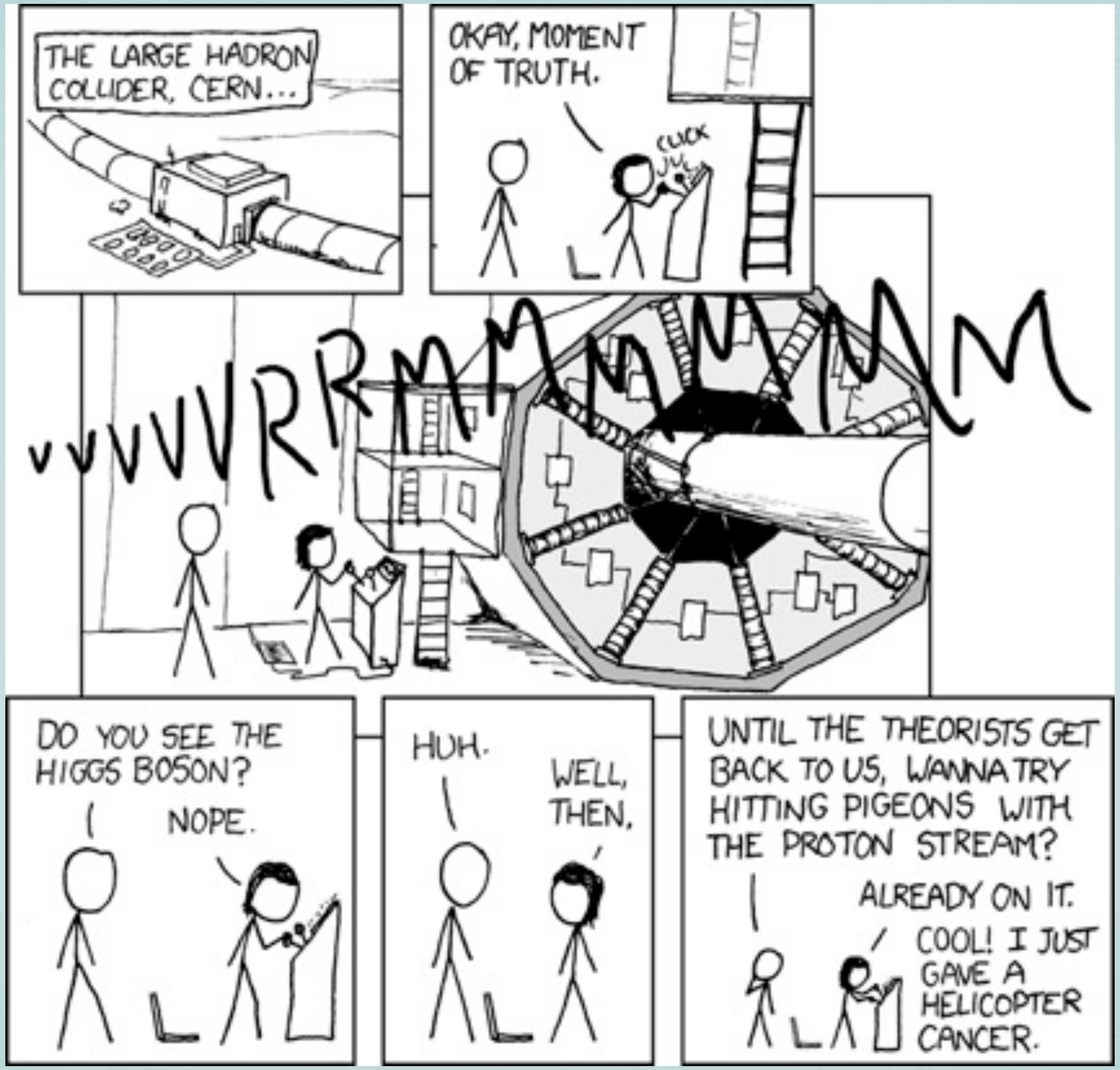
SEARCHING FOR THE HIGGS BOSON



2011 CTEQ Summer School
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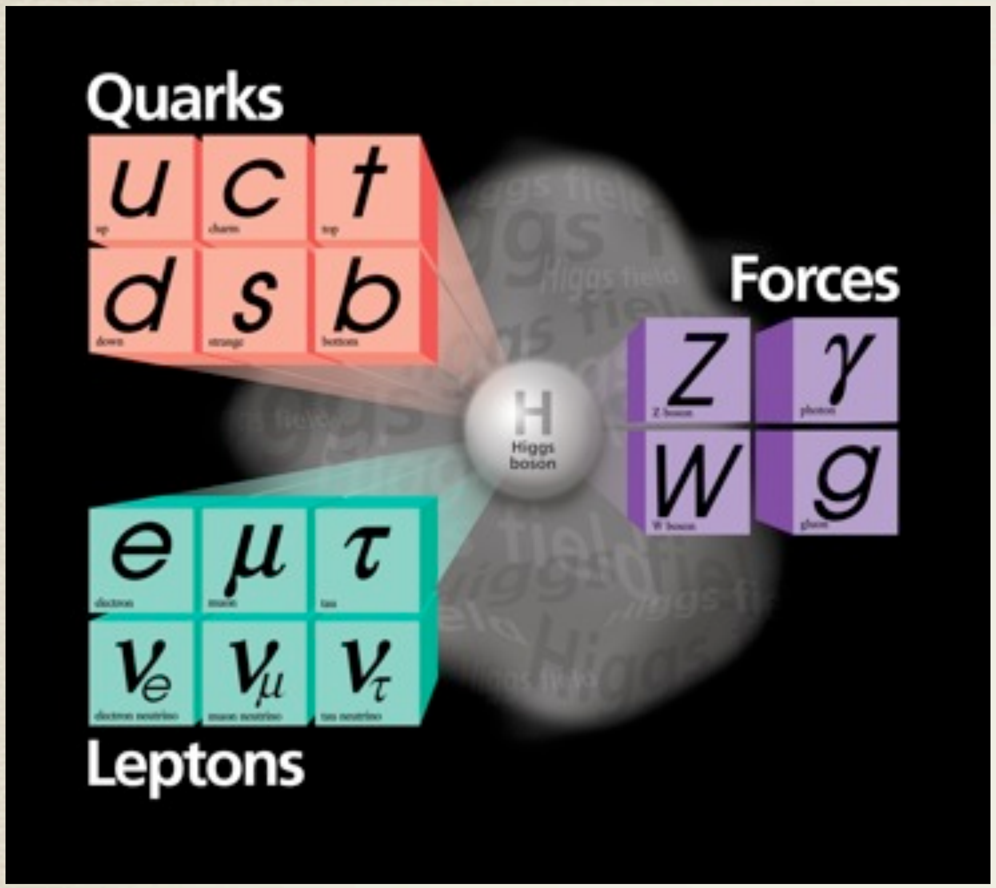
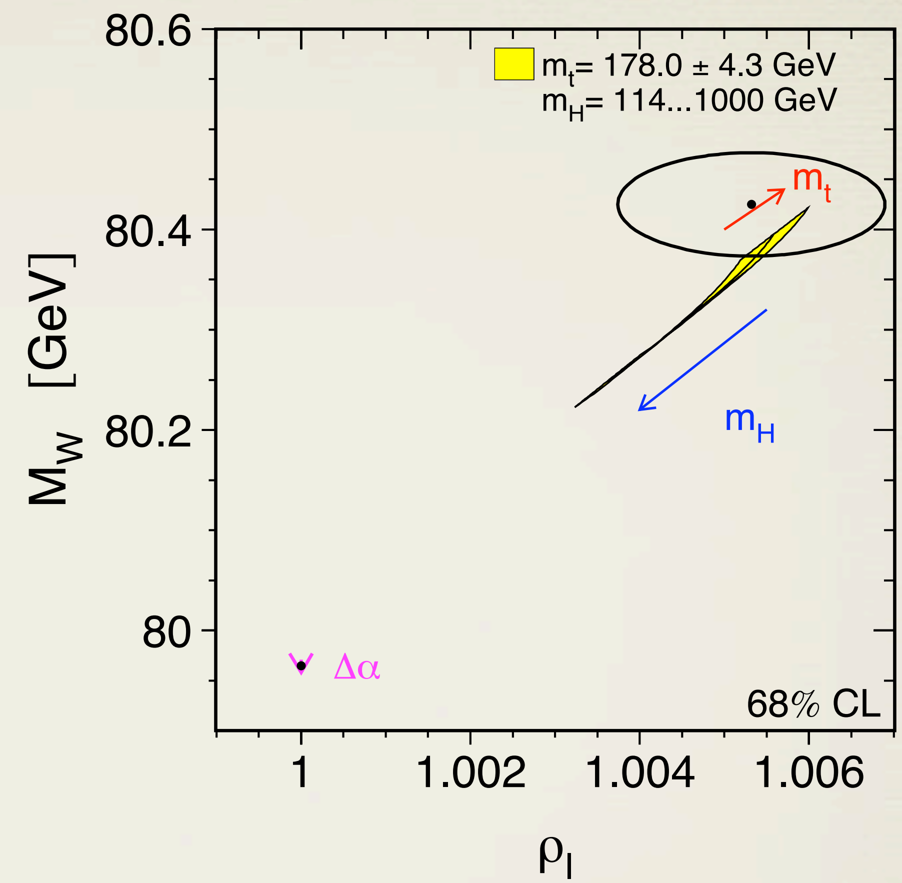
Outline

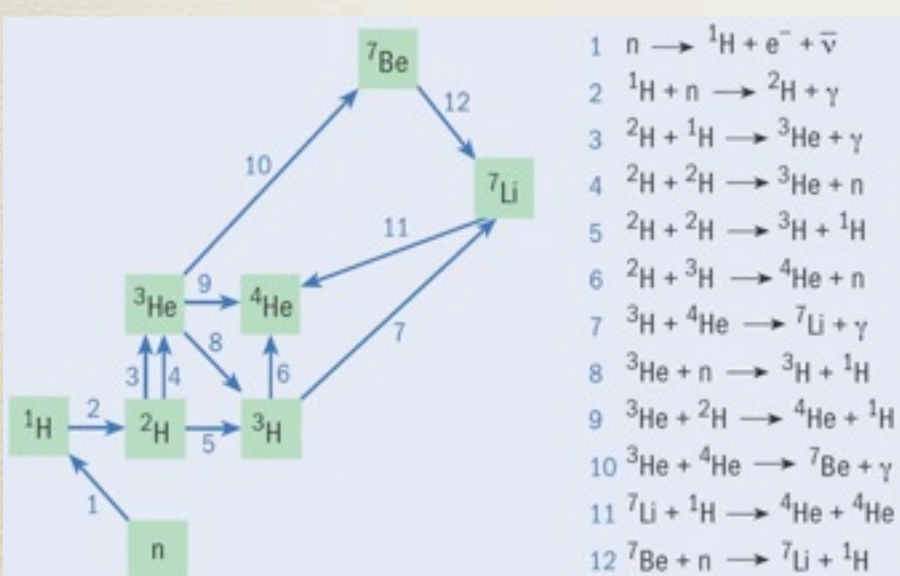
- Quick review of the SM and the Higgs mechanism
- Constraining the Higgs: theoretical constraints and electroweak precision
- A phenomenological profile: decays of the Higgs boson
- Production mechanisms at e^+e^- and hadron colliders
- Searches at the Tevatron and the LHC
- Appendix: gluon-fusion Higgs production, a case study in QCD

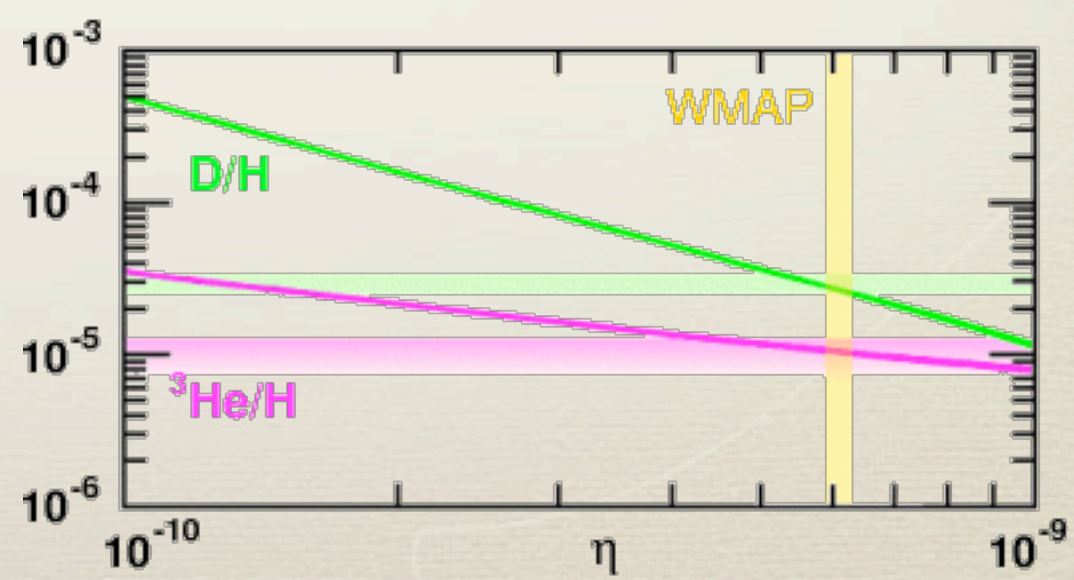
Mostly SM, but will try to mention possible deviations

Success of the Standard Model









Building a gauge theory

- Guiding principle in construction of SM is *gauge symmetry*
- Pick a gauge group
- Assign matter fields (fermions, scalars) to a *representation* of the gauge group, e.g., the *fundamental* N-component vector for SU(N)
- To make the matter Lagrangian gauge invariant, replace $\partial_\mu \rightarrow D_\mu$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + \mathcal{L}_{matter}\left(\Psi, D_\mu\Psi\right)$$

gives Feynman rules
for gauge self-interactions

governs gauge-matter
interactions

Problems with mass

- The Lagrangian of the SM:

$$
\begin{aligned}
\mathcal{L}_{gauge+ferm} &= -\frac{1}{4}\overbrace{B_{\mu\nu}B^{\mu\nu}}^{U(1)_Y} - \frac{1}{4}\overbrace{W^a_{\mu\nu}W_a^{\mu\nu}}^{SU(2)_L} - \frac{1}{4}\overbrace{G^a_{\mu\nu}G_a^{\mu\nu}}^{SU(3)_C} \\
&+ \underbrace{\sum_f i\bar{f}\,\not{D}f}_{f=Q_L,u_R,d_R,L_L,e_R}
\end{aligned}
$$

- We know the $W^\pm$, Z bosons have mass, but this is not allowed by gauge symmetry

$$
\mathcal{L}^{SU(2)}_{mass} = \frac{1}{2}m^2 W^a_\mu W^\mu_a \Rightarrow \Delta\mathcal{L}^{SU(2)}_{mass} \neq 0 \;\; \text{under G.T.}
$$

- Similarly, fermion mass terms are not allowed by $SU(2)_L$ or $U(1)_Y$

$$
\mathcal{L}^{ferm}_{mass} = -m\;\underbrace{\left[\bar{f}_R f_L + \bar{f}_L f_R\right]}_{\text{transforms as } SU(2)_L \text{ doublet, } \sum Y \neq 0}
$$

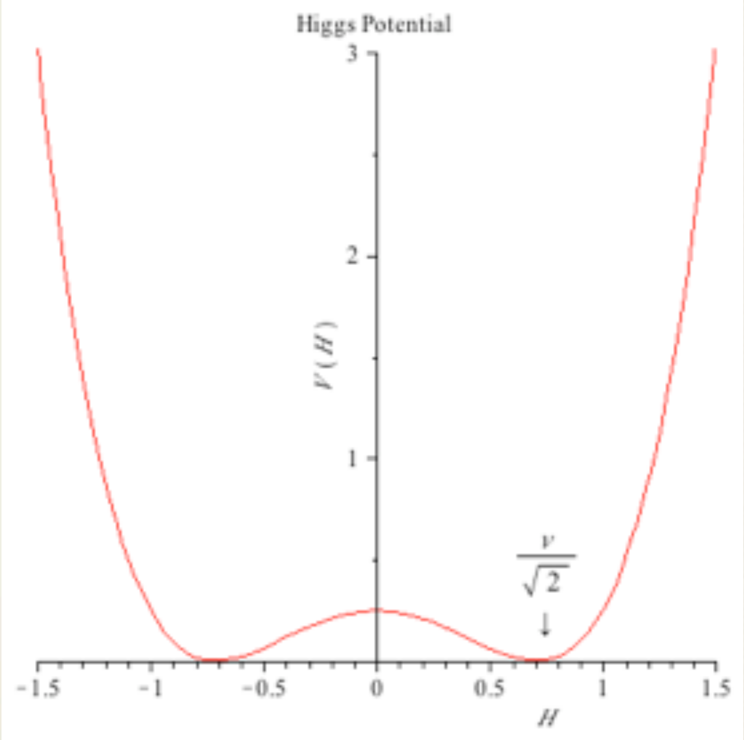
Spontaneous symmetry breaking

- The solution: Lagrangian is symmetric, ground state isn't ⇒ *spontaneous symmetry breaking*
- Complex scalar transforming as (1,2,1/2) under $SU(3)_C \times SU(2)_L \times U(1)_Y$

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger D^\mu H - \overbrace{\lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2}^{V(H)}$$

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$D^\mu = \partial^\mu - igW_a^\mu \frac{\sigma^a}{2} - ig' B^\mu \frac{1}{2}$$



$$\text{Vacuum expectation value: } \langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\text{Expand around vev: } H = \begin{pmatrix} \phi^+ \\ \frac{v + h + i\chi}{\sqrt{2}} \end{pmatrix}$$

(φ^+,χ can be removed by G.T., set to zero)

The Higgs mechanism

- Work out the kinetic part of Higgs Lagrangian

$$
\begin{aligned}
D_\mu H &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial_\mu h \end{pmatrix} - \frac{i}{2}\left[\frac{v+h}{\sqrt{2}}\right]\begin{pmatrix} \sqrt{2} g W^+_\mu \\ \sqrt{g^2+g'^2} Z_\mu \end{pmatrix} \\
(D^\mu H)^\dagger D_\mu H &= \frac{1}{2}\partial_\mu h \partial^\mu h + \left(1+\frac{h}{v}\right)^2 \left(\underbrace{\frac{g^2 v^2}{4}}_{M_W^2} W^{\mu+} W^-_\mu + \frac{1}{2} \underbrace{\frac{(g^2+g'^2) v^2}{4}}_{M_Z^2} Z_\mu Z^\mu \right) \\
Z_\mu &= c_W W^3_\mu - s_W B_\mu,\ A_\mu = s_W W^3_\mu + c_W B_\mu,\quad W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \\
c_W &= \frac{g}{\sqrt{g^2+g'^2}},\quad s_W = \frac{g'}{\sqrt{g^2+g'^2}}
\end{aligned}
$$

- $W^\pm$, Z acquire mass by "eating" φ^+, χ

$$\boxed{\text{Prediction: } \rho = \frac{M_W^2}{M_Z^2 c_W^2} = 1}$$

(tree-level; more later)

Fermion masses

- Yukawa interactions with Higgs doublets give fermions mass

$$\begin{aligned} \mathcal{L}_{Yuk} &= -\lambda_d \bar{Q}_L H d_R - \lambda_u \bar{Q}_L (i\sigma_2 H^*) u_R - \lambda_e \bar{L}_L H e_R + \text{h.c.} \\ &\Rightarrow -\left(1+\frac{h}{v}\right) \sum_{f=u,d,e} m_f \bar{f} f \quad \text{with} \quad m_f = \frac{\lambda_f v}{\sqrt{2}} \end{aligned}$$

(matrix in generation space, implicitly diagonalized at price of V_{CKM} in charged currents)

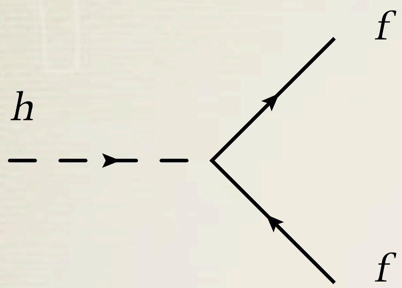
- Sum of all pieces so far give the SM Lagrangian:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge+ferm} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yuk}$$

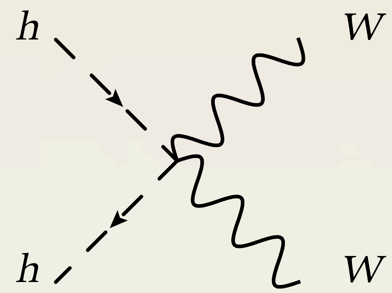
- The single Higgs doublet is just the simplest way to break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$; EWSB could be more intricate.
But this is the benchmark to compare other theories against.

Feynman rules

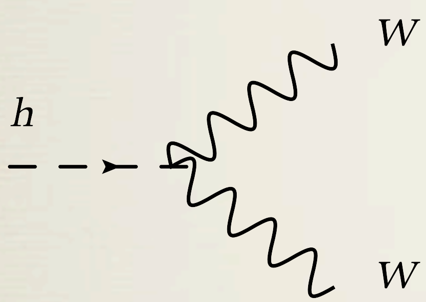
- Work out the experimental predictions with Feynman rules:



$$= -i\frac{m_f}{v}$$



$$= 2i\frac{M_W^2}{v^2}g_{\mu\nu}$$



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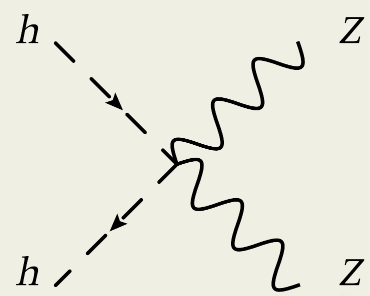
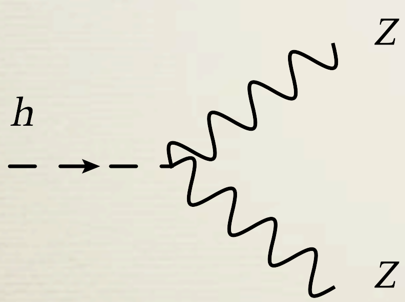


$$= 2i\frac{M_W^2}{v^2}g_{\mu\nu}$$



$$= 2i\frac{M_Z^2}{v}g_{\mu\nu}$$

From muon decay,
$v^2=1/(G_F\sqrt{2}) \Rightarrow v\approx 246$ GeV

- Only scalars with vevs have linear HVV couplings

Test the consequences of the Higgs mechanism

Unitarity of S-matrix

- Conservation of probability in QFT:

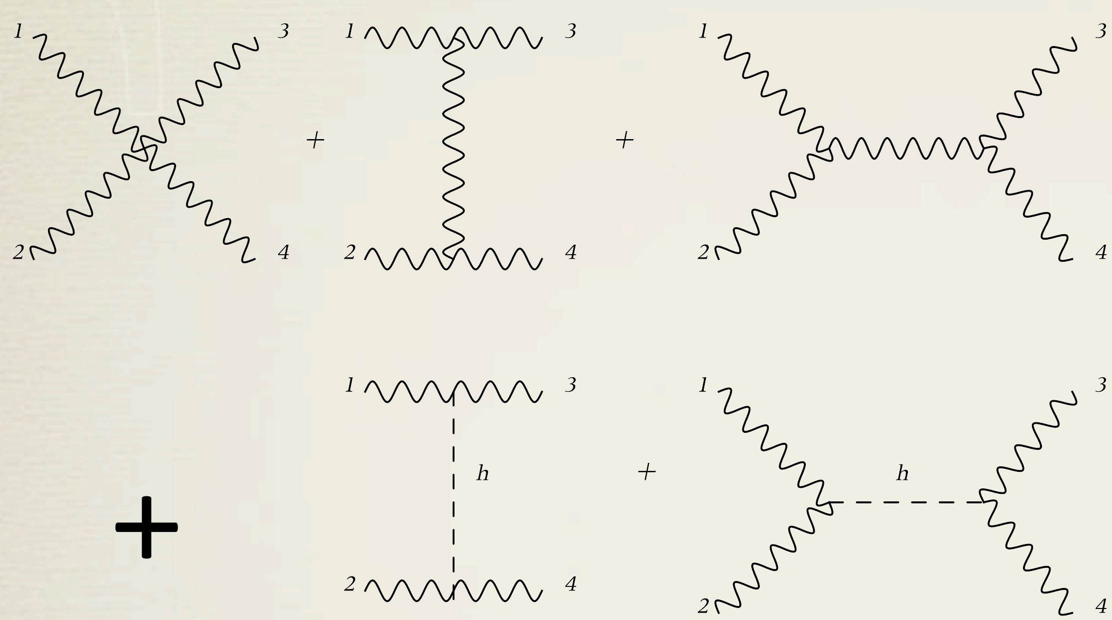
$$S^\dagger S = 1 \;\Rightarrow\; \sigma = \frac{1}{s}\mathrm{Im}\underbrace{\{\mathcal{M}(\theta=0)\}}_{\text{forward scattering}}$$

