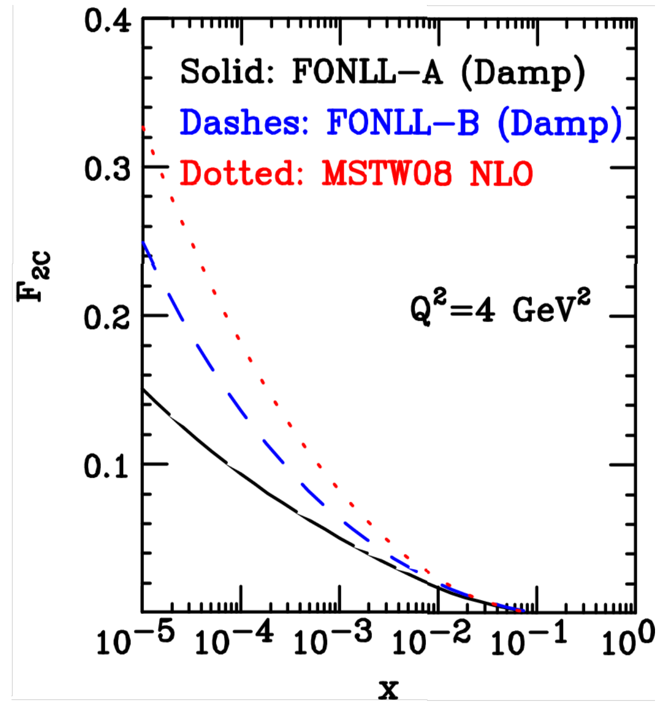
Comparison of ACOT & TR Schemes



Les Houches Comparative Study







ACOT & S-ACOT
essentially identical

... it's all in the kinematics

FONNL & S-ACOT
numerically similar

chi(χ) prescription enforces threshold

MSTW09
uses different
threshold definition

different scheme
different intermediate result

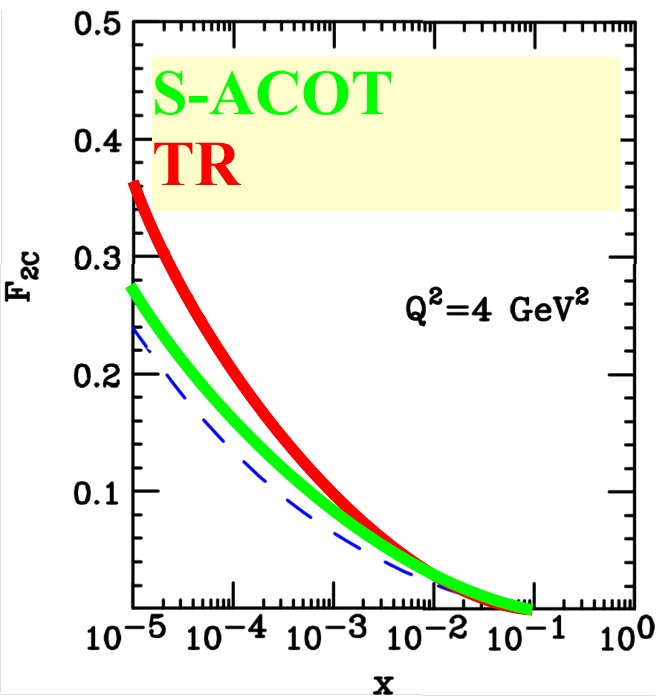
A comment about schemes

Essential to match PDF with (hard) cross section in proper schemes!!!

		Consistent Schemes		Mixed Schemes	
Set	# pts	6HQ	6M	6M⊗GM	6HQ⊗ZM
ZEUS	104	0.91	0.98	2.84	3.72
H1	484	1.02	1.04	1.50	1.22
TOTAL	1925	1.04	1.06	1.26	1.30

χ^2/DOF

$\delta\chi^2 \approx 420$ $\delta\chi^2 \approx 500$



Just because the PDFs or (hard) cross sections do not match, for a consistent scheme, the physical observable should be invariant to $O(\alpha_S^{N+1})$