- Decompose into Legendre polynomials

$$\begin{aligned}
\mathcal{M} &= 16\pi\sum_{l=0}^{\infty}(2l+1)P_l(c_\theta)a_l \\
a_l &= \frac{1}{32\pi}\int_{-1}^{1} dc_\theta\, P_l(c_\theta)\mathcal{M} \\
\Rightarrow \sigma &= \frac{16}{s}\sum_{l=0}^{\infty}(2l+1)|a_l|^2 \\
&\Rightarrow\; |a_L|^2 = \mathrm{Im}(a_l) \\
&\Rightarrow\; \mathrm{Re}(a_l) \le 1/2
\end{aligned}$$

WW scattering

- Longitudinal modes: $\varepsilon_L = (p/M, 0 , 0, E/M)$ (boost from (0,0,0,1))



$$a_0(W_L W_L \to W_L W_L) \to -\frac{s}{32\pi v^2}$$

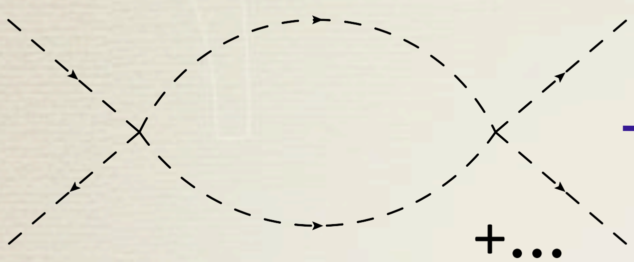
$$a_0(W_L W_L \to W_L W_L) \to -\frac{M_H^2}{8\pi v^2}$$

Exercise: Derive these values for a_0

- Probability not conserved without Higgs; with, M_H<900 GeV (perturbative argument)
- Other mechanisms possible: for example, exchanges of Kaluza-Klein modes in extra-dimensional Higgsless models Csaki et al 2004

Theoretical constraints

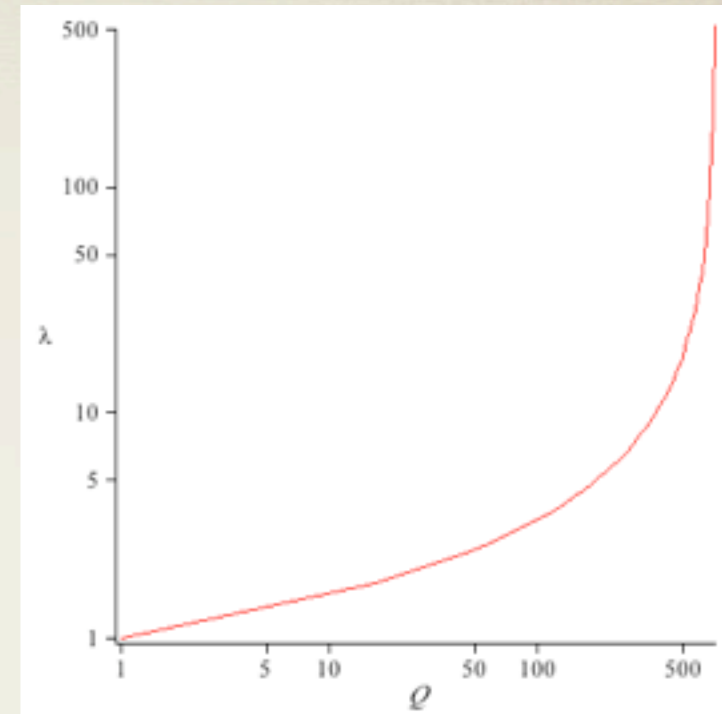
- Landau pole of λh^4 coupling



+...

$$\lambda(Q) = \frac{M_H^2}{2v^2}\frac{1}{1 - \frac{3}{4\pi^2}\frac{M_H^2}{v^2}\ln\frac{Q}{v}}$$

(large λ limit)

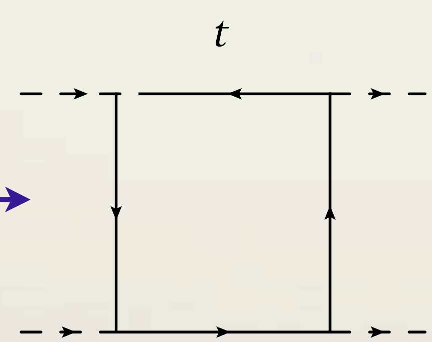


Breaks down at some Q
For validity up to Q=Λ (λ<∞),
upper bound on M_H

Exercise: Derive this RG equation

- Shape of Higgs potential: λ>0 ⇒ lower bound on M_H

top quark drives coupling negative

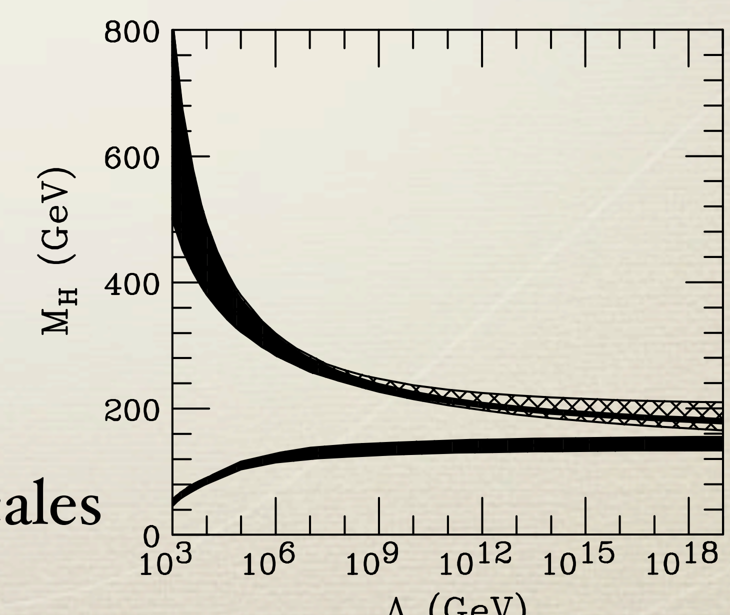


+...

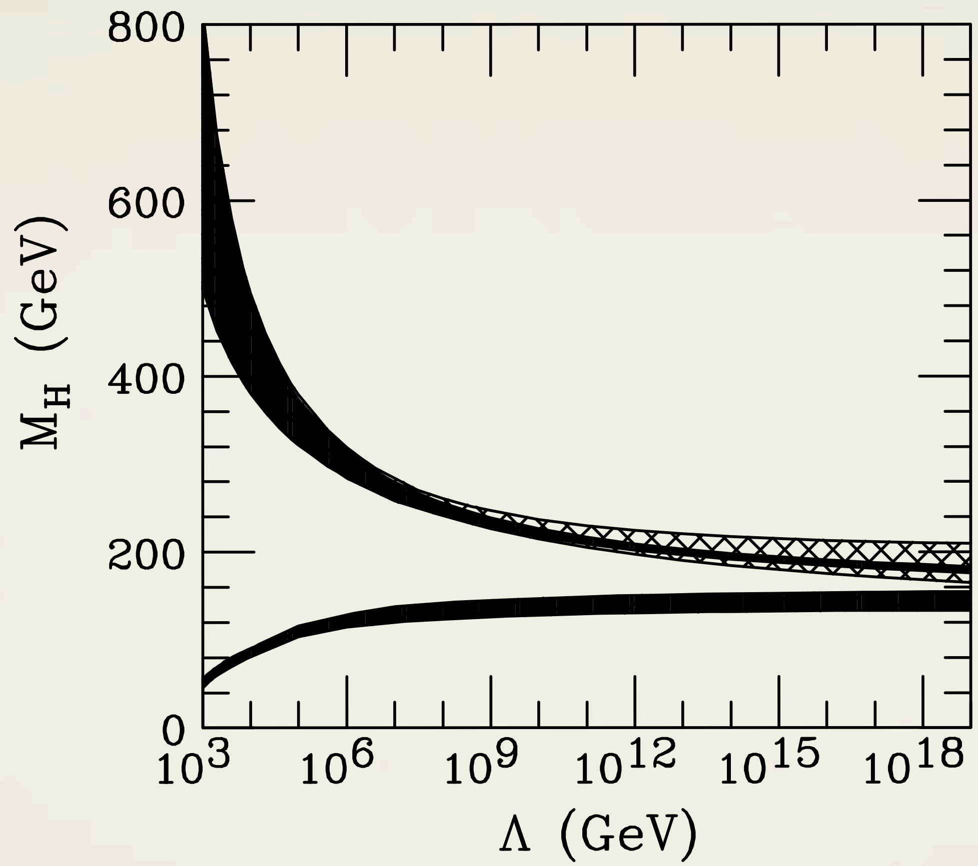
$$\lambda(Q) = \lambda_0 - \frac{\frac{3y_t^4}{8\pi^2}\ln\frac{Q}{Q_0}}{1 - \frac{9y_t^2}{16\pi^2}\ln\frac{Q}{Q_0}}$$

(small λ limit)

Validity of SM to high scales restricts allowed M_H



Theoretical constraints



- Theory tells us to expect the SM Higgs in the range 100 GeV ≤ M_H ≤1 TeV
- What does experiment say?

Electroweak precision

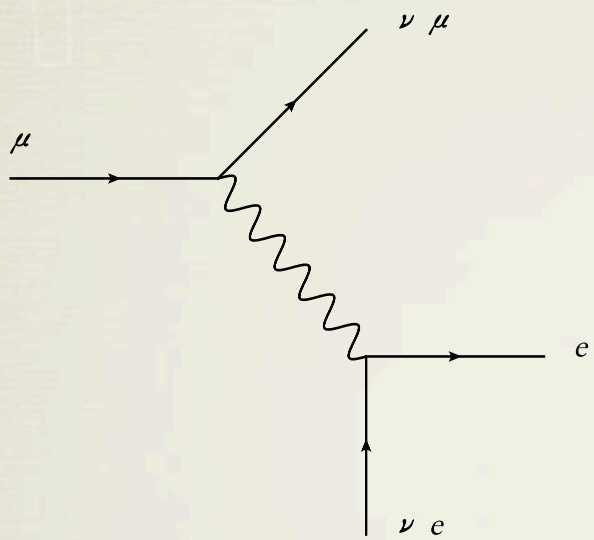
- Can experimentally probe properties of the Higgs directly (try to produce at a collider) or indirectly (through quantum effects)
- LEP+SLC: millions of $e^+e^- \to Z \to f\bar{f}$, high-precision measurements of SM electroweak parameters $\Rightarrow$ effect of Higgs?
- Study one-loop predictions of SM
- Basic idea in renormalizable theory: fix most precisely known quantities, calculation others in terms of them

$$\begin{aligned} G_F &= 1.166367(5) \times 10^{-5}\ \mathrm{GeV}^{-2} \quad \text{(muon decay)} \\ \alpha^{-1} &= 137.035999679(94) \quad \text{(low-energy experiments)} \\ M_Z &= 91.1875(21) \quad \text{(LEP)} \end{aligned}$$

- Example: we'll outline prediction for M_W

Muon decay

- Muon-decay at tree-level:



$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{e^2}{8M_W^2 s_W^2} \quad (m_{e,\mu}=0) \\ s_W^2 &= 1-\frac{M_W^2}{M_Z^2} \quad \text{(on-shell scheme)} \\ \Rightarrow \frac{G_F}{\sqrt{2}} &= \frac{\pi\alpha}{2M_W^2\left(1-M_W^2/M_Z^2\right)} \\ \Rightarrow M_W^2 &= \frac{M_Z^2}{2}\left\{1+\left[1-\frac{2\sqrt{2}\pi\alpha}{G_F M_Z^2}\right]^{1/2}\right\} \\ &\approx 80.94\ \text{GeV} \end{aligned}$$

⇒ experiment gets 80.4 GeV!

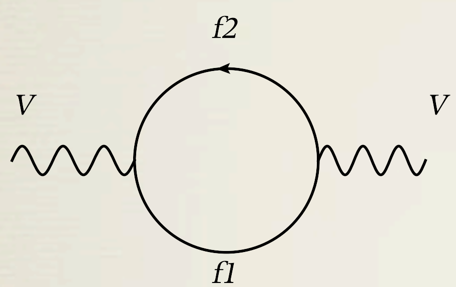
- Keep only leading corrections (m_t, M_H, running of α; others defined as ‘small’)

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{e^2}{8M_W^2 s_W^2}(1+\Delta r) \\ \Rightarrow M_W^2 &= \frac{M_Z^2}{2}\left\{1+\left[1-\frac{2\sqrt{2}\pi\alpha\,(1+\Delta r)}{G_F M_Z^2}\right]^{1/2}\right\} \end{aligned}$$

Δϱ and non-decoupling

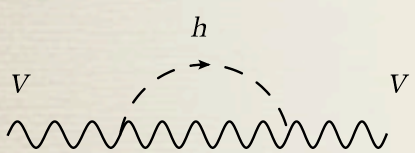
Δr receives important contribution from gauge-boson self-energies

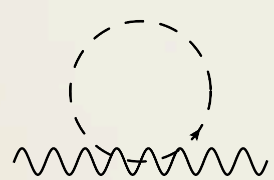
$$\boxed{\begin{aligned} \Delta r &= \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho \\ \Delta\rho &= \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} \end{aligned}}$$



$$\Delta\rho_{ferm} = \overbrace{\frac{3G_F m_t^2}{8\pi^2\sqrt{2}}}^{\text{quadratic in } m_t} + \text{subleading terms}$$

Exercise: Derive these





$$\Delta\rho_{Higgs} = -\frac{3G_F M_Z^2 s_W^2}{4\pi^2\sqrt{2}} \underbrace{\ln\frac{M_H}{M_Z}}_{\text{logarithmic in } M_H} + \text{ subleading terms}$$

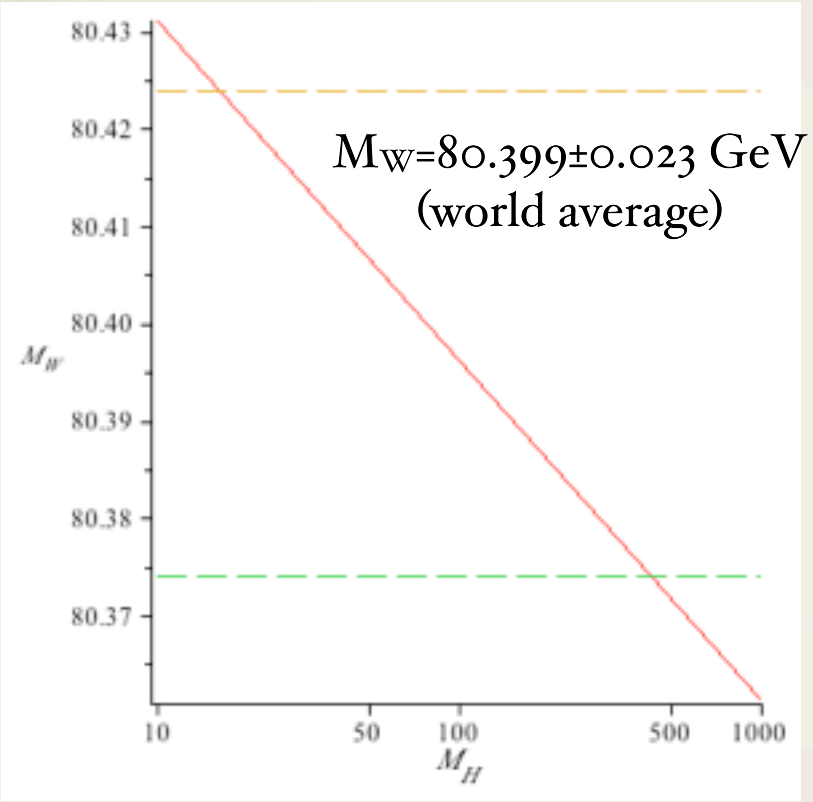
Decoupling theorem holds only if dimensionful parameters made large

$$m_t = \frac{\lambda_t v}{\sqrt{2}} \quad \Rightarrow \quad m_t \to \infty, \ \ v \text{ fixed} \Rightarrow \lambda_t \to \infty$$

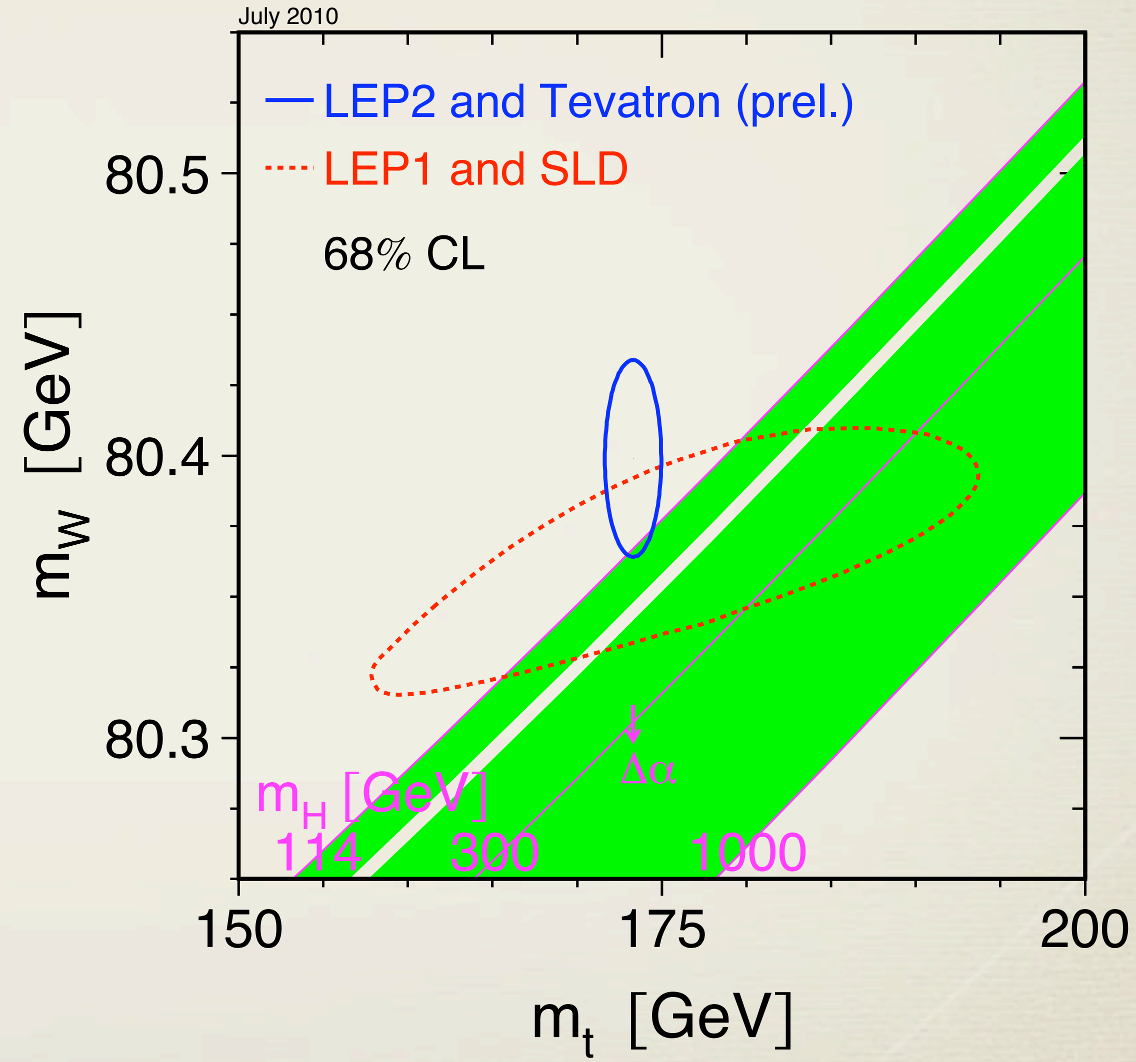
$$M_H^2 = 2\lambda v^2 \quad \Rightarrow M_H \quad \to \infty, \ \ v \text{ fixed} \Rightarrow \lambda \to \infty$$

Bounding the Higgs mass

- Logarithmic dependence on M_H allows M_W to bound it (but very sensitive to the top-quark mass)



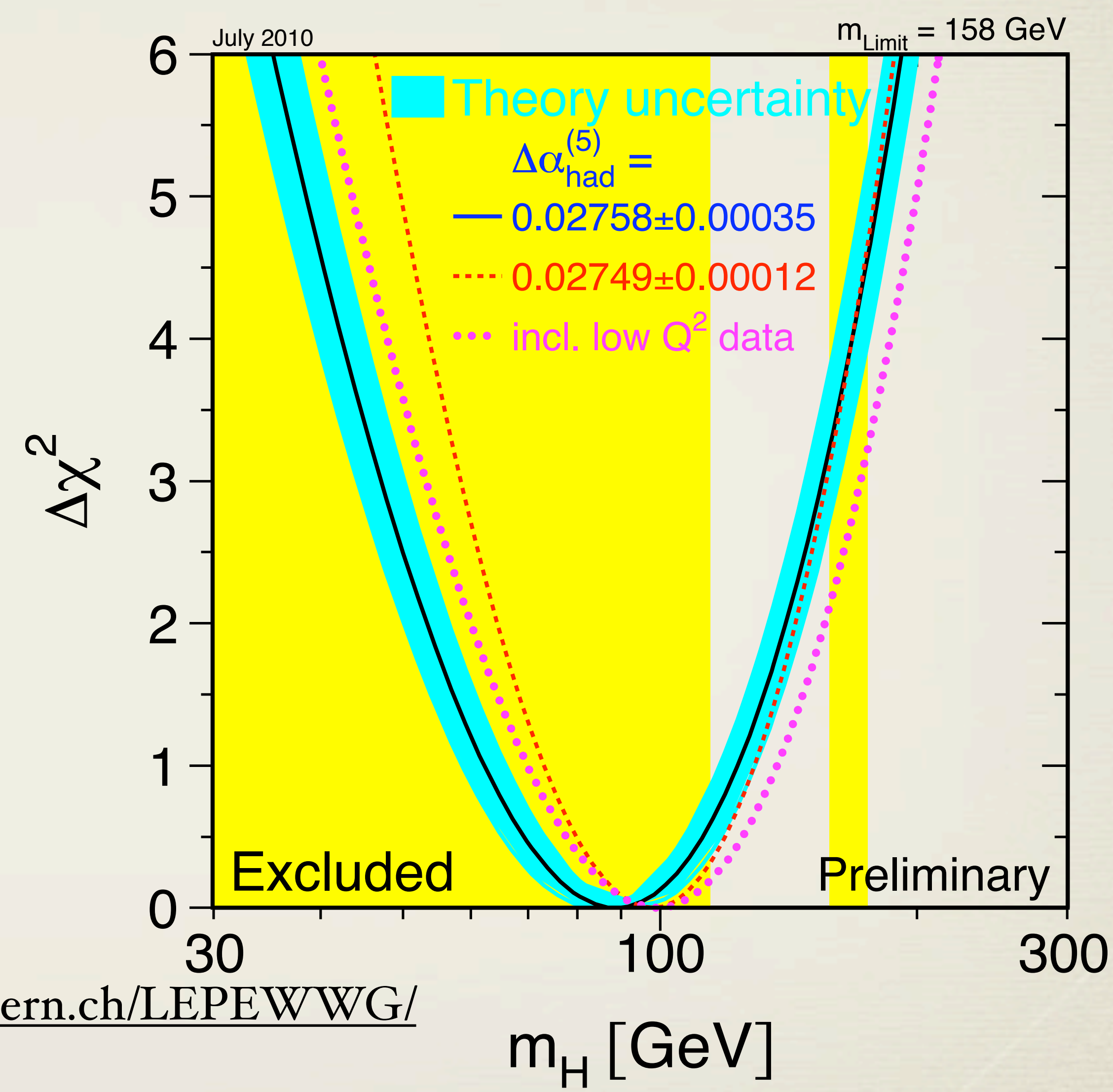
(Refinements needed for real comparison to data; important ln(m_t) and other terms; see PDG and refs within)



$$M_W^{tree} = 80.94 \text{ GeV} \Rightarrow M_W^{1-loop} = 80.39 \text{ GeV} \ (M_H = 120 \text{ GeV})$$

Global EW fit

- Do the same for large set of LEP-SLC measurements



http://lepewwg.web.cern.ch/LEPEWWG/

SM Higgs mass: M_H<~160 GeV from EW precision measurements

S, T, and hiding a heavy Higgs

- How robust are these bounds? Consider corrections that are *oblique:* affect only gauge boson propagators

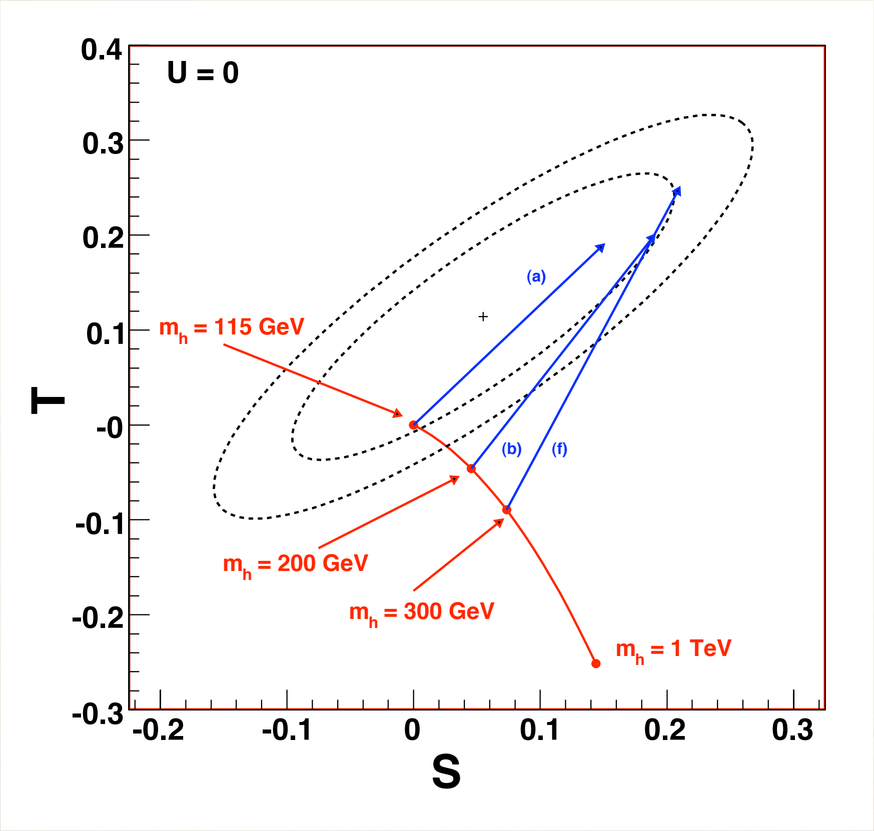
$$\alpha T = \frac{\Pi_{WW}(0)}{M_W^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2} = \Delta\rho$$

$$\frac{\alpha}{4s_W^2 c_W^2} S = \frac{\Pi_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Pi_{ZZ}(0)}{M_Z^2}$$

(Not a complete basis, but these are often the most important ones)

- Calculate for reference M_H, propagate through all EW parameters

Example: 4th generation
Kribs et al. 2007



$$\Delta\rho_{new} = \frac{3G_F \overbrace{\Delta m^2_{ferm}}^{\text{doublet mass splitting}}}{8\pi^2\sqrt{2}} - \frac{3G_F M_Z^2 s_W^2}{4\pi^2\sqrt{2}} \ln\frac{M_H}{M_H^{ref}}$$

$\Rightarrow$ increase M_H, cancel with Δm

Need direct searches!

Decays of the Higgs boson

Higgs decays

- Since $g_{Hxx} \sim m_x$, Higgs tends to decay to heaviest kinematically accessible states (with many important caveats...)
- Tree-level decays to various massive final states:

$$\Gamma_{qq} = N_c \frac{G_F}{4\sqrt{2}\pi} M_H m_f^2 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}, \quad \Gamma_{ll} = \frac{G_f}{4\sqrt{2}\pi} M_H m_f^3 \left(1 - \frac{4m_f^2}{M_H^2}\right)^{3/2}$$

$$\Gamma_{VV} = \frac{G_F}{8\sqrt{2}\pi n_V} M_H^3 \left(1-4x\right)^{1/2} \left(1 - 4x + 12x^3\right) \text{ with } x = \frac{M_V^2}{M_H^2}, n_W = 1, n_Z = 2$$

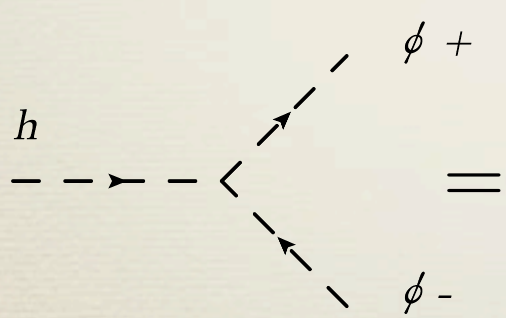
- Threshold structure depends on spin, CP (3/2→1/2 for CP-odd A)
- Note $\Gamma_{ff} \sim M_H$, while $\Gamma_{VV} \sim (M_H)^3 \Rightarrow$ when W, Z channels open, Higgs becomes very broad
- For light Higgs ($M_H \leq 130$ GeV), expect bb, ττ, cc to be important

Equivalence theorem

- Growth of VV width comes from longitudinal gauge modes

$$\begin{aligned}
\mathcal{A}(h \to W_L^+ W_L^-) &= 2\frac{M_W^2}{v}\epsilon_L^+ \cdot \epsilon_L^-, \quad \epsilon_L^\pm = \frac{E}{M_W}\left(\pm\beta_W, \vec{0}, 1\right) \\
\mathcal{A}(h \to W_L^+ W_L^-) &\to -\frac{M_H^2}{v} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right) \\
\Gamma_{WW} &= \frac{1}{16\pi M_H}|\mathcal{A}|^2 \to \frac{G_F M_H^3}{8\pi\sqrt{2}} + \mathcal{O}\left(\frac{M_V^2}{M_H^2}\right)
\end{aligned}$$

- In the high energy limit, longitudinal mode interactions equivalent to those of eaten scalar ⇒ *Goldstone boson equivalence theorem*

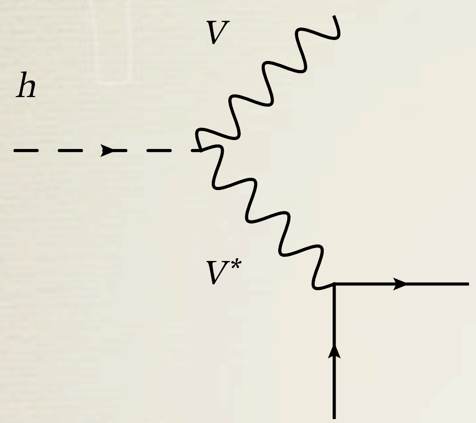


$$= -i\frac{M_H^2}{v} \qquad \mathcal{A}(h \to \phi^+\phi^-) = -\frac{M_H^2}{v}$$

Exercise: Work out from L_{Higgs}

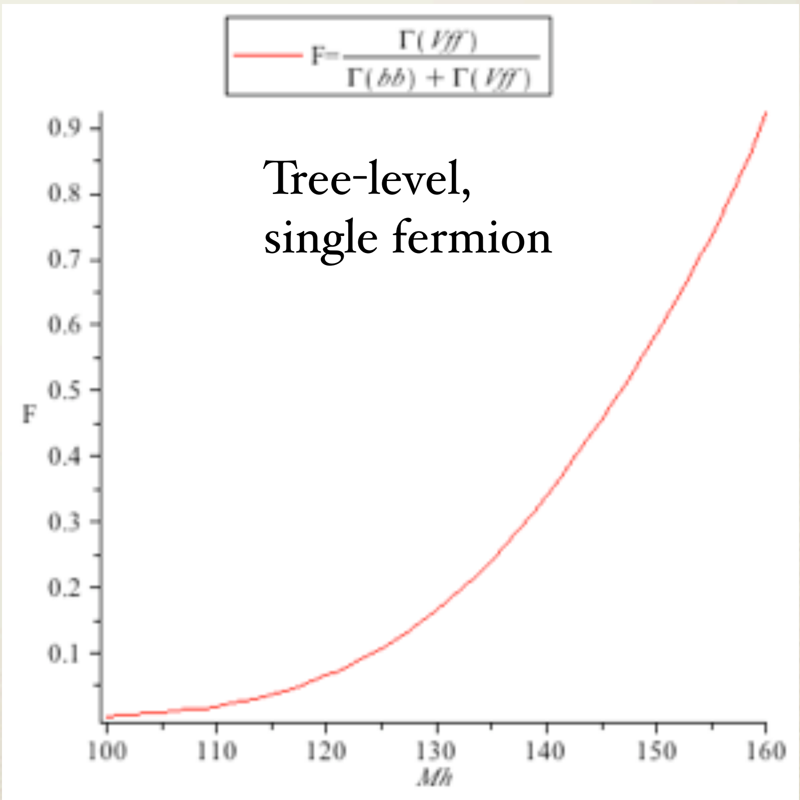
Three-body decays

- Since $M_{W,Z} >> m_{b,c,\tau}$, $H \rightarrow VV^* \rightarrow Vff$ important for $M_H < 2M_{W,Z}$



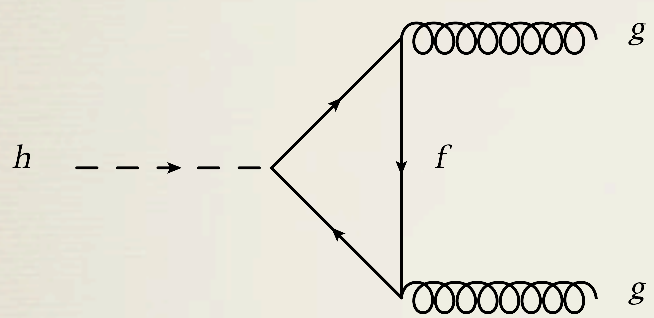
$$
\begin{aligned}
\Gamma_{W\bar{f}f} &= \frac{3G_F^2 M_W^4}{16\pi^3} M_H \left\{ \frac{3(1-8x+20x^2)}{\sqrt{4x-1}} \arccos\left(\frac{3x-1}{2x^{3/2}}\right) \right. \\
&\quad \left. -\frac{1-x}{2x}(2-13x+47x^2) - \frac{3}{2}(1-6x+4x^2)\ln x \right\} \\
x &= M_W^2/M_H^2
\end{aligned}
$$

- Important mode even down at $M_H \approx 130$ GeV since f=e,μ



Loop-induced H→gg

- Can we leverage the large Htt, HVV couplings at low M_H?
- Two important cases: h→gg (production more important), h→γγ



$+$ *crossed diagram*

$$\Gamma_{gg} = \frac{G_F \alpha_s^2 M_H^3}{36\pi^3\sqrt{2}} \left| \frac{3}{4} \sum_Q \mathcal{F}_{1/2}(\tau_Q) \right|^2 \quad \text{with } \tau_Q = \frac{M_H^2}{4m_Q^2}$$

$$\mathcal{F}_{1/2}(\tau) = \frac{2}{\tau^2}\left[\tau + (\tau - 1) f(\tau)\right]$$

$$f(\tau) = \begin{cases} \arcsin^2\sqrt{\tau} & \tau \leq 1 \\ -\frac{1}{4}\left[\ln\frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}} - i\pi\right]^2 & \tau > 1 \end{cases}$$

$$\tau \to 0 \quad \Rightarrow \quad \mathcal{F}_{1/2} \to \frac{4}{3}$$

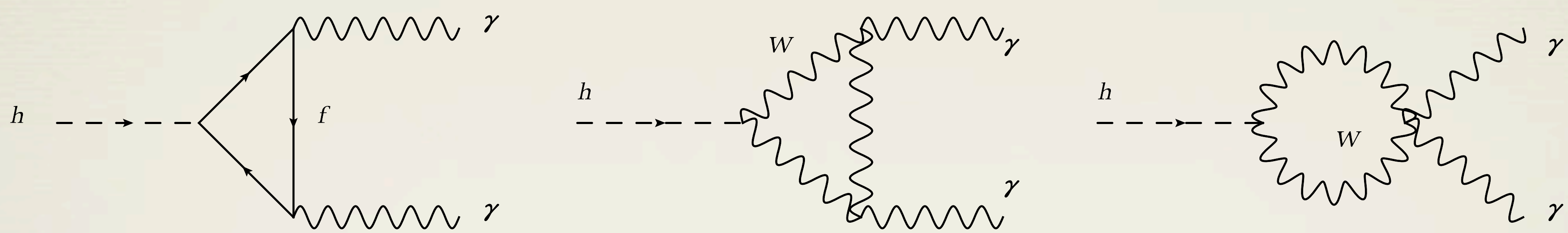
$$\tau \to \infty \quad \Rightarrow \quad \mathcal{F}_{1/2} \to -\frac{2m_Q^2}{M_H^2} \ln\frac{M_H^2}{m_Q^2}$$

•Independent of m_f when $m_f \to \infty$ ⇒ true for *any* heavy fermion that gets its mass entirely from Higgs

Exercise: Derive $m_t \to \infty$ result from direct integration

Loop-induced H→γγ

- Crucial for low-mass Higgs search at LHC



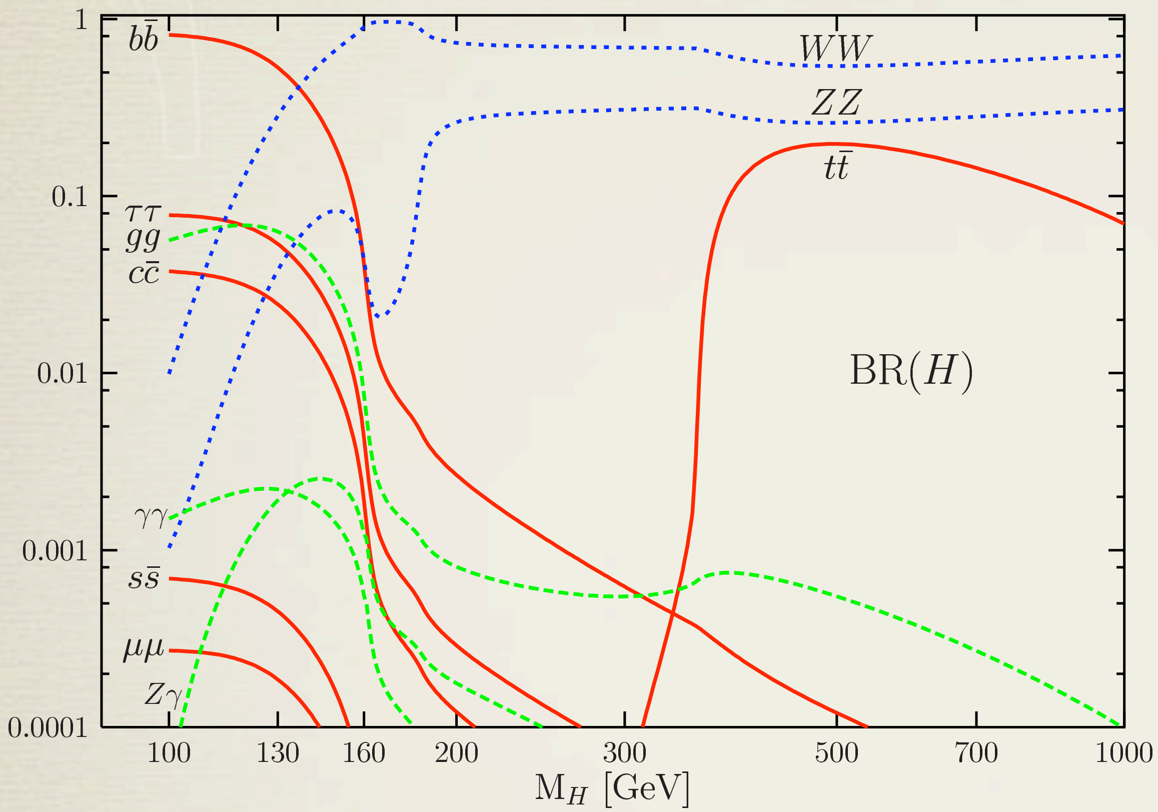
$$\Gamma_{\gamma\gamma} = \frac{G_F \alpha^2 M_H^3}{128\pi^3\sqrt{2}} \left| \sum_f N_c Q_f^2 \mathcal{F}_{1/2}(\tau_f) + \mathcal{F}_1(\tau_W) \right|^2$$

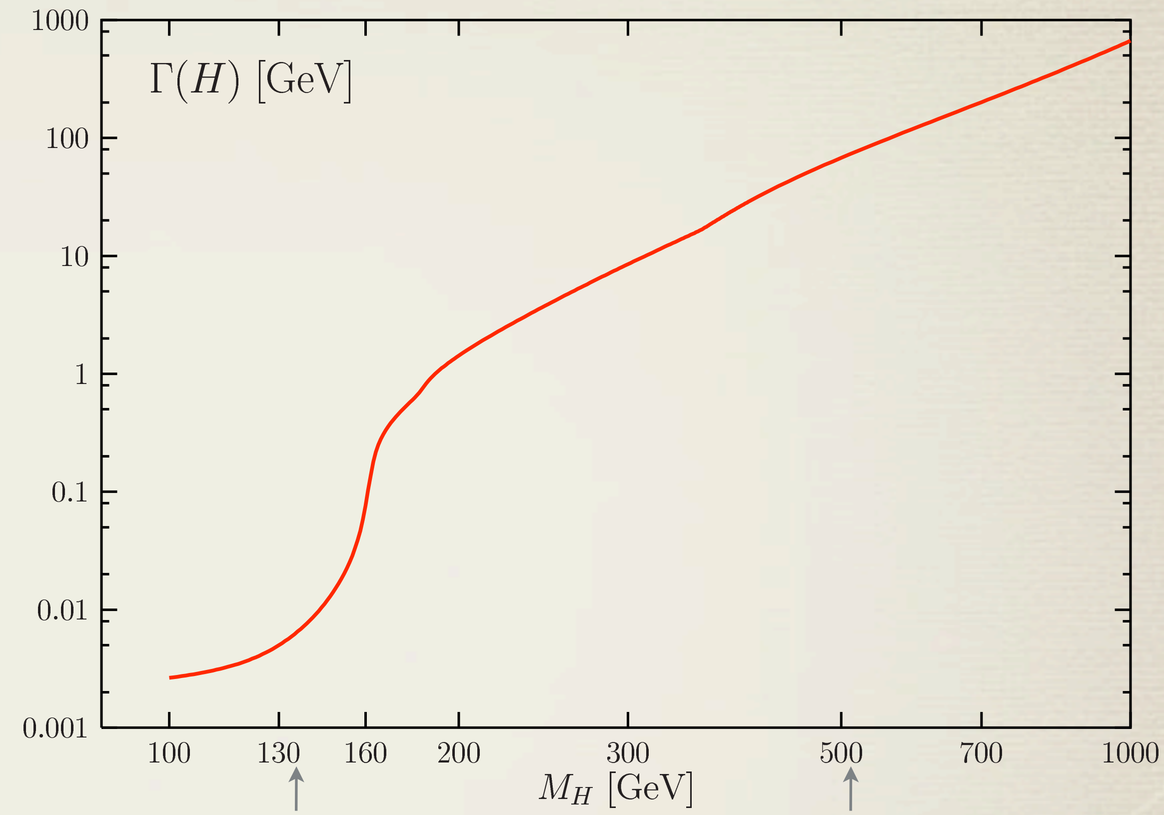
$$\mathcal{F}_1(\tau) = -\frac{1}{\tau^2}\left[2\tau^2 + 3\tau + 3(2\tau - 1) f(\tau)\right]$$

$\tau \to 0 \Rightarrow \mathcal{F}_1 \to -7$

W contribution larger than top-quark, they interfere destructively

Putting it all together





$\Gamma/M \sim 10^{-5}$ $\Gamma/M \sim 0.2$

Most important channels:

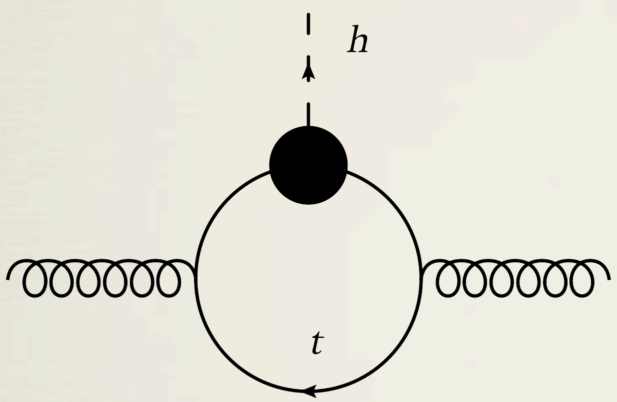
<u>$M_H \leq 130$ GeV</u>: bb, ττ, γγ (clean signature)

<u>$M_H \geq 130$ GeV</u>: WW, ZZ

(boundaries are rough)

Refinement: low-energy theorems

- Can exactly calculate QCD corrections to h→gg, γγ (two-loop diagrams plus real radiation for gg decay) Djouadi, Spira, Zerwas early 1990s
- Useful, illuminating alternative approach for $2m_t > M_H$



$$\frac{i}{\not{k} - m_t} \quad \rightarrow \quad \frac{i}{\not{k} - m_t}\frac{-im_t}{v}\frac{i}{\not{k} - m_t} = i\frac{m_t}{v}\left(\frac{1}{\not{k} - m_t}\right)^2$$
$$= \frac{m_t}{v}\frac{\partial}{\partial m_t}\frac{i}{\not{k} - m_t}$$

Generates both diagrams in the $M_H \rightarrow 0$ limit

- Diagrammatically, clear that Higgs interaction comes from derivatives of the top part of the gluon self-energy:

$$\mathcal{M}(hgg) \underbrace{=}_{p_H \rightarrow 0} \frac{m_t}{v}\frac{\partial}{\partial m_t}\mathcal{M}(gg)$$

Effective Lagrangian

- Integrate out top quark to produce effective Lagrangian

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \mathcal{L}_{top} \\
\Rightarrow \underbrace{G^{a\prime}_{\mu}}_{\text{EFT field}} &= \zeta_3 \underbrace{G^a_{\mu}}_{\text{full QCD}} \\
\Rightarrow \mathcal{L}_{EFT} &= -\frac{\zeta_3}{4} G^{a\prime}_{\mu\nu} G_a^{\mu\nu\prime}
\end{aligned}
$$

Equate propagators in full QCD and EFT

$$
\begin{aligned}
-\frac{ig_{\mu\nu}}{p^2}\zeta_3 &= -\frac{ig_{\mu\nu}}{p^2} \underbrace{[1+\Pi_t(0)]}_{m_t^2 \gg p^2} \\
\Rightarrow \zeta_3 &= 1 + \Pi_t(0) \\
\Rightarrow \mathcal{L}_{EFT} &= -\frac{[1+\Pi_t(0)]}{4} G^{a\prime}_{\mu\nu} G_a^{\mu\nu\prime}
\end{aligned}
$$

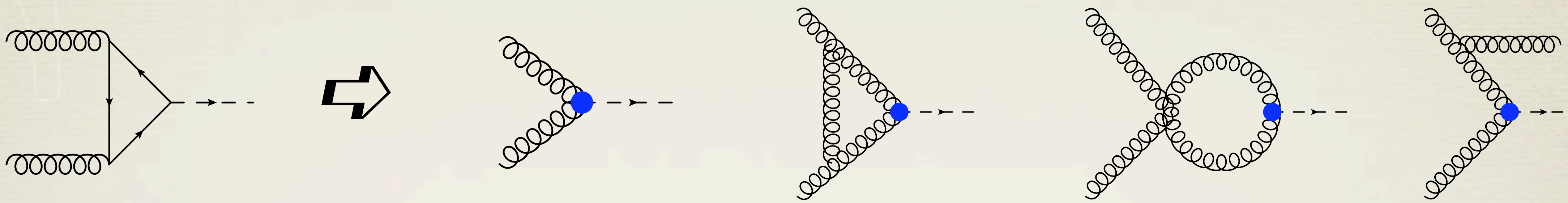
- Can generate hgg amplitudes from derivatives of gg amplitudes:

$$
\begin{aligned}
\mathcal{L}^{hgg}_{EFT} &= -\frac{m_t}{4v}\left(\frac{\partial}{\partial m_t}\Pi_t(0)\right) h\, G^{a\prime}_{\mu\nu} G_a^{\mu\nu\prime} \\
\Rightarrow \Pi_t(0) &= \frac{\alpha_s}{6\pi}\left[\frac{\bar{\mu}^2}{m_t^2}\right]^{\epsilon} \frac{\Gamma(1+\epsilon)}{\epsilon} \\
\Rightarrow & \boxed{\mathcal{L}^{hgg}_{EFT} = \frac{\alpha_s}{12\pi}\frac{h}{v} G^{a\prime}_{\mu\nu} G_a^{\mu\nu\prime}}
\end{aligned}
$$

Much of the Tevatron/ LHC phenomenology based on this franework

QCD in the EFT

- Reduces 2-loop calculation ⇒ 1-loop; separates m_t dependence



- Systematically improvable to all orders in α_s

$$\mathcal{L}^{hgg}_{EFT} = -C_1 \frac{h}{v} G^{a\prime}_{\mu\nu} G^{\mu\nu\prime}_{a}$$

$$C_1 = -\frac{1}{12}\frac{\alpha_s}{\pi}\left\{1 + \frac{\alpha_s}{\pi}\left(\frac{11}{4} - \frac{1}{6}\ln\frac{\mu^2}{m_t^2}\right) + \dots\right\}$$

Correction to h→light hadrons:
(must include qq at higher orders)

$$\begin{aligned} K &= 1 + 17.9167\, a_s' + 152.5 (a_s')^2 + 381.5 (a_s')^3 \\ &= 1 + 0.65038 + 0.20095 + 0.01825. \end{aligned}$$

Large!

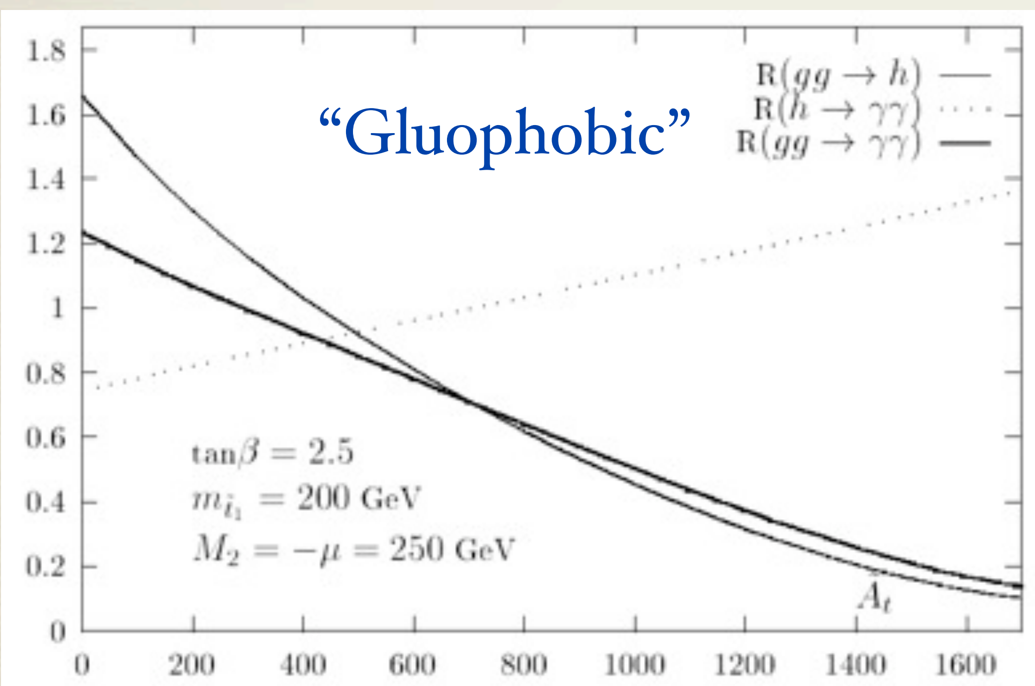
Baikov, Chetyrkin 2006

- Can do same for h→γγ decay, for W contribution also

For references and subtleties, see Chetyrkin et al.1997, Kniehl, Spira 1995

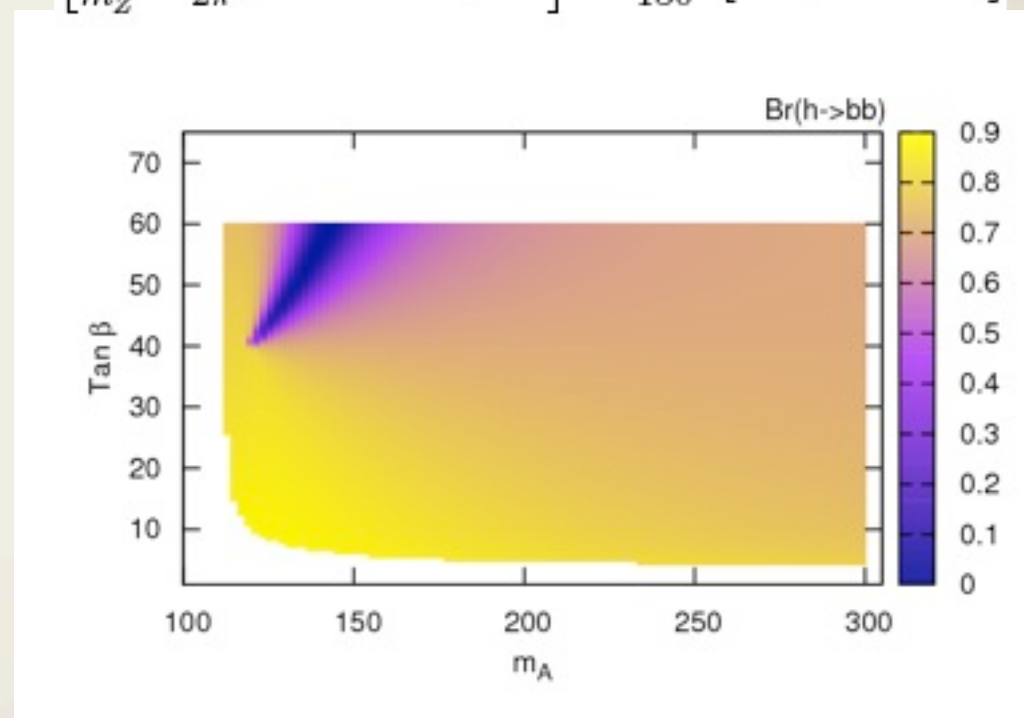
Decays beyond the SM

- MSSM: cancellations between top/squark loops in gg, γγ modes
- MSSM: opposite signs for A_t, μ terms ⇒ suppresses h→bb



Djouadi, hep-ph/9806315

$$\left[\frac{m_A^2}{m_Z^2} - \frac{1}{2\pi^2}(3\bar{\mu}^2 - \bar{\mu}^2\bar{A}_t^{\,2}) + 1\right] \simeq \frac{\tan\beta}{150}\left[\bar{\mu}\bar{A}_t(2\bar{A}_t^{\,2} - 11)\right]$$



Draper, Liu, Wagner 0905.4721

Decays beyond the SM

- <u>NMSSM</u>: decays to light CP-odd scalar can produce final states h→aa→bbττ,ττττ,ττγγ,...
- <u>Extended scalar sectors</u>: decays to stable scalars (dark matter) can make Higgs invisible decaying

m_{h_1}/m_{a_1} (GeV)	Branching Ratios $h_1 \to b\bar{b}$	$h_1 \to a_1 a_1$	$a_1 \to \tau\bar{\tau}$	$n_{\rm obs}/n_{\rm exp}$ units of 1σ	$s95$	N_{SD}^{LHC}
98.0/2.6	0.062	0.926	0.000	2.25/1.72	2.79	1.2
100.0/9.3	0.075	0.910	0.852	1.98/1.88	2.40	1.5
100.2/3.1	0.141	0.832	0.000	2.26/2.78	1.31	2.5
102.0/7.3	0.095	0.887	0.923	1.44/2.08	1.58	1.6
102.2/3.6	0.177	0.789	0.814	1.80/3.12	1.03	3.3
102.4/9.0	0.173	0.793	0.875	1.79/3.03	1.07	3.6
102.5/5.4	0.128	0.848	0.938	1.64/2.46	1.24	2.4
105.0/5.3	0.062	0.926	0.938	1.11/1.52	2.74	1.2

Dermisek, Gunion 2005

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}m_S^2 S^2 - \frac{k}{2}|H|^2 S^2 - \frac{h}{4!}S^4$$

h→SS decays can dominate

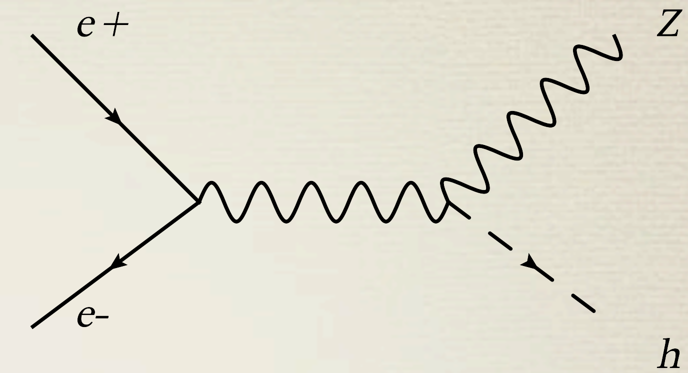
Burgess, Pospelov, ter Veldhuis 2001;
Davoudiasl et al.2004

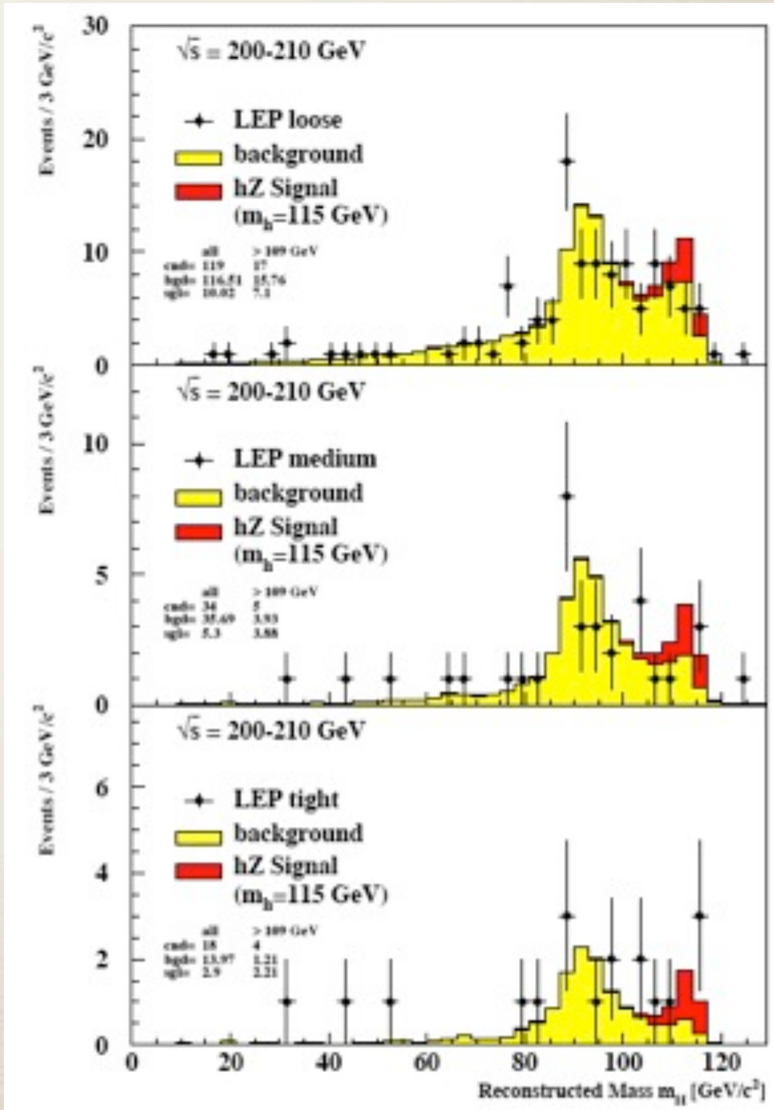
Many deviations, some drastic, from SM predictions possible!

Producing the Higgs boson

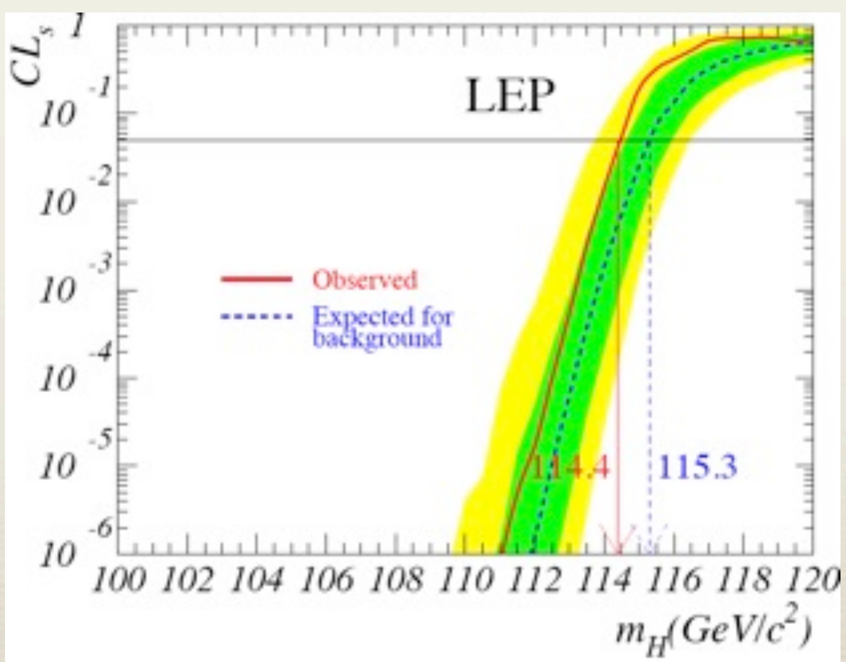
Direct searches at LEP

- LEP2 ran at energies reaching √s ≤ 209 GeV
- Dominant production process: $e^+e^- \rightarrow HZ$
- SM analysis utilizes the following channels:
  - h→bb, Z→qq
  - h→bb, Z→νν
  - h→bb, Z→ll (l=e,μ)
  - h→bb, Z→ττ
  - h→ττ, Z→qq



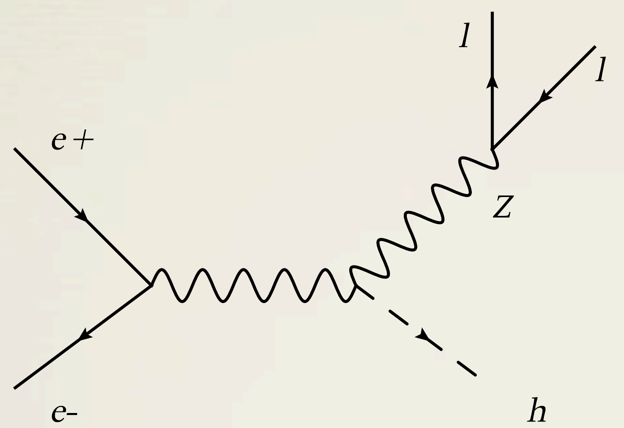


$M_H > 114.4$ GeV



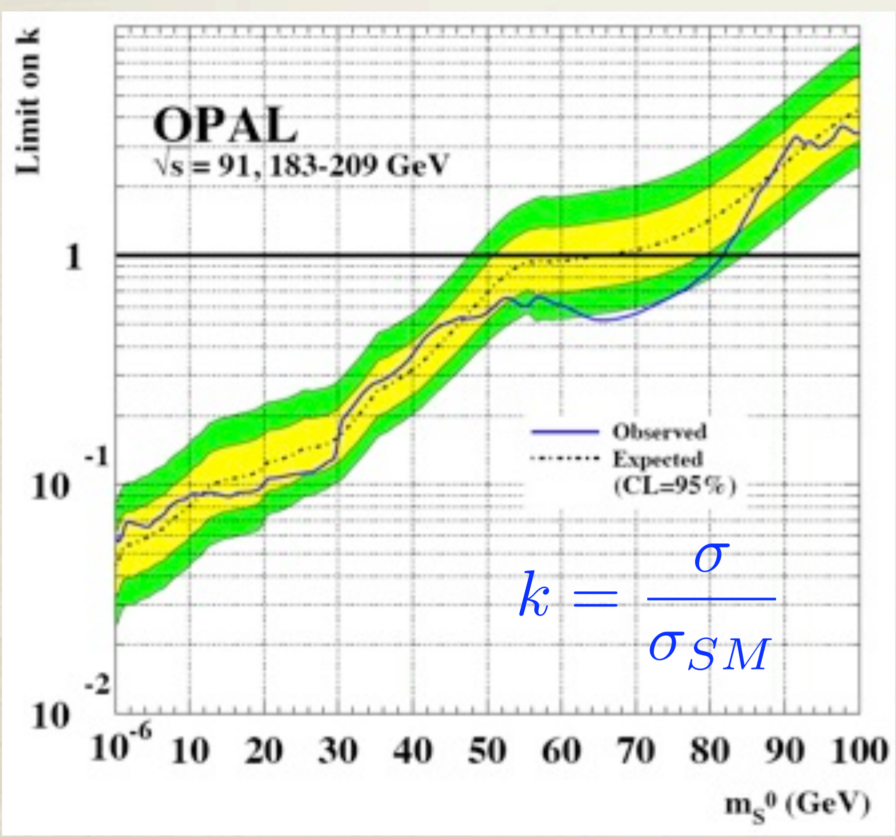
Model-independent search

- This is optimized for SM decays, any way to remove this bias?



Measure two leptons in final state, demand they reconstruct to Z mass

$$
\begin{aligned}
p_{e^+} + p_{e^-} &= p_{l^+} + p_{l^-} + p_X \\
&= p_{ll}^{rec} + p_X \\
\Rightarrow M_X^2 &= s - 2E_{ll} + M_{ll}^2 \\
\text{Predicted peak} &: \quad M_X^2 = M_H^2
\end{aligned}
$$



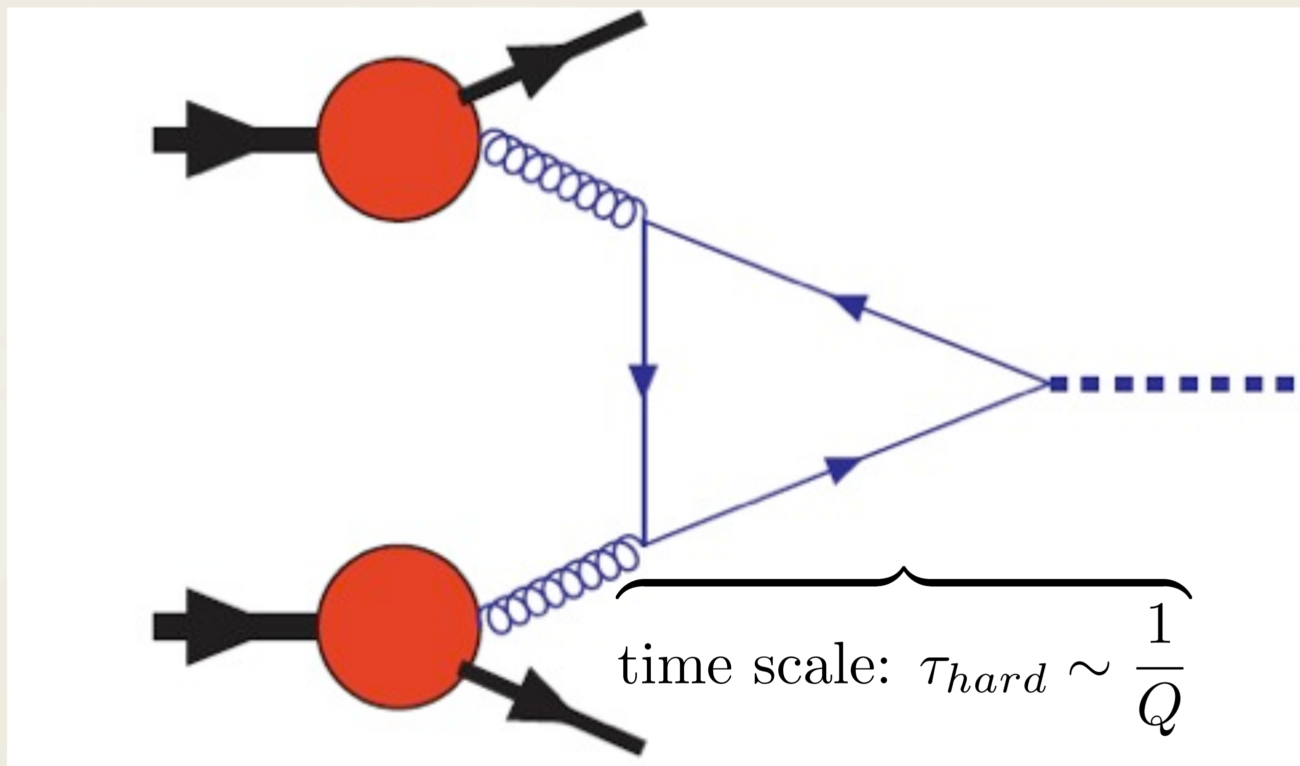
Limits hold for *any* decay mode

Many other searches designed for specific models

Hadron collider basics

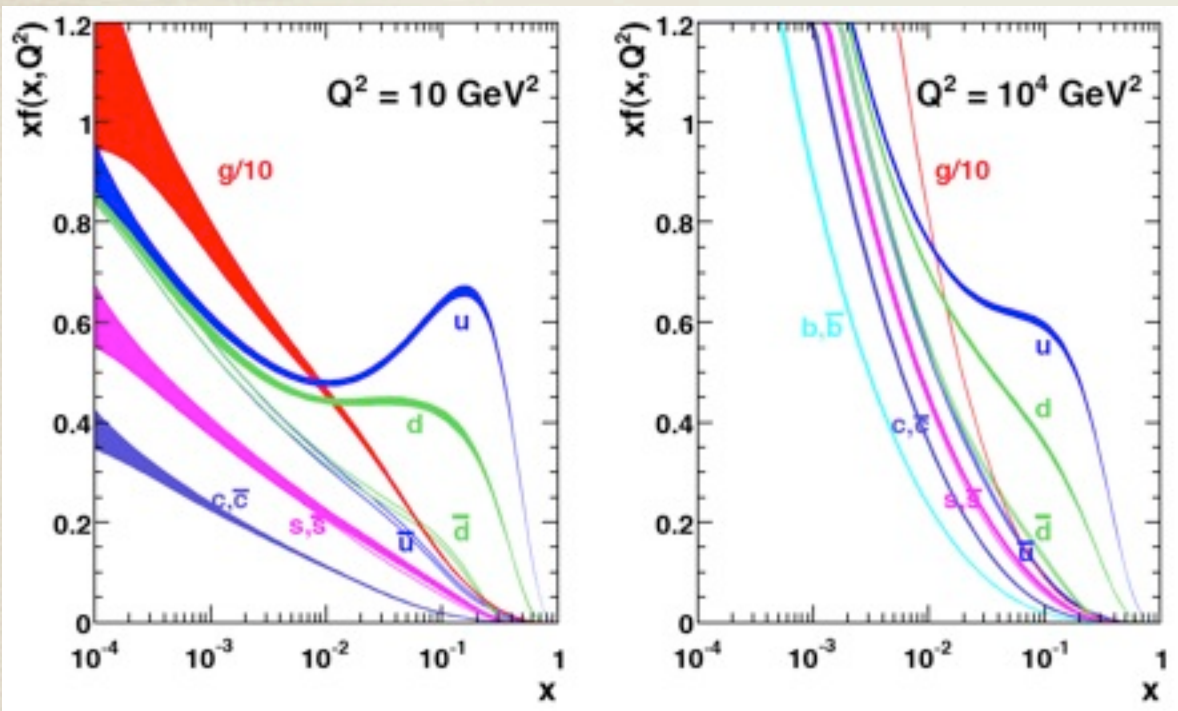
- The basic picture of hadronic collisions: factorize long and short time processes

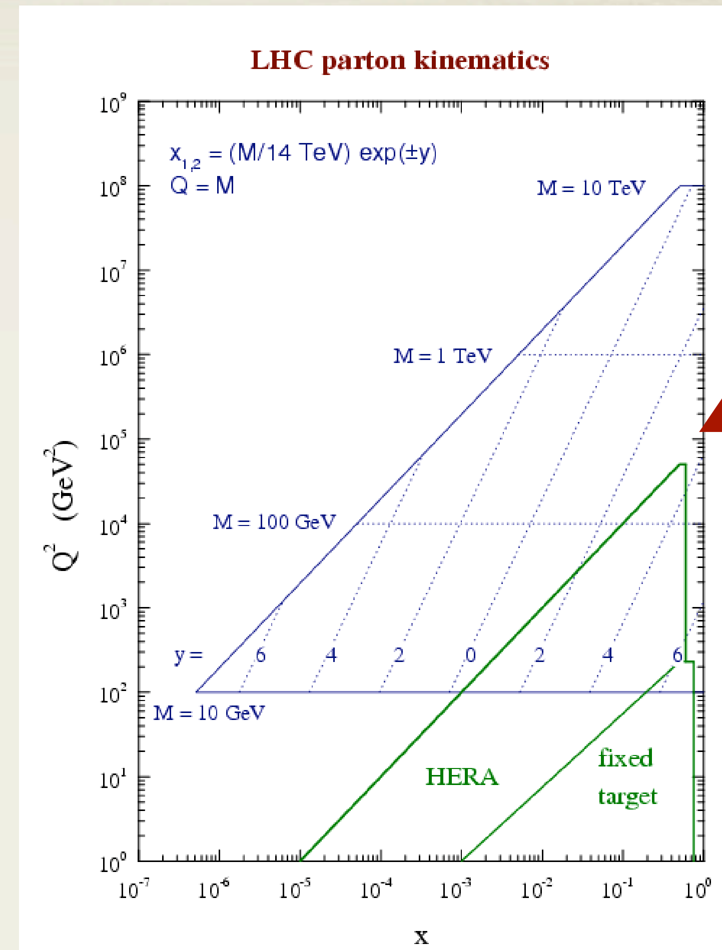
$$\underbrace{\text{time scale: } \tau_{proton} \sim \frac{1}{\Lambda_{QCD}}}$$



$$\sigma_{h_1 h_2 \to X} = \int dx_1 dx_2 \underbrace{f_{h_1/i}(x_1; \overbrace{\mu_F^2}^{\text{factorization scale}}) f_{h_1/j}(x_2; \mu_F^2)}_{PDFs} \underbrace{\sigma_{ij \to X}(x_1, x_2, \mu_F^2, \{q_k\})}_{\text{partonic cross section}} + \underbrace{\mathcal{O}\left(\frac{\Lambda_{QCD}}{Q}\right)^n}_{\text{power corrections}}$$

Parton distribution functions





DGLAP

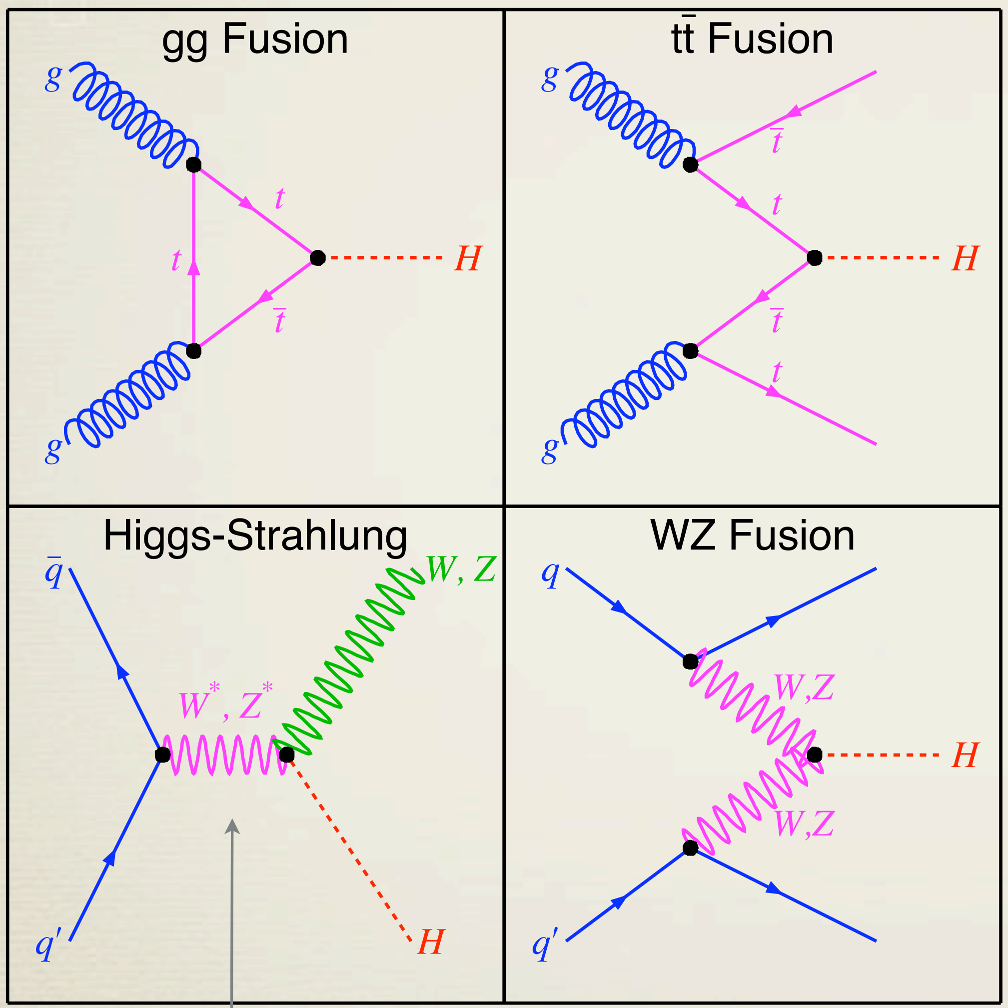
$x \sim M_H/\sqrt{s}$

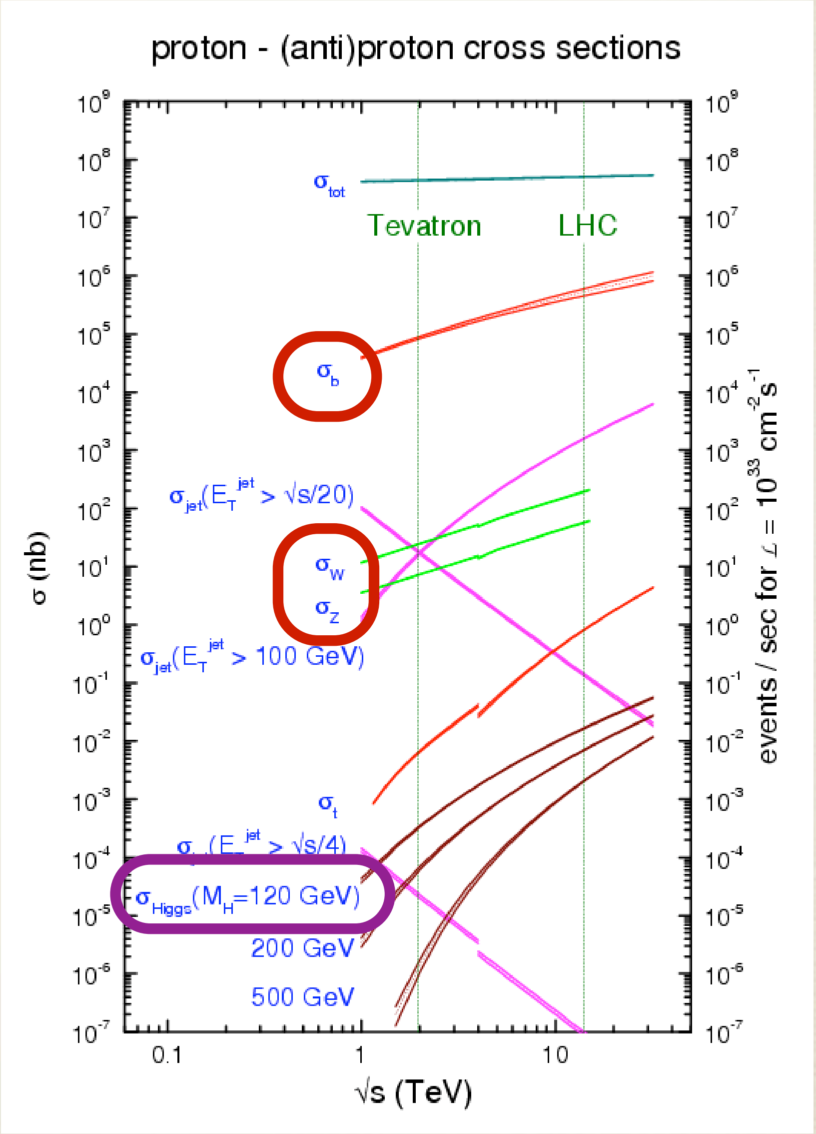
LHC **Tevatron**

Lots of gluons, at LHC especially

Higgs at hadron colliders

- Clearly want to use large gluon luminosity; W, Z assisted production another option



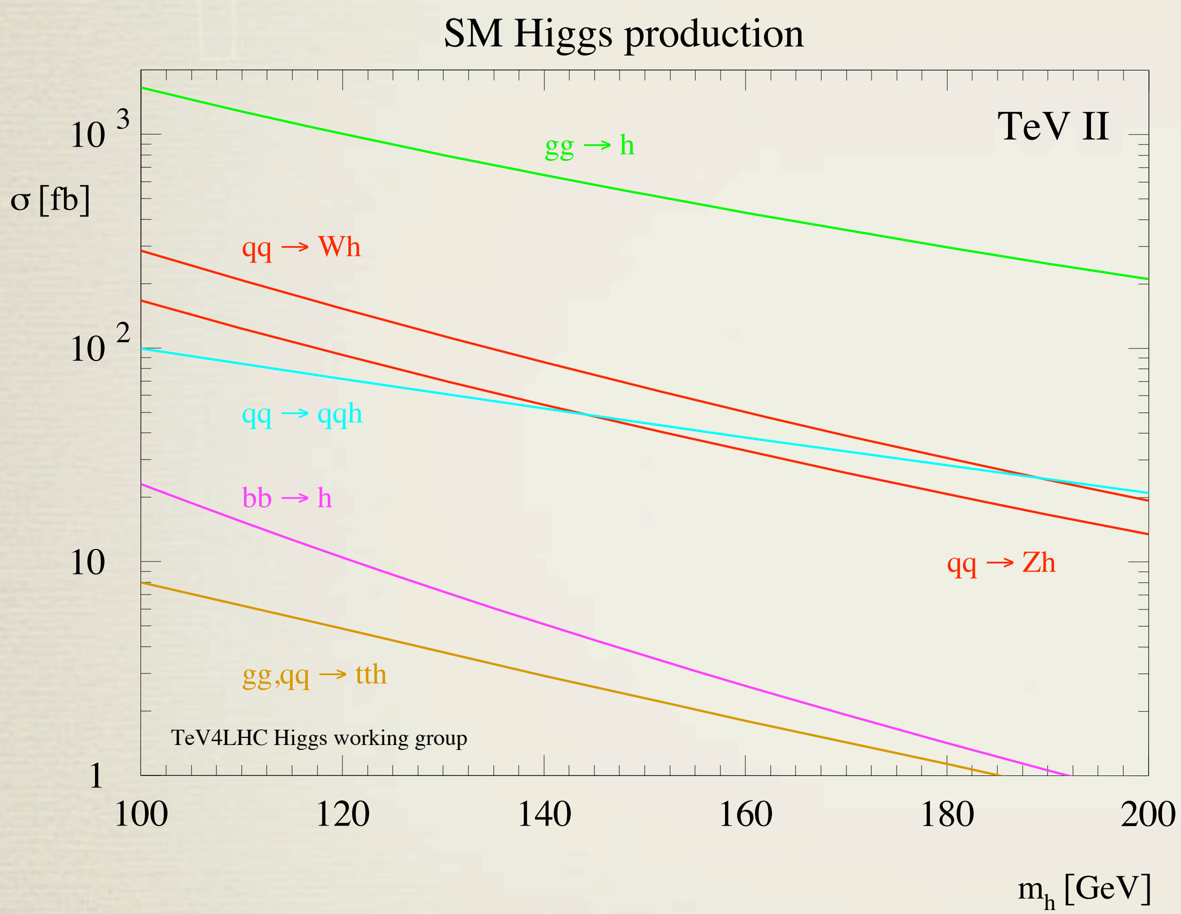


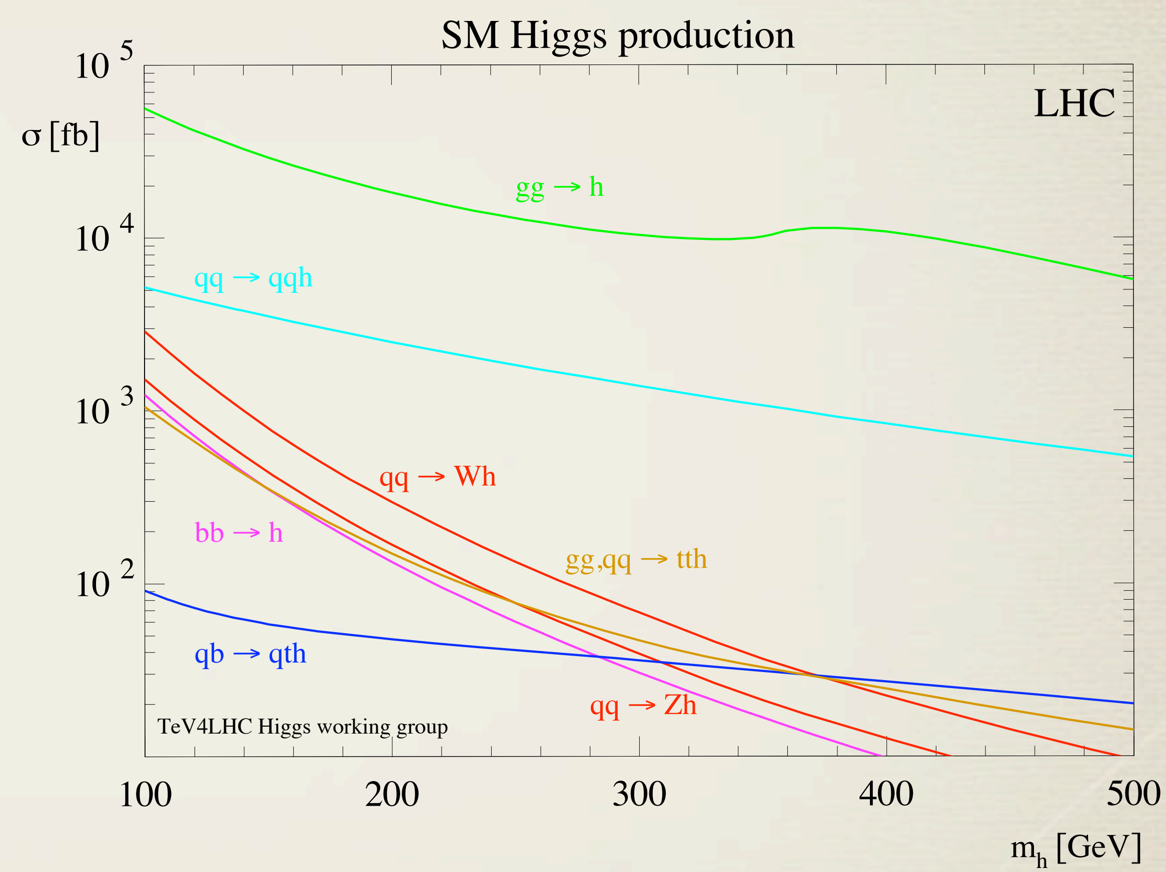
Can't do LEP search, √s not fixed at hadron machine

Any hadron collider search must confront backgrounds

Overview of Higgs cross sections

- Gluon-fusion dominant at both colliders; WH next at Tevatron, WBF at LHC

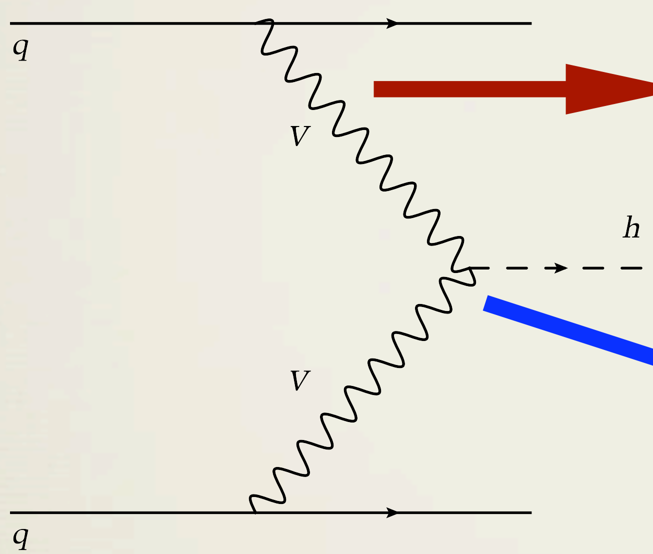




- <u>SUSY</u>: $g_{bbh} \sim \tan^2\beta$; becomes dominant at $\tan\beta \sim 10$

Weak boson fusion: effective W/Z

- Important throughout large region of Higgs mass and in many decay modes; forward jets give experimental handle
- First approximation: inclusive cross section for $M_H >> M_{W,Z}$



$$\frac{1}{(p_1-p_3)^2 - M_V^2} \quad \Rightarrow \quad \text{peaked at } p_1 \cdot p_3 = 0$$

$$\Rightarrow \tau_{q\to V} \quad \sim \quad 1/M_V$$

$$\tau_{VV\to h} \sim 1/M_H \ll \tau_{q\to V}$$

- Should be able to factorize, think of V as a parton in q

$$\sigma_{qq\to VV\to h} = \int dz_1 dz_2\, f_{q/V_1}(z_1) f_{q/V_2}(z_2)\, \sigma_{VV\to h}$$

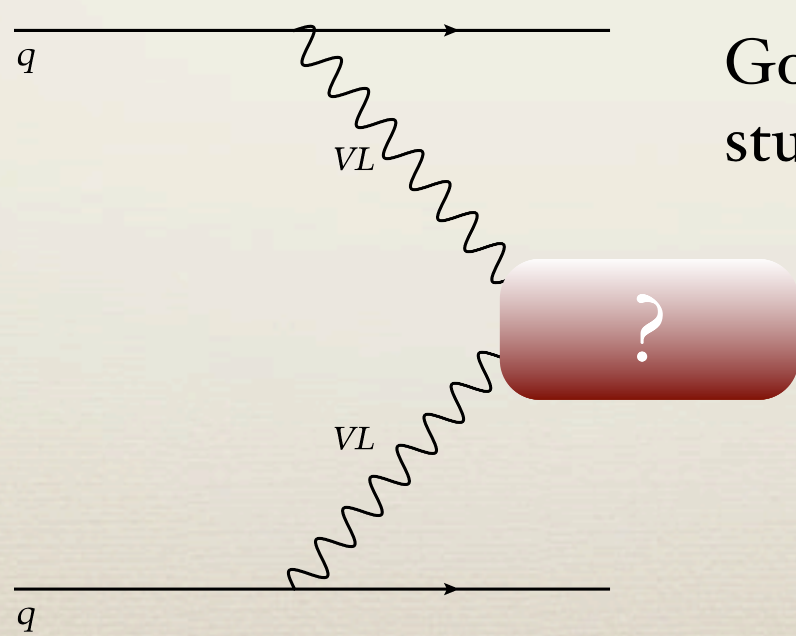
WBF + the equivalence theorem

- Can derive when $M_V \ll \sqrt{s}$ (small angle scattering dominated)

$$\begin{aligned}
\sigma_{q_1 q_2 \to VV \to h} &= \int_{2M_V/\sqrt{\hat{s}}}^{1} dz_1 \int_{2M_V/\sqrt{\hat{s}}}^{1} dz_2\, f_{q/V_L}(z_1) f_{q/V_L}(z_2)\, \sigma_{V_L V_L \to h}(z_1 z_2 \hat{s}) \\
\sigma_{V_L V_L \to h}(x) &= \frac{\pi}{36}\, g^2_{HVV}\, \frac{x}{M_V^2} \delta(x - M_H^2) \\
f_{q/V_L}(z) &= \frac{g_v^2 + g_a^2}{4\pi^2} \frac{1-z}{z}
\end{aligned}$$

$\boxed{\text{Exercise: Derive this}}$

- Angular momentum cons. prevents emission of transverse boson with forward quark: $\bar{u}^{\pm}(p\hat{z})\, \not{\epsilon}\, u^{\pm}(p'\hat{z}) \Rightarrow \text{Set } \not{\epsilon} = \gamma^{1,2} \Rightarrow \xi^{\dagger}_{\pm} \sigma^{1,2} \xi_{\pm} = 0$

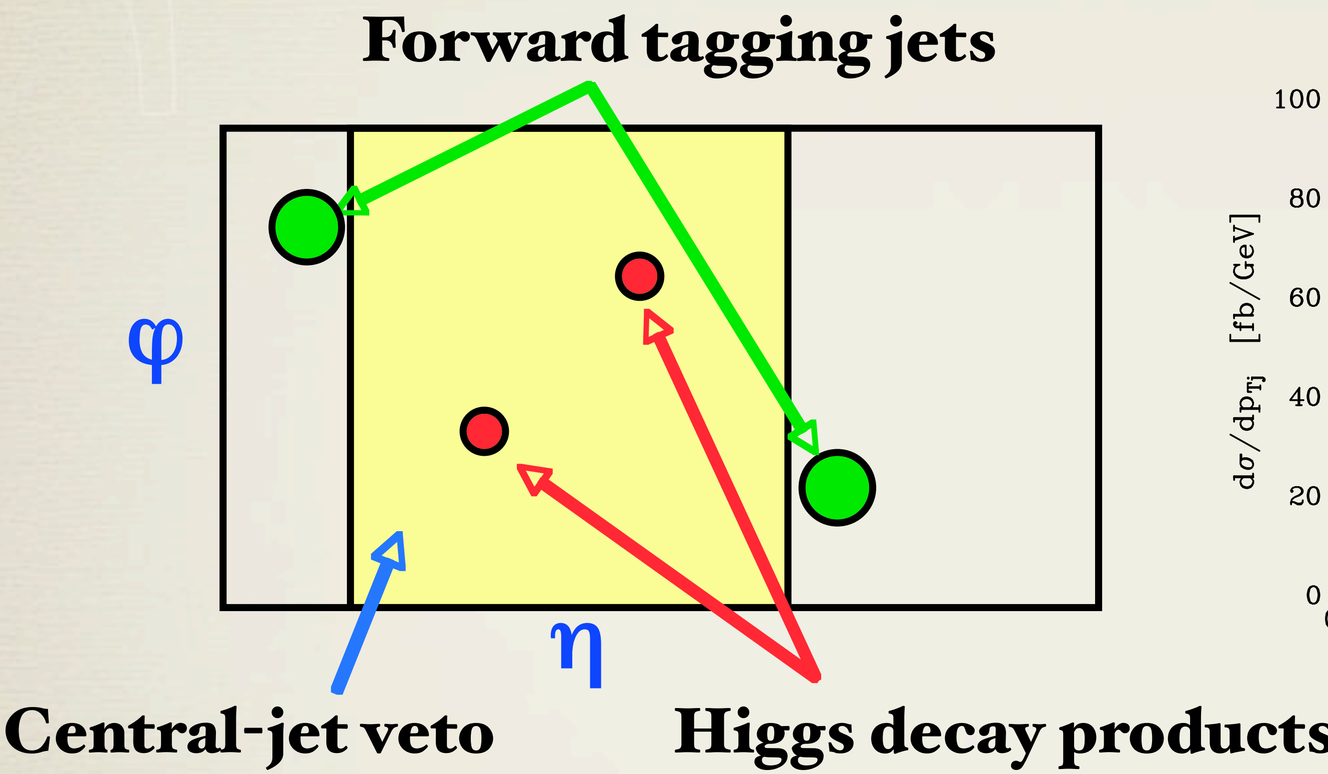


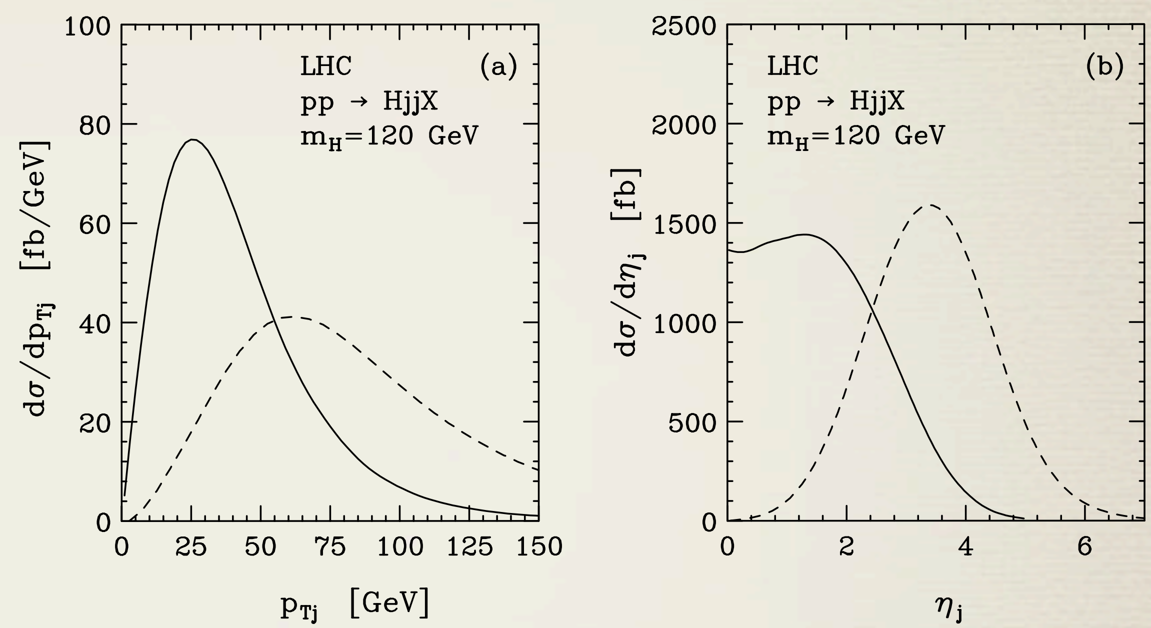
Good channel to study strong EWSB

Dawson 1984; Chanowitz, Gaillard 1985

Kinematics of WBF

- Two energetic (p_T~40 GeV) jets with large rapidity separation





Rainwater, Zeppenfeld 1999 and many others... check refs+citations

- Extra gluon emission suppressed; impose central jet veto

$$\mathcal{M}(q_1 q_2 \to q_3 q_4 h + g) \quad \propto \quad \mathcal{M}(q_1 q_2 \to q_3 q_4 h)\, T^a \left\{ \frac{p_3 \cdot \epsilon_g^a}{p_3 \cdot p_g} + \frac{p_4 \cdot \epsilon_g^a}{p_4 \cdot p_g} - \frac{p_1 \cdot \epsilon_g^a}{p_1 \cdot p_g} - \frac{p_2 \cdot \epsilon_g^a}{p_2 \cdot p_g} \right\}$$

$$\to \quad 0 \;\; \text{since} \;\; p_1 \parallel p_3, \;\; p_2 \parallel p_4$$

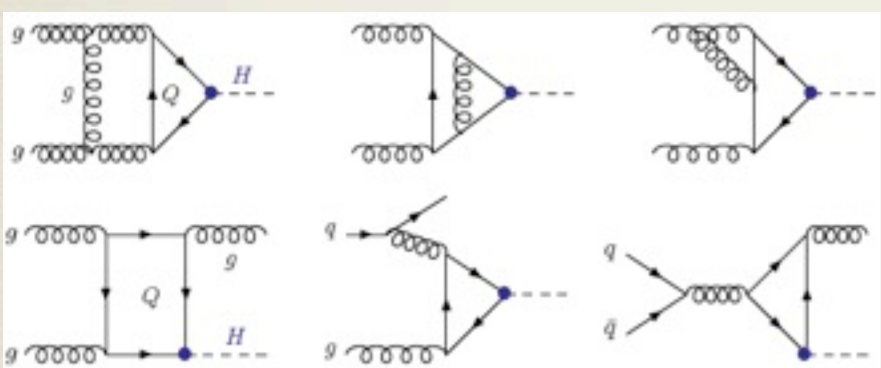
Exercise: Derive this

Gluon fusion production

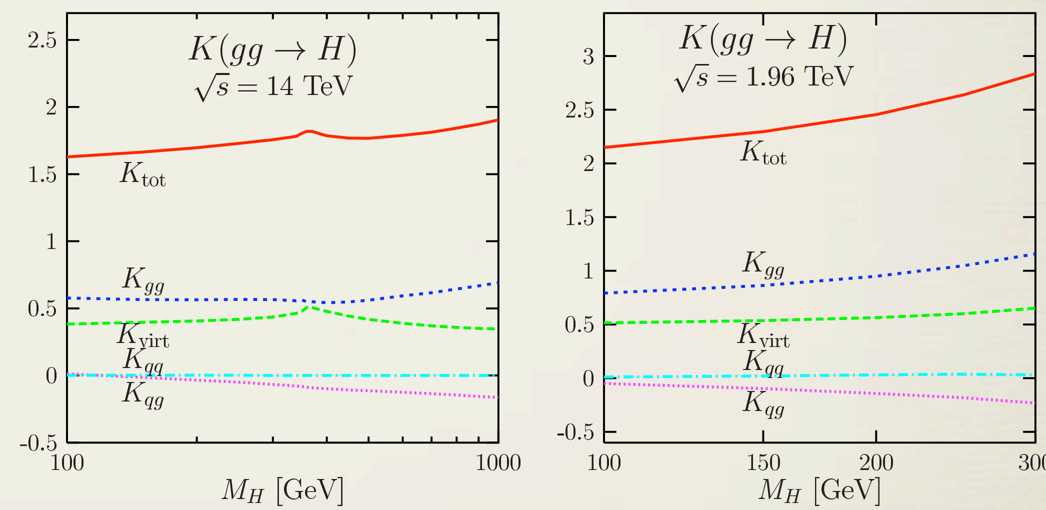
- Largest mode at Tevatron and LHC; through top-quark loops

$$\sigma^{LO}_{gg\to h} = \frac{G_F\alpha_s^2}{288\pi\sqrt{2}}\left|\frac{3}{4}\sum_Q \mathcal{F}_{1/2}(\tau_Q)\right|^2 \delta(1-z), \quad \tau_Q = \frac{M_H^2}{4m_Q^2}. \quad z = \frac{M_H^2}{\hat{s}}$$

- NLO QCD corrections require 2-loop virtual, 1-loop real-virtual



Dawson; Djouadi, Graudenz, Spira, Zerwas 1991, 1995



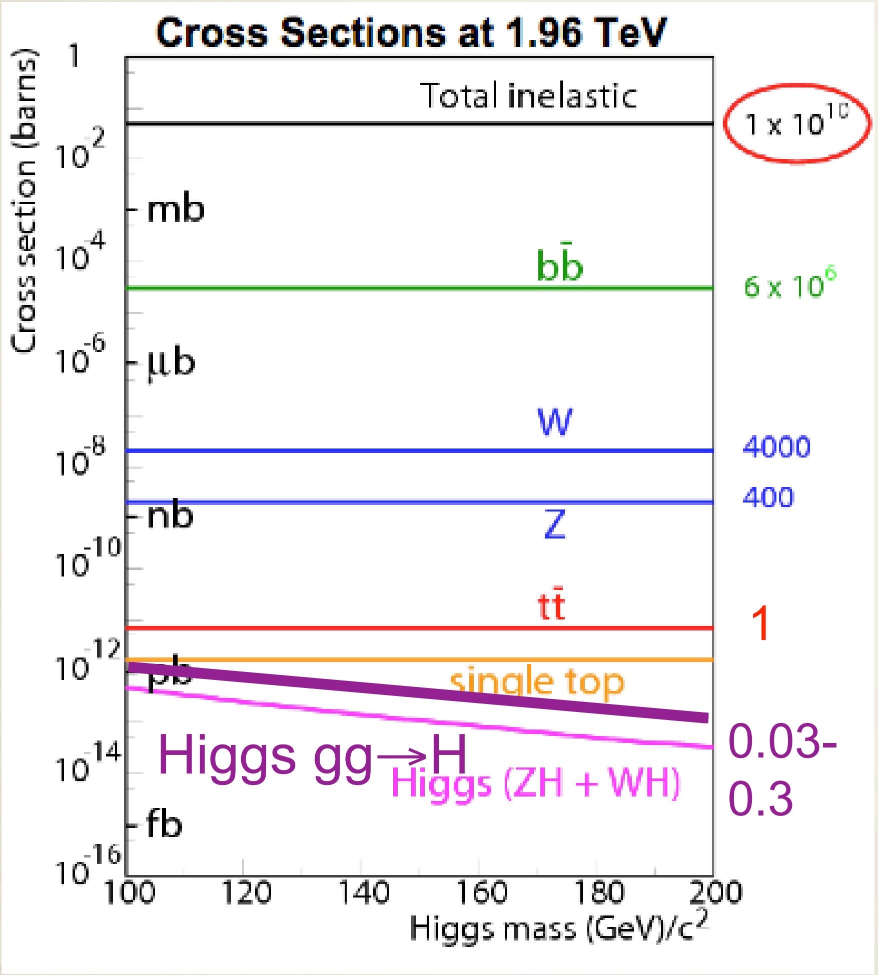
- Can reach $K_{NLO}=\sigma_{NLO}/\sigma_{LO}\approx 2$ at LHC, 3 at Tevatron!

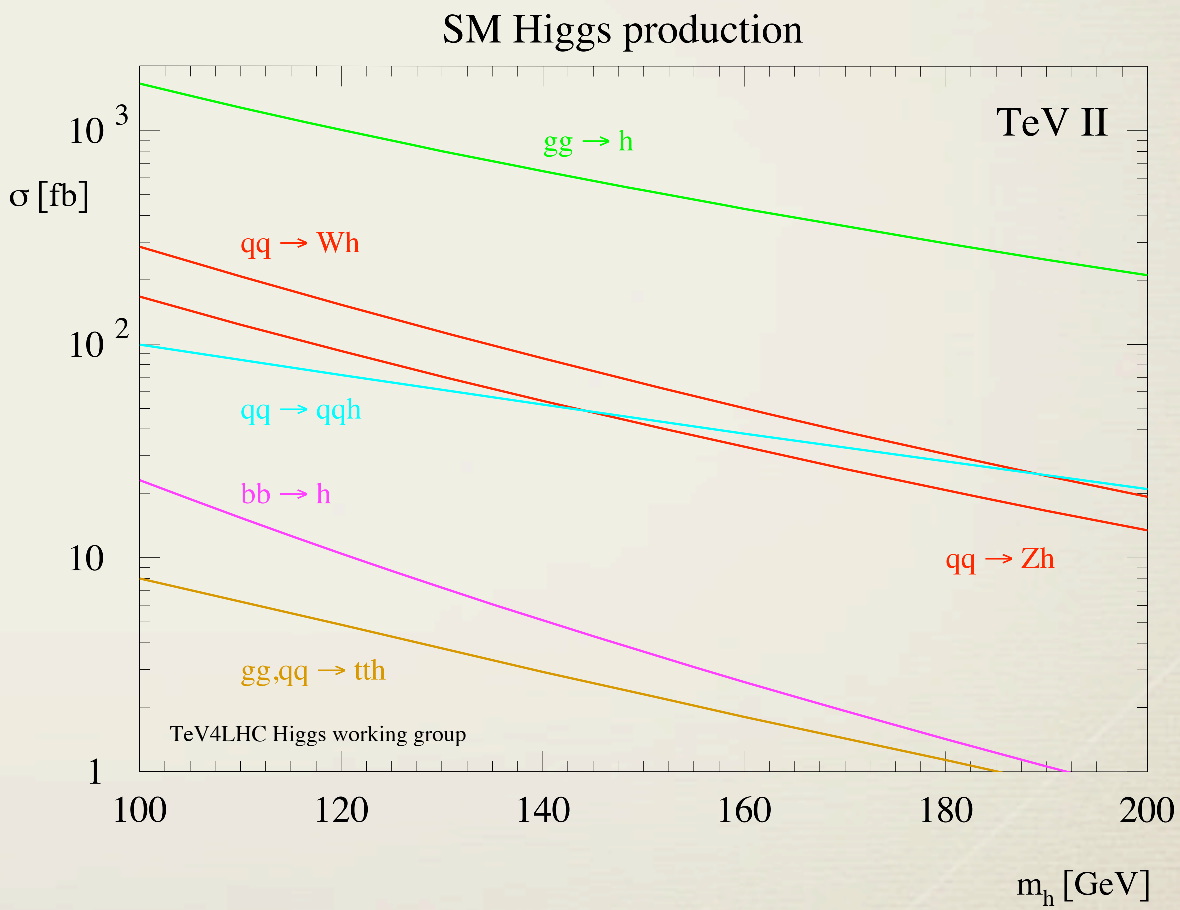
Work through calculation in Appendix slides

Searches at the Tevatron and LHC

Tevatron analysis overview

- Inclusive gg→h→bb not feasible at low masses
- WBF only slightly adds to analyses designed for other channels



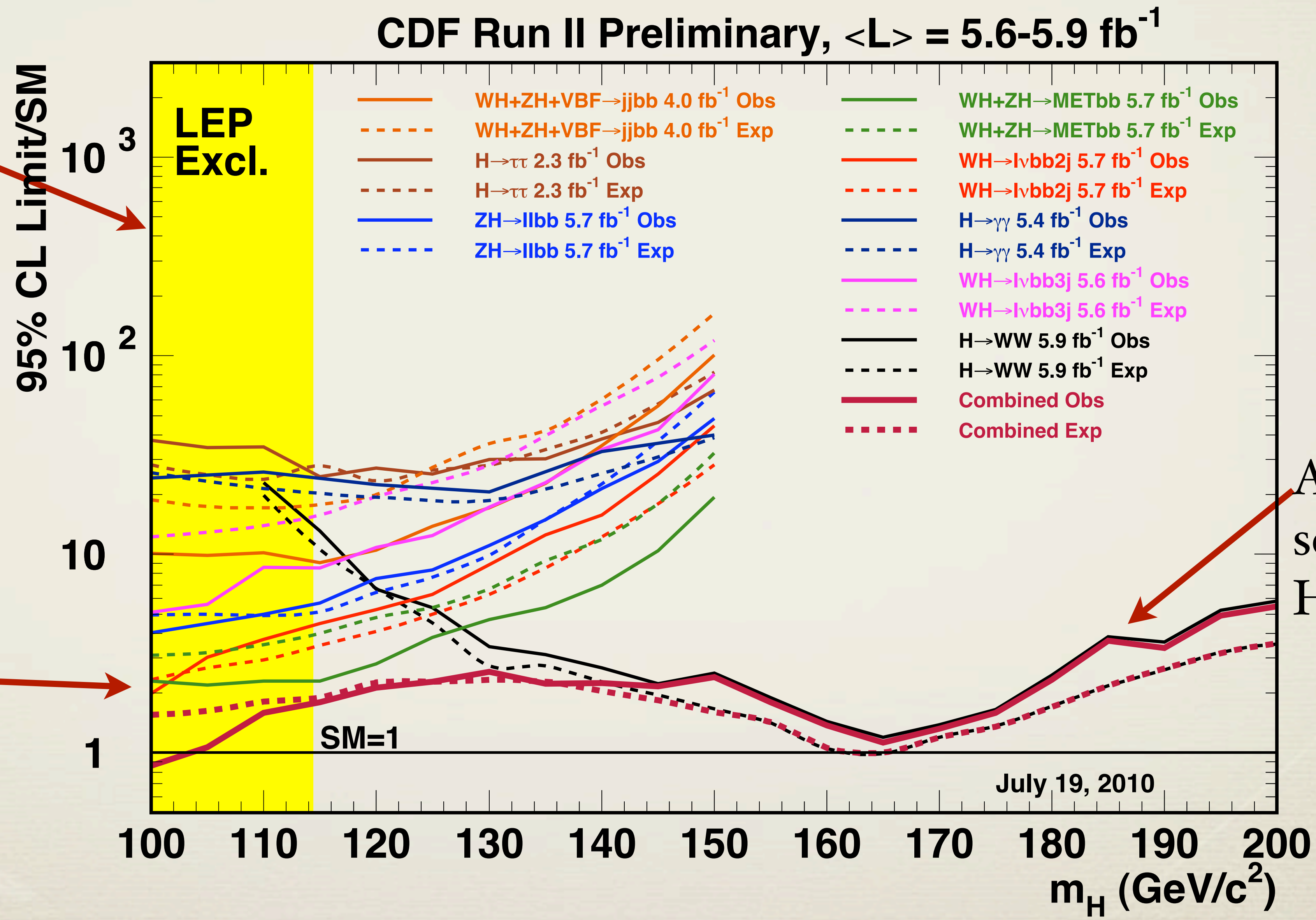


Combined exclusion limit

- No observation, so collaborations report 95% C.L. exclusion by combining many possible channels

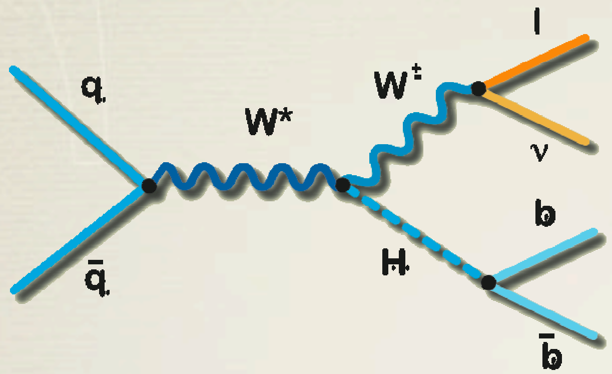
Cross section excluded, normalized to SM

Most sensitive: Wh,Zh→lνbb,ννbb



At high mass, all sensitivity from H→WW

Wh→lνbb analysis



Basic acceptance cuts :

- $p_T^l > 20$ GeV
- $\not{E}_T > 20$ GeV
- 2-3 jets, 1-2 b-tags
- $p_T^j > 20$ GeV

Process	1 tag	2 tags
All Pretag Cands.	50644.0 ± 0.0	57174.0 ± 0.0
WW	56.2 ± 6.2	0.4 ± 0.1
WZ	23.0 ± 1.7	4.8 ± 0.5
ZZ	0.8 ± 0.1	0.2 ± 0.0
TopLJ	121.3 ± 17.1	23.8 ± 3.9
TopDil	48.8 ± 6.8	14.1 ± 2.3
STopT	64.0 ± 9.3	1.8 ± 0.3
STopS	40.6 ± 5.7	12.8 ± 2.1
Z+jets	37.4 ± 5.5	2.1 ± 0.3
Total MC	392.0 ± 35.0	59.9 ± 7.5
Wbb	538.7 ± 162.5	70.3 ± 22.5
Wcc/Wc	489.1 ± 150.9	6.8 ± 2.3
Total HF	1027.8 ± 312.3	77.1 ± 24.7
Mistags	458.0 ± 57.9	2.2 ± 0.6
Non-W	135.5 ± 54.2	9.0 ± 3.6
Total Prediction	2013.3 ± 324.1	148.2 ± 26.1
WH100	9.5 ± 0.8	2.9 ± 0.3
WH105	8.6 ± 0.7	2.7 ± 0.3
WH110	7.6 ± 0.6	2.4 ± 0.3
WH115	6.3 ± 0.5	2.0 ± 0.2
WH120	4.9 ± 0.4	1.6 ± 0.2
WH125	4.0 ± 0.3	1.3 ± 0.2
WH130	3.1 ± 0.3	1.0 ± 0.1
WH135	2.3 ± 0.2	0.7 ± 0.1
WH140	1.5 ± 0.1	0.5 ± 0.1
WH145	1.0 ± 0.1	0.3 ± 0.0
WH150	0.7 ± 0.1	0.2 ± 0.0
Observed	1998.0 ± 0.0	156.0 ± 0.0

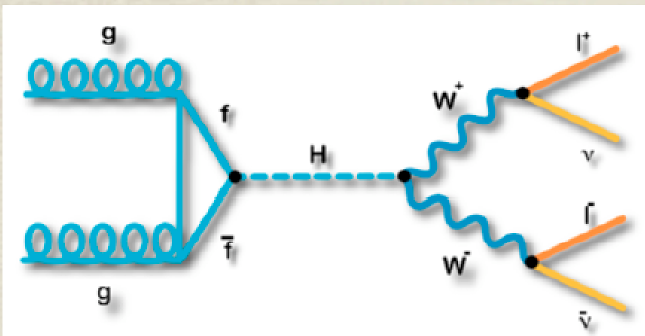
From CDF, after event selection

``Estimating the background contribution after applying the event selection to the WH candidate sample is an elaborate process"

W+jets: normalization from data; heavy-flavor fraction from ALPGEN for shape (tree-level)+data for norm.; check with NLO

Combined theory +experiment error

h→WW→lνlν



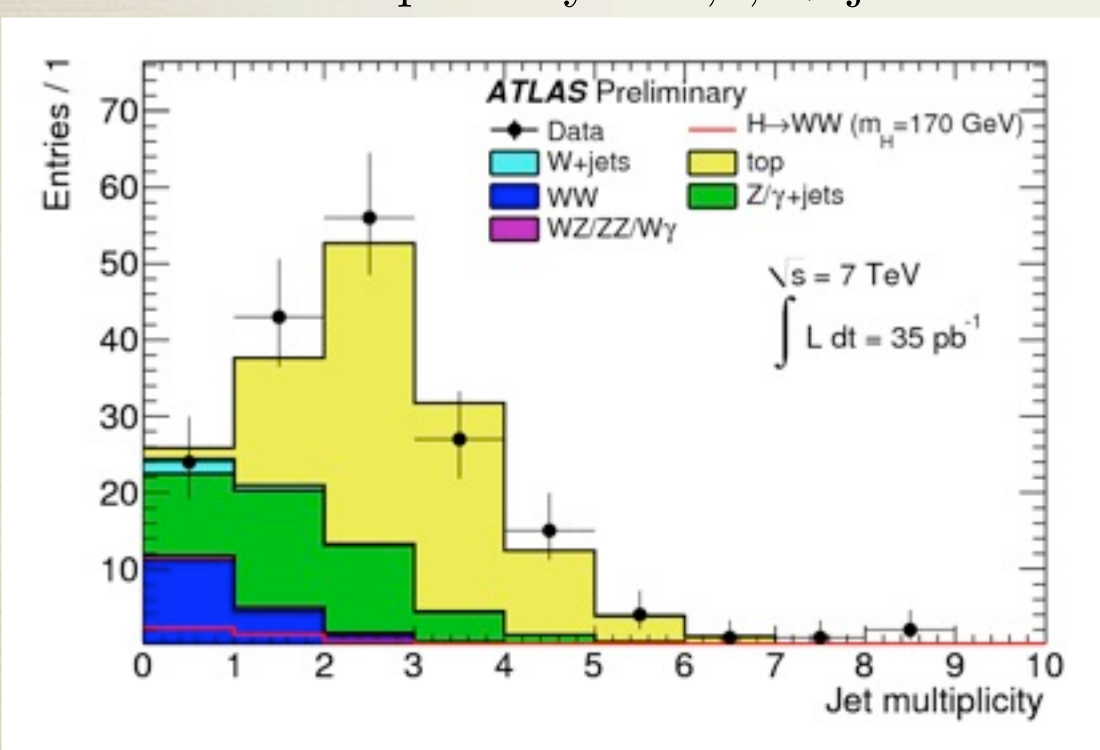
Basic acceptance cuts (CDF) :

- $p_T^{l1} > 20$ GeV
- $p_T^{l2} > 10$ GeV
- $\not{E}_T > 15 - 25$ GeV

  (for various final states)
- Look separately at 0,1,2+ jet bins

from D0

	ee pre-selection	ee final	$e\mu$ pre-selection	$e\mu$ final
$Z \to ee$	218695 ± 704	108 ± 14	280.6 ± 3.3	$0.0^{+0.1}_{-0.0}$
$Z \to \mu\mu$	–	–	274.6 ± 0.9	5.8 ± 0.1
$Z \to \tau\tau$	1135 ± 16	1.4 ± 0.5	3260 ± 3	7.3 ± 0.1
$t\bar{t}$	131.4 ± 1.4	39.9 ± 0.8	272.0 ± 0.3	82.5 ± 0.2
W+jets	241 ± 5	98 ± 3	183 ± 4	78.6 ± 2.8
WW	172.2 ± 2.6	66.8 ± 1.6	421.2 ± 0.1	154.7 ± 0.1
WZ	112.5 ± 0.2	9.68 ± 0.05	20.5 ± 0.1	6.6 ± 0.1
ZZ	98.2 ± 0.2	7.68 ± 0.07	5.3 ± 0.1	0.60 ± 0.01
Multijet	1351 ± 55	$1.7^{+2.0}_{-1.7}$	279 ± 168	$1.1^{+9.6}_{-1.1}$
Signal ($M_H = 165$ GeV)	9.45 ± 0.01	6.13 ± 0.01	17.1 ± 0.01	12.2 ± 0.1
Total Background	221937 ± 707	332 ± 15	4995 ± 168	337 ± 10
Data	221530	336	4995	329

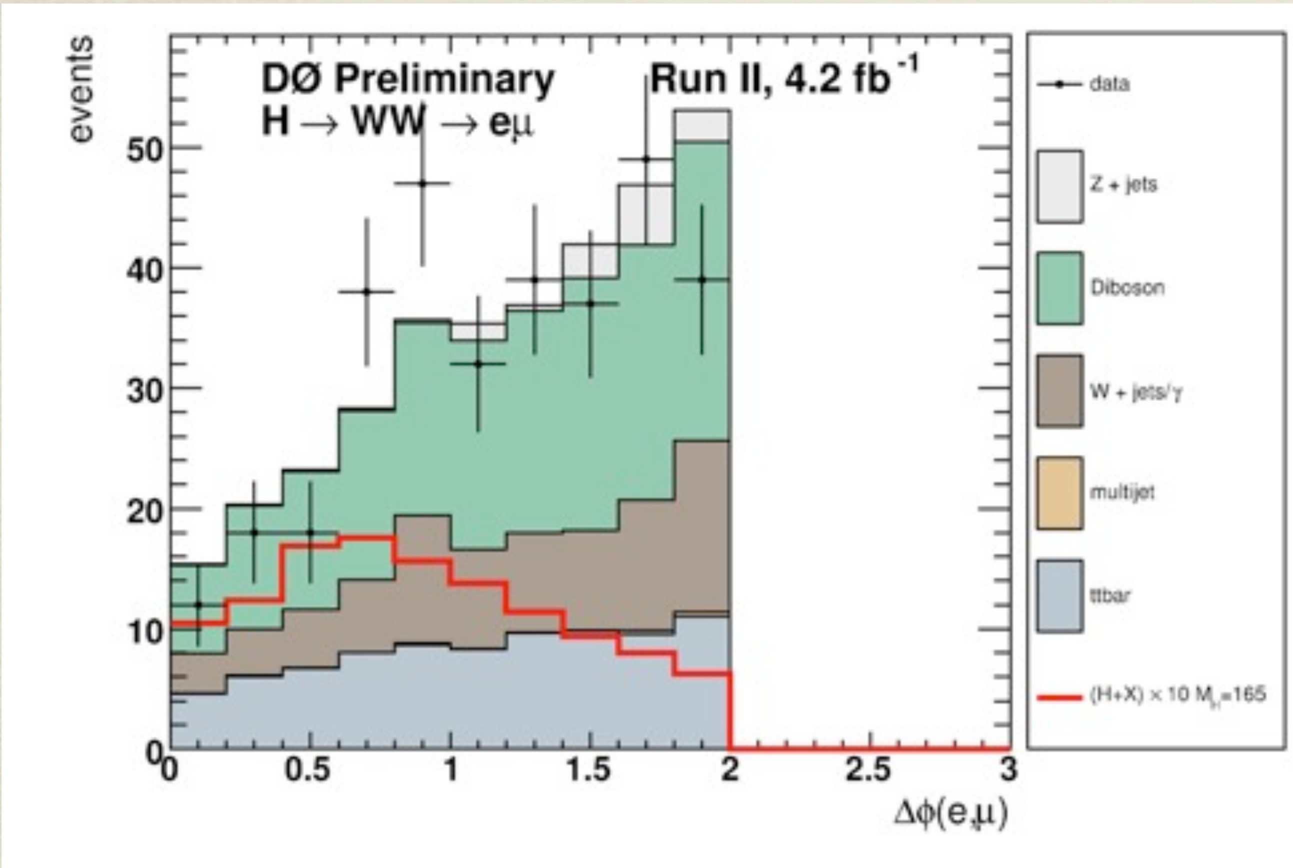


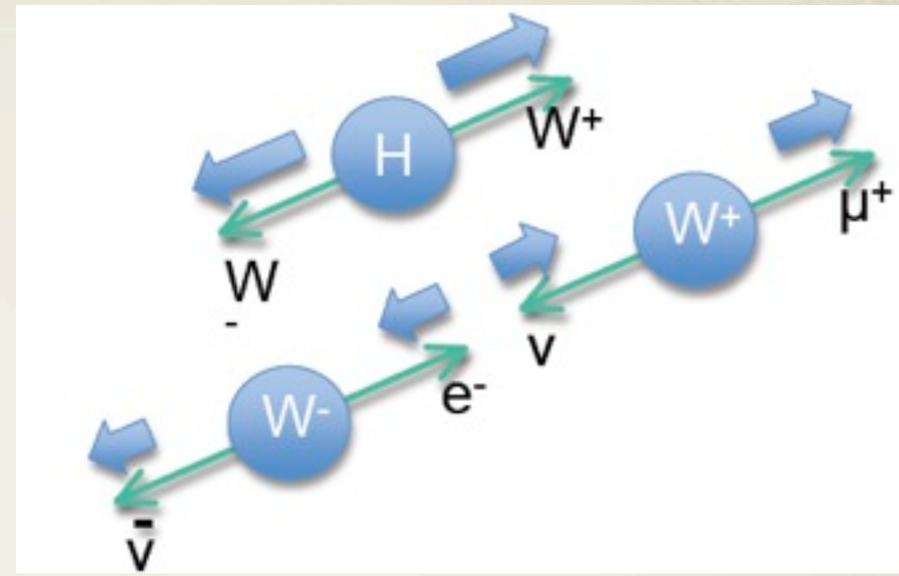
<u>tt</u>: affects 2-jet bin; rate taken from NNLO calculations

<u>W+jets</u>: jet fakes lepton; from ALPGEN+data-driven methods

<u>WW</u>: taken from MC@NLO

Kinematic discriminants

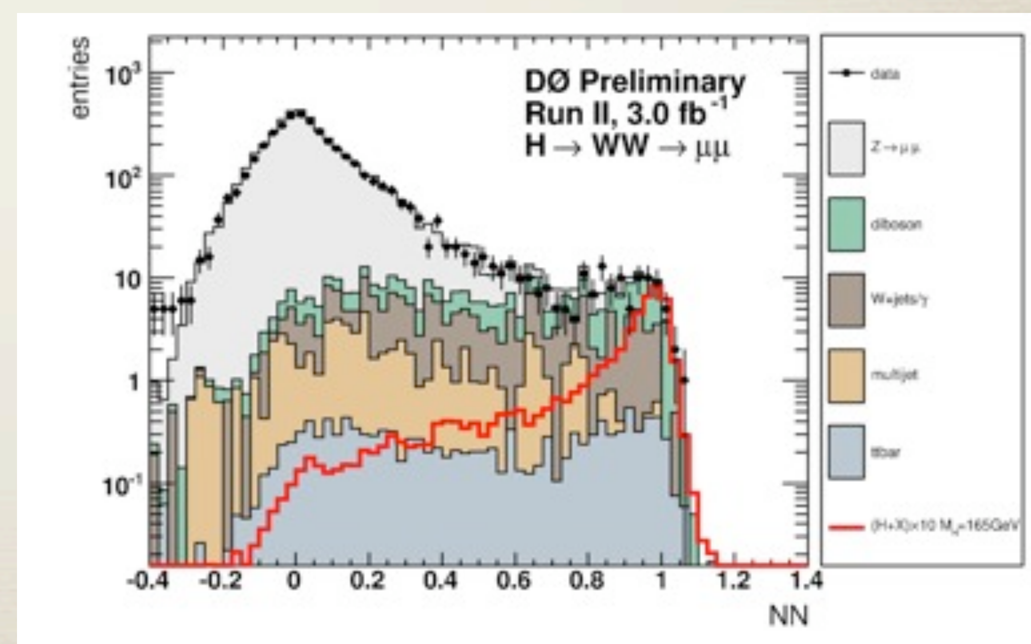




A primary handle for 0-jet bin: $\Delta\varphi_{ll}$

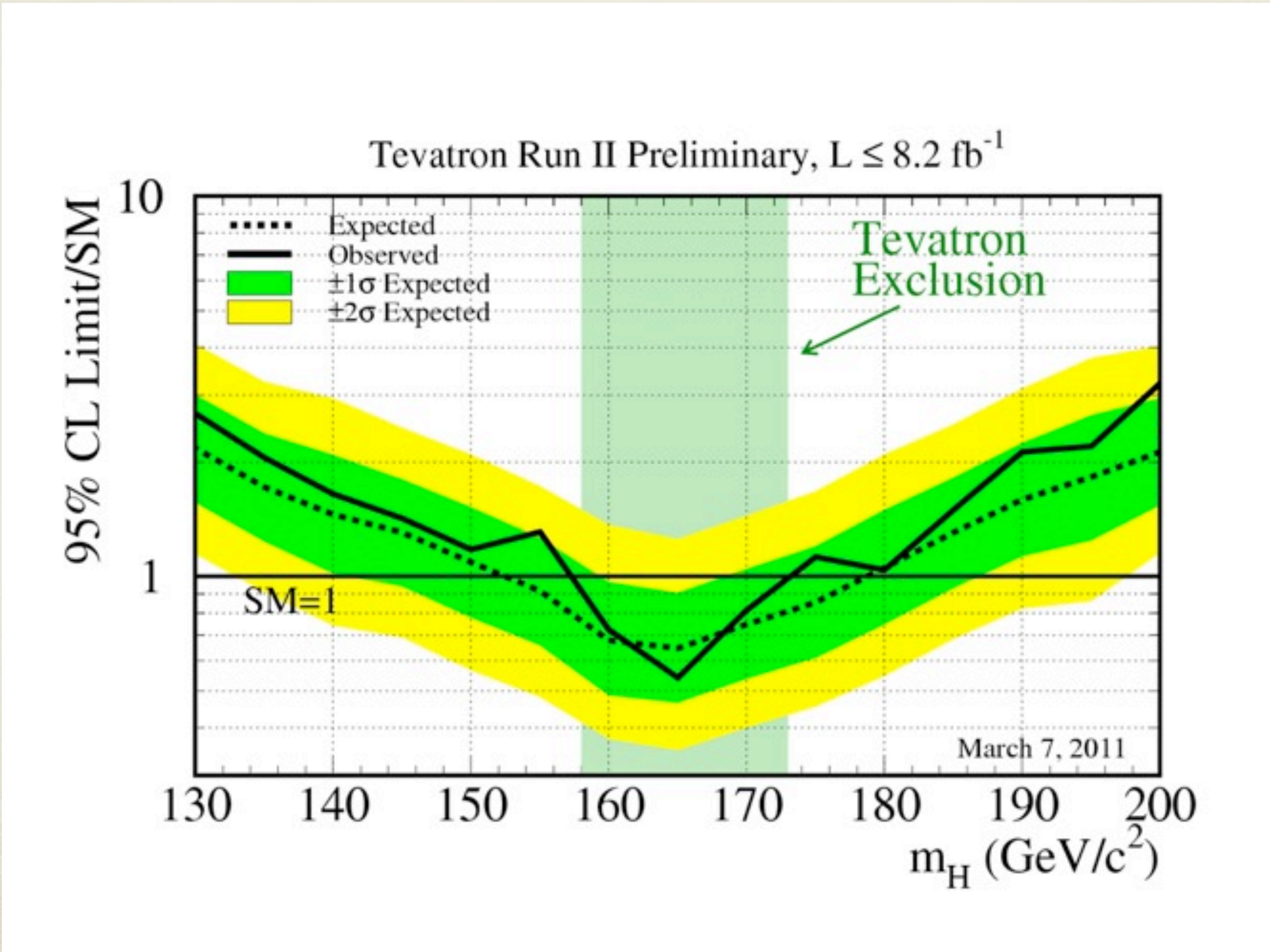
Spin correlation: leptons in same direction

NN Analysis Variables	
p_T of leading lepton	$p_T(\ell_1)$
p_T of trailing lepton	$p_T(\ell_2)$
Minimum of both lepton qualities	$\min(q_{\ell 1}, q_{\ell 2})$
Vector sum of the transverse momenta of the leptons:	$p_T(\ell_1) + p_T(\ell_2)$
Scalar sum of the transverse momenta of the jets:	$H_T = \sum_i \lvert p_T(\mathrm{jet}_i)\rvert$
Invariant mass of both leptons	$M_{\mathrm{inv}}(\ell_1, \ell_2)$
Minimal transverse mass of one lepton and $\not{E}_T$	$M_T^{\min}$
Missing transverse energy	$\not{E}_T$
Scalar transverse energy	E_T^{scalar}
Azimuthal angle between selected leptons	$\Delta\phi(\ell_1, \ell_2)$
Solid angle between selected leptons ($e\mu$ only)	$\Delta\Theta(\ell_1, \ell_2)$
ΔR between selected leptons ($e\mu$ only)	$\Delta R(\ell_1, \ell_2)$
Azimuthal angle between leading lepton and $\not{E}_T$	$\Delta\phi(\not{E}_T, \ell_1)$
Azimuthal angle between trailing lepton and $\not{E}_T$	$\Delta\phi(\not{E}_T, \ell_2)$



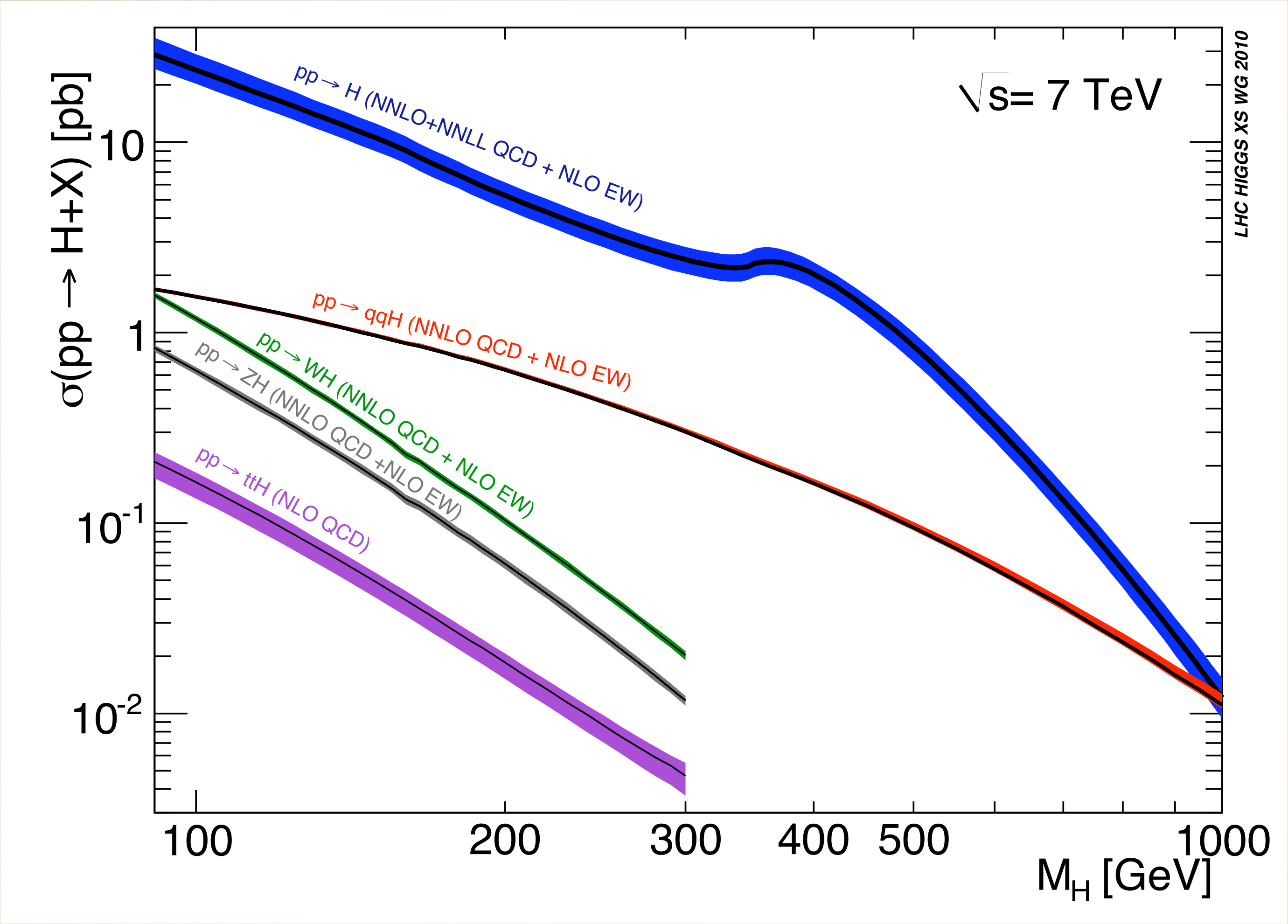
High-mass exclusion

- Combine CDF+D0 exclusion limits: 158 ≤ M_H ≤ 173 GeV at 95% CL

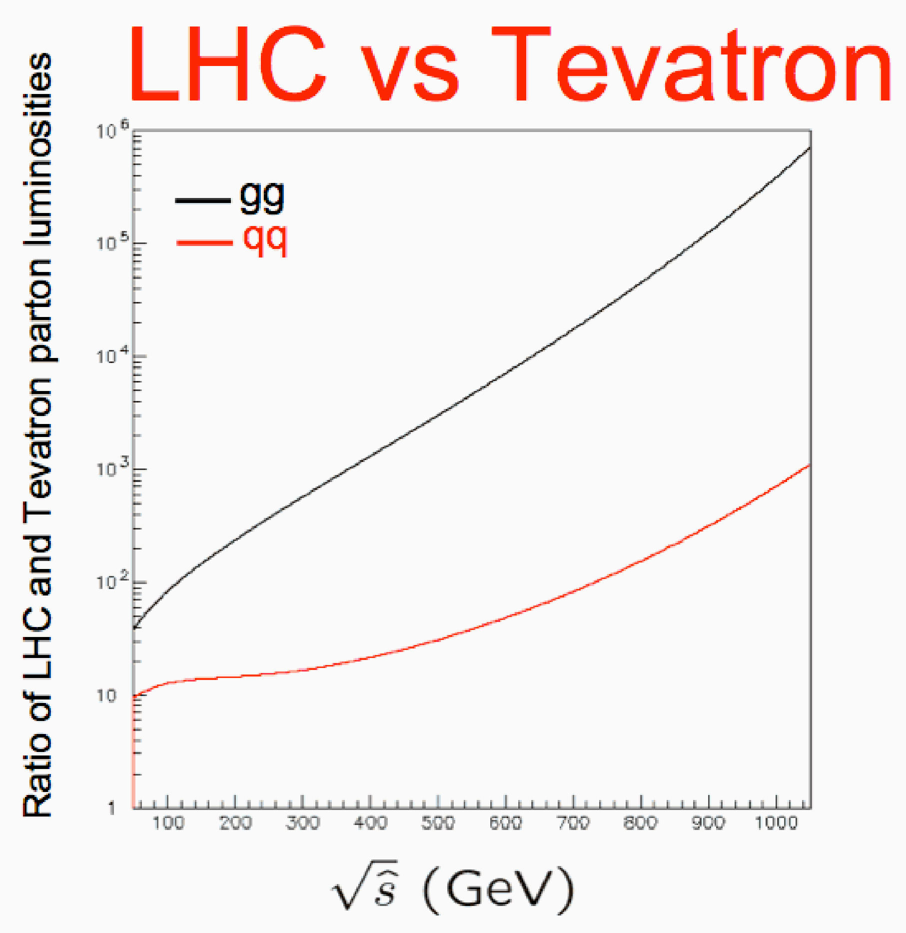


LHC physics overview

- Qualitative change; gluons now overwhelm scattering rate

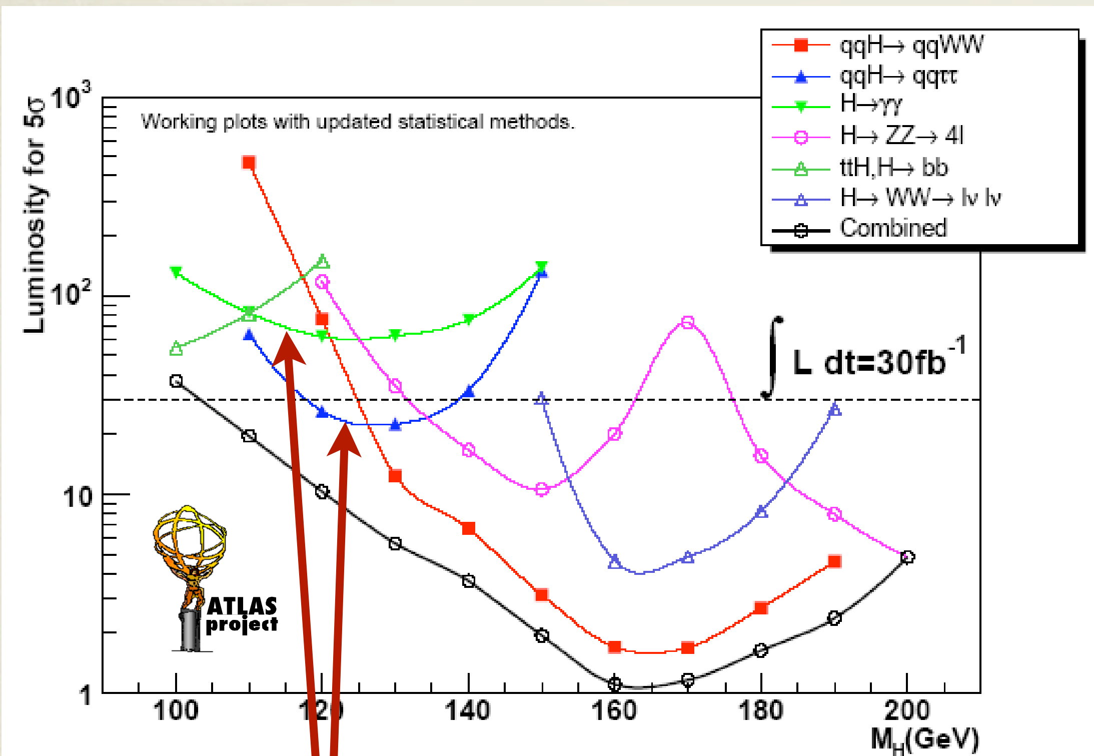


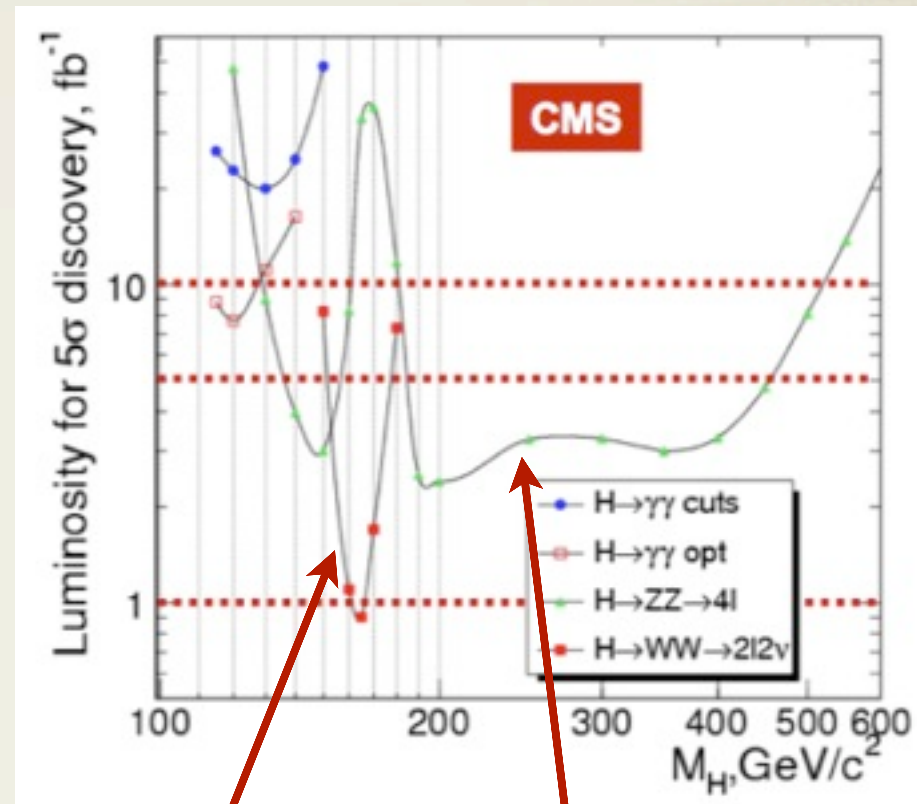
LHC Higgs cross section working group, 2011



LHC Higgs summary

- Entire mass range covered, much with multiple modes





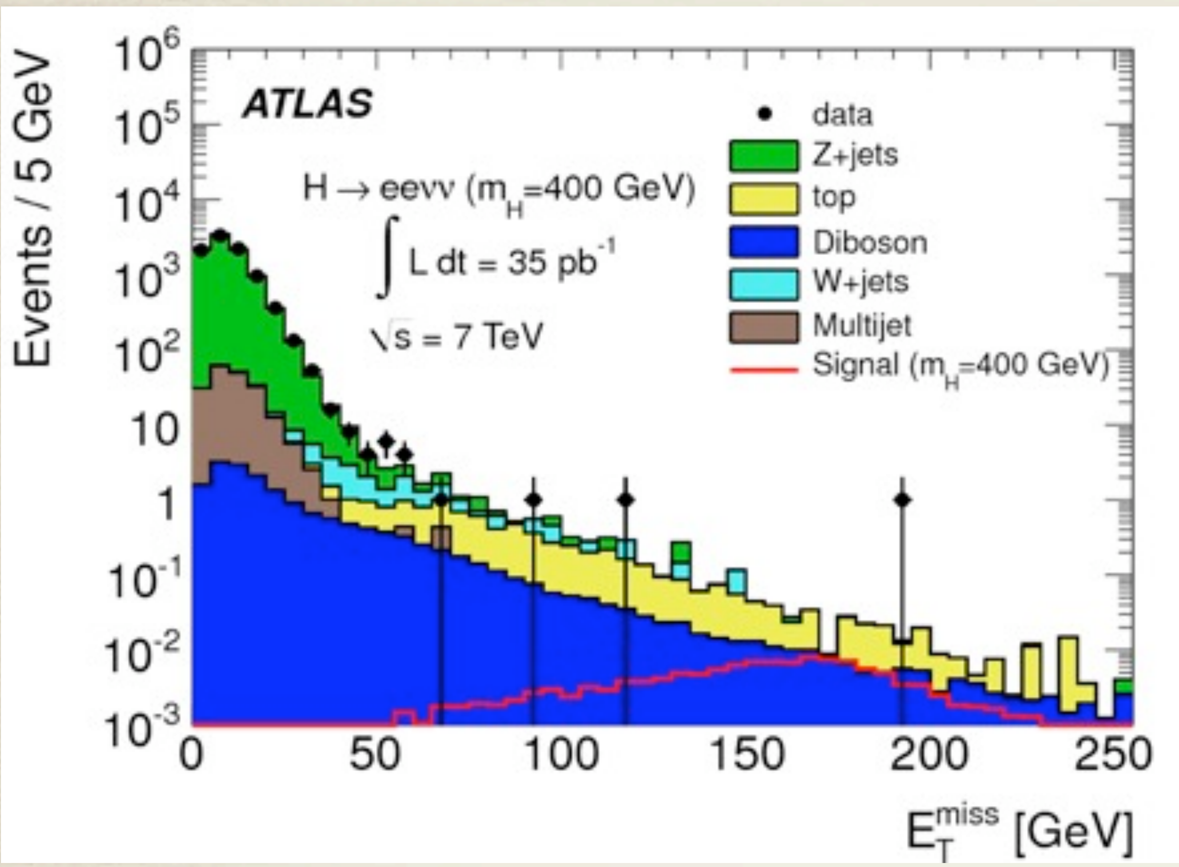
h→γγ, WBF h→ττ cover low mass range

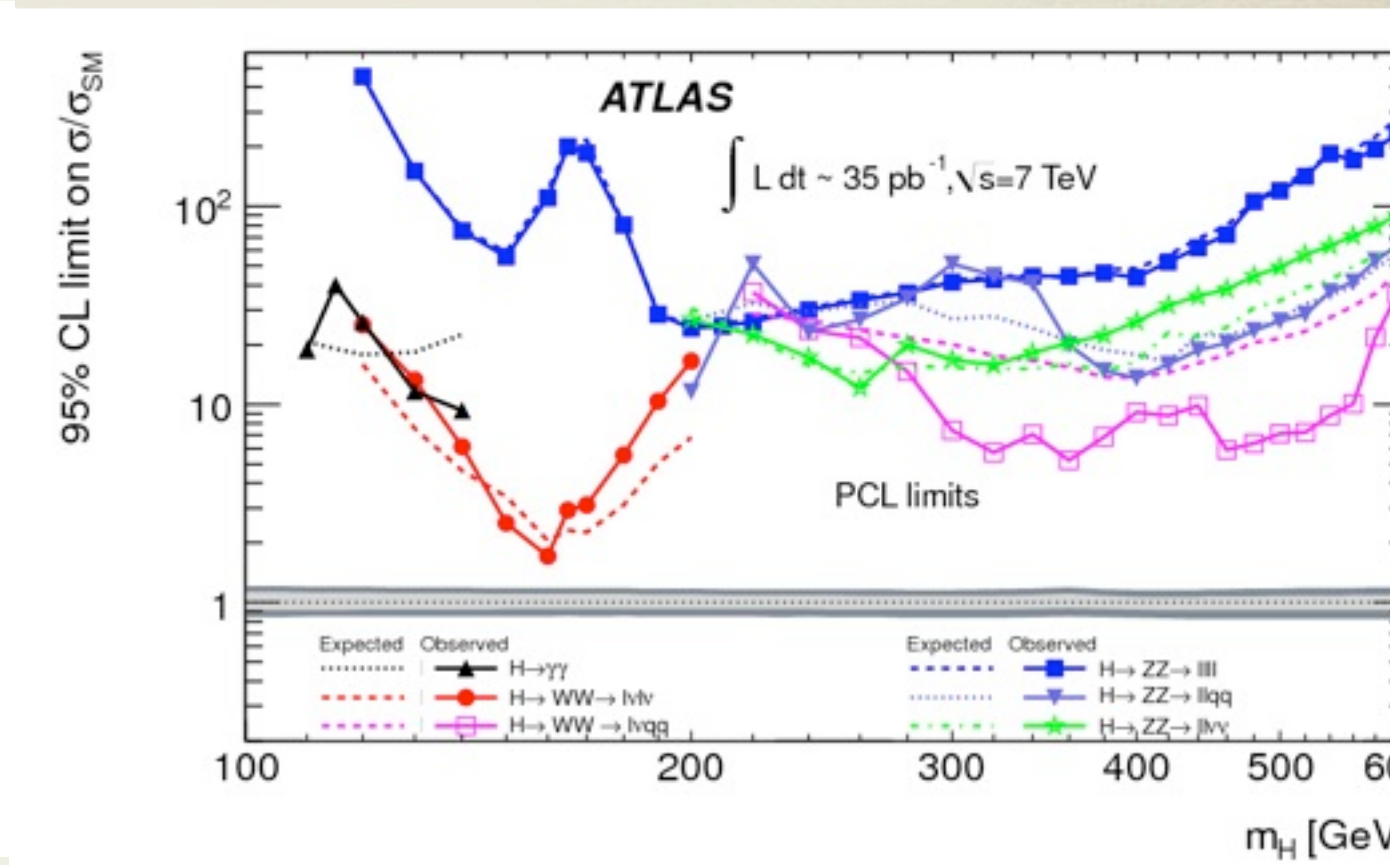
h→ZZ→4l assures discovery over entire high-mass range

h→WW→lνlν again important at LHC

M_H>130 GeV: only few fb^{-1}
M_H<130 GeV: 20-30 fb^{-1}

$h \to ZZ \to l_1 l_1 l_2 l_2$





- tt background estimated from Monte Carlo
- Data from control regions used for Z+jets

- Dominant mode in the 220-280 GeV region

Global analysis of couplings

- Observe Higgs in many modes: gluon-fusion, WBF, W/Z+h (not discovery at LHC, but after M_H known using boosted methods Butterworth et al., 0802.2470)

$$\sigma_p \times BR(h \to xx) = \underbrace{\left(\frac{\sigma_p}{\Gamma_p}\right)_{SM}}_{\text{NP effects cancel, calculate}} \times \underbrace{\frac{\Gamma_p \Gamma_x}{\Gamma}}_{\text{measure}}$$

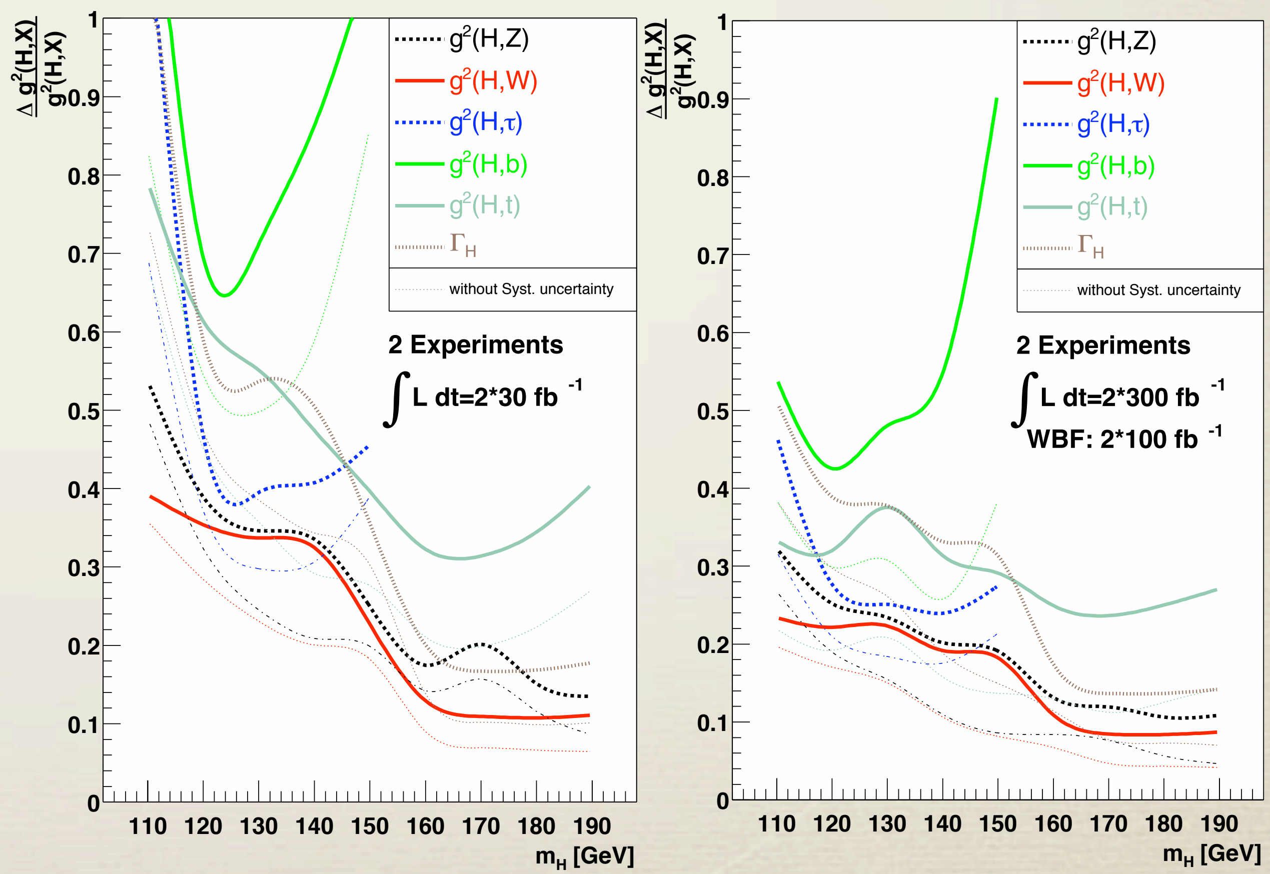
Scaling degeneracy if total width unknown:

$$\Gamma_i \to f\,\Gamma_i, \Gamma \to f^2\,\Gamma$$

Mild assumption: $g^2_{hVV} < 1.05 \times g^2_{hVV,SM}$

Allows any # of scalar doublets, new particles in loops, small contributions of scalar triplets

⇒Assumption+VBF measurement of $(\Gamma_V)^2/\Gamma$ breaks degeneracy



Duhrssen et al. 2004; Lafaye et al. 2009

Conclusions

- We must find a Higgs boson or something else which consistently breaks EW symmetry
- Phenomenology of Higgs intricate and highly dependent on its mass; detailed experimental program needed to find it
- Very sensitive to quantum effects; better have a good handle on QCD
- Stay tuned for LHC search results this summer!
- Potential to determine whether the Higgs is SM or not with LHC measurements

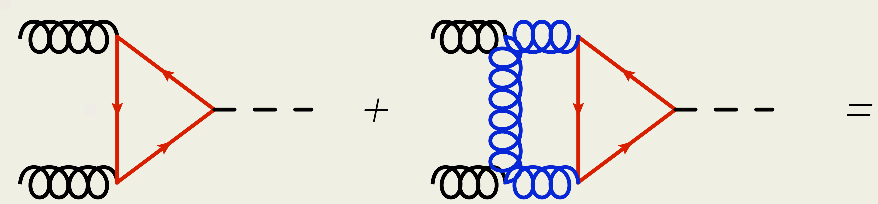
Appendix: Higgs production at NLO

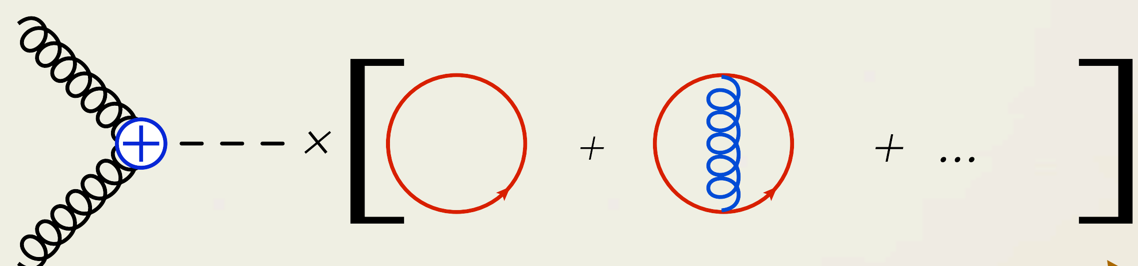
Effective interactions

- Can get exact 2-loop NLO corrections without effective interaction (Djouadi, Graudenz, Spira, Zerwas 1995), but next term too tough

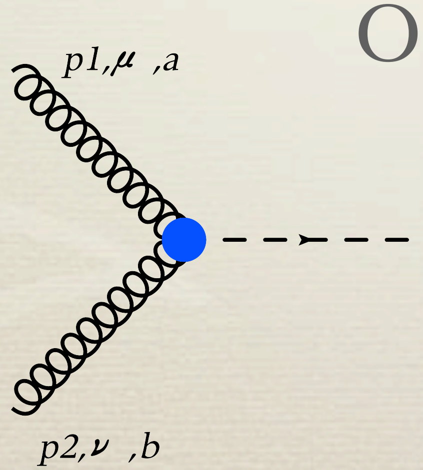
Effective field theory: exploit heavy mass of virtual particles

Two scales: M_{Higgs}, m_{top}





Only M_{Higgs} Only m_{top} $O(M^2_{\mathrm{Higgs}}/4m^2_{\mathrm{top}})$



$$= -i\frac{\alpha_s}{3\pi v}\left\{1+\frac{11}{4}\frac{\alpha_s}{\pi}\right\}\delta^{ab}\left[p_1\cdot p_2 g^{\mu\nu}-p_1^\nu p_2^\mu\right]$$

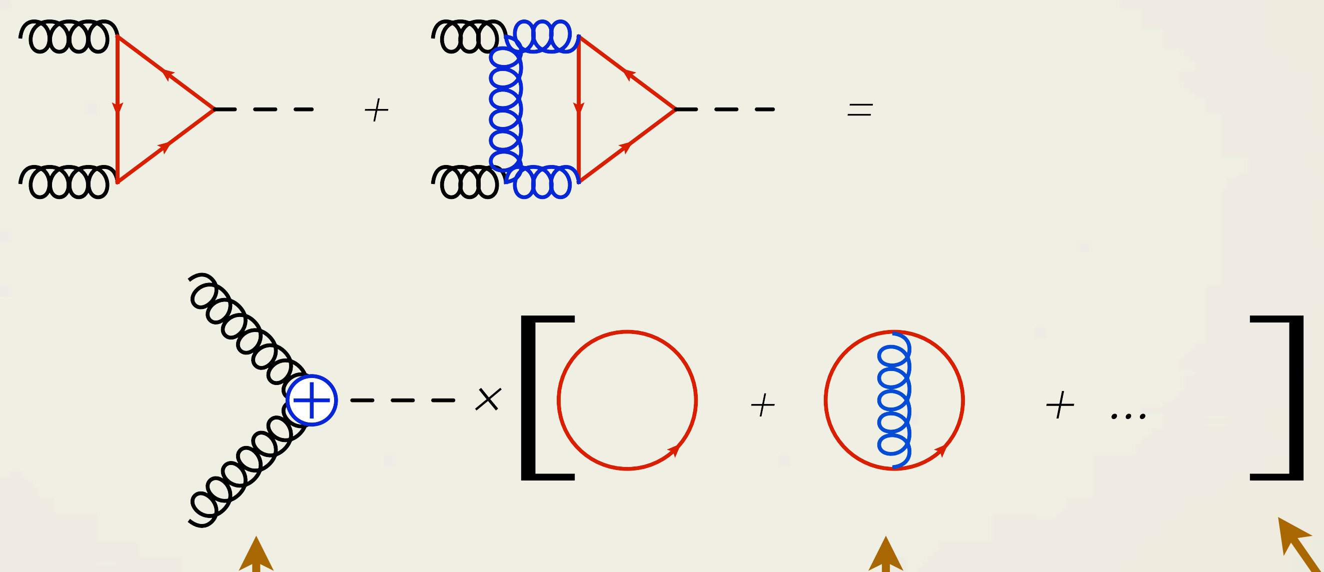
(also cubic, quartic gluon vertices)

Effective interactions

- Can get exact 2-loop NLO corrections without effective interaction (Djouadi, Graudenz, Spira, Zerwas 1995), but next term too tough

Effective field theory: exploit heavy mass of virtual particles

Two scales: M_{Higgs}, m_{top}



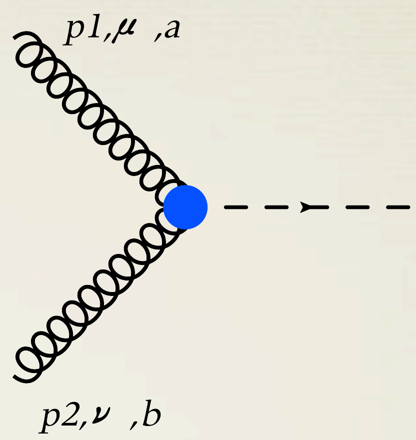
Only M_{Higgs} Only m_{top} $O(M^2_{\text{Higgs}}/4m^2_{\text{top}})$

$$\mathcal{L}_{eff} = \frac{\alpha_s}{12\pi}\left\{1 + \frac{11}{4}\frac{\alpha_s}{\pi}\right\}\frac{h}{v}F^a_{\mu\nu}F_a^{\mu\nu}$$

Tree-level

($z = m_h^2/x_1 x_2 s$)

$$\sigma_{h_1 h_2 \to h} = \int dx_1\, dx_2 f_g(x_1) f_g(x_2)\, \hat{\sigma}(z) + \text{smaller partonic channels}$$



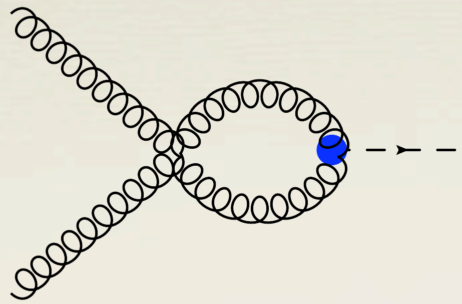
$$|\bar{\mathcal{M}}|^2 = \frac{1}{256(1-\epsilon)^2} \times |\mathcal{M}|^2 = \frac{\hat{s}^2}{576 v^2 (1-\epsilon)} \left(\frac{\alpha_s}{\pi}\right)^2$$

$$\frac{PS}{2\hat{s}} = \frac{\pi}{\hat{s}^2} \delta(1-z) \quad (\text{with } \hat{s} = x_1 x_2 s)$$

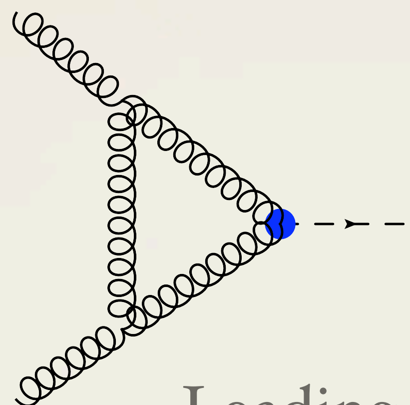
$$\hat{\sigma}_0(z) = \sigma_0\, \delta(1-z) = \frac{\pi}{576 v^2} \left(\frac{\alpha_s}{\pi}\right)^2 \delta(1-z)$$

Gluon-fusion: virtual

- Virtual:



$$= \sigma_0 \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon} \left\{-\frac{13}{4\epsilon} - \frac{83}{12}\right\} \delta(1-z)$$



$$= \sigma_0 \frac{\alpha_s}{\pi} \Gamma(1+\epsilon) \left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon} \left\{-\frac{3}{\epsilon^2} + \frac{1}{4\epsilon} + \frac{47}{12} + 2\pi^2\right\} \delta(1-z)$$

Leading soft+collinear singularity; emitting gluons from gluons gives color factor C_A=3

- UV renormalization: counterterm for α_s at leading order

Full d-dimensional LO

First term in beta-function

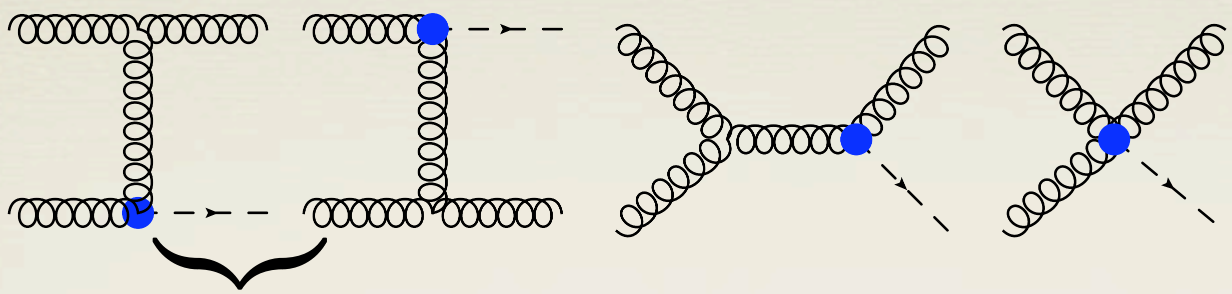
$$= \hat{\sigma}_0^{(d)}(z) \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \frac{1}{\epsilon} \left[-2\, b_0\right]$$

$$= \sigma_0 \frac{\alpha_s}{\pi} \frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}} \left\{-\frac{11}{2} + \frac{N_F}{3}\right\} \left[\frac{1}{\epsilon} + 1\right] \delta(1-z)$$

Number of light fermions

Gluon-fusion: real radiation

- Real:



singular

$$\text{Phase space} \quad : \quad \frac{1}{2\hat{s}}\int \frac{d^d p_g}{(2\pi)^d}\frac{d^d p_H}{(2\pi)^d}\,(2\pi)\delta(p_g^2)\,(2\pi)\delta(p_H^2 - M_H^2)\,(2\pi)^d\delta^{(d)}(p_1+p_1-p_g-p_H)$$

$$= \frac{1}{16\pi\hat{s}}\frac{s^{-\epsilon}}{(4\pi)^{-\epsilon}\Gamma(1-e)}(1-z)^{1-2\epsilon}\int_0^1 d\lambda\,\lambda^{-\epsilon}(1-\lambda)^{-\epsilon}$$

$$\Rightarrow \quad \hat{t} = (p_1-p_g)^2 = -\hat{s}(1-z)\lambda, \quad \hat{u} = (p_2-p_g)^2 = -\hat{s}(1-z)(1-\lambda)$$

$$|\bar{\mathcal{M}}|^2 = 24\,\alpha_s\sigma_0\left\{\frac{(1-2\epsilon)}{(1-\epsilon)}\frac{M_H^8+\hat{s}^4+\hat{t}^4+\hat{u}^4}{\hat{s}\hat{t}\hat{u}} + \frac{\epsilon}{2(1-\epsilon)^2}\frac{(M_H^4+\hat{s}^2+\hat{t}^2+\hat{u}^2)^2}{\hat{s}\hat{t}\hat{u}}\right\}$$

$$\Downarrow$$

$$\Rightarrow (1-z)^{-1-2\epsilon}\lambda^{-1-\epsilon}(1-\lambda)^{-1-\epsilon}$$

singular regulator

$\lambda \rightarrow 0,1$: collinear

$z \rightarrow 1$: soft

Real radiation and plus dists.

- Extract singularities using plus distribution expansion

$$\lambda^{-1-\epsilon} = -\frac{1}{\epsilon}\delta(\lambda) + \frac{1}{[\lambda]_+} - \epsilon\left[\frac{\ln\lambda}{\lambda}\right]_+ + \mathcal{O}(\epsilon^2), \quad \text{etc.}$$

$$\int_0^1 dx\, f(x)[g(x)]_+ = \int_0^1 dx\,[f(x) - f(0)]\, g(x)$$

$$= \sigma_0 \frac{\alpha_s}{\pi}\underbrace{\left(\frac{\hat{s}}{4\pi\mu^2}\right)^{-\epsilon}}_{1/\Gamma(1-\epsilon)} \left\{ \overbrace{\left[\frac{3}{\epsilon^2} + \frac{3}{\epsilon}\right]\delta(1-z)}^{\text{cancels virtual poles}} - \frac{6}{\epsilon}\frac{1}{[1-z]_+} + \frac{6z(z^2-z+2)}{\epsilon}\right.$$

$$+ (3-\pi^2)\,\delta(1-z) - \frac{6}{[1-z]_+} + 12\left[\frac{\ln(1-z)}{1-z}\right]_+$$

$$\left. - 12z(z^2-z+2)\ln(1-z) - \frac{11}{2} + \frac{57z}{2} - \frac{45z^2}{2} + \frac{23z^3}{2}\right\}$$

Remaining terms

- PDF renormalization: counterterm for initial-state collinear sings.

one for each PDF

$$= 2\,\hat{\sigma}_0^{(d)}(z)\frac{\alpha_s}{2\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\frac{1}{\epsilon}P_{gg}(z) \longrightarrow$$

subtracting P_{gg} from PDF same as adding to partonic cross section

$$= \sigma_0\frac{\alpha_s}{\pi}\frac{\Gamma(1+\epsilon)}{(4\pi)^{-\epsilon}}\left\{\underbrace{\left(\frac{11}{2}-\frac{N_F}{3}\right)\delta(1-z)}_{\text{cancels UV counterterm}}+\underbrace{\frac{6}{[1-z]_+}-6z(z^2-z+2)}_{\text{cancels real radiation}}\right\}\left[\frac{1}{\epsilon}+1\right]$$

- Effective Lagrangian correction: $= \sigma_0\dfrac{\alpha_s}{\pi}\dfrac{11}{2}\delta(1-z)$

Gluon fusion: final result

- Arrive at the final NLO correction

$$\Delta\sigma = \sigma_0 \frac{\alpha_s}{\pi} \left\{ \left(\frac{11}{2}+\pi^2\right)\delta(1-z) + 12\left[\frac{\ln(1-z)}{1-z}\right]_+ - 12z(-z+z^2+2)\ln(1-z) \right.$$
$$\left. - \frac{11}{2}(1-z)^3 + 6\ln\frac{\hat{s}}{\mu^2}\left[\frac{1}{[1-z]_+} - z(z^2-z+2)\right]\right\} \quad (M^2/s \leq z \leq 1)$$

(integration over PDFs⇒integration over z)

  - First source of large correction: $11/2+\pi^2 \Rightarrow 50\%$ increase
  - Second source: shape of PDFs enhances *threshold* logarithm

$$\sigma_{had} = \tau\int_\tau^1 dz\,\frac{\sigma(z)}{z}\,\mathcal{L}\left(\frac{\tau}{z}\right)$$
$$\mathcal{L}(y) = \int_y^1 dx\,\frac{y}{x}\,f_1(x)f_2(y/x) \quad \text{(partonic luminosity)}$$

Assume $f_i \sim (1-x)^b$; plot L for various b
Look for peak near z≈1

⇒Sharp fall-off of gluon PDF enhances correction