Jets and jet definitions

Davison E. Soper
University of Oregon

CTEQ, Madison, July 2011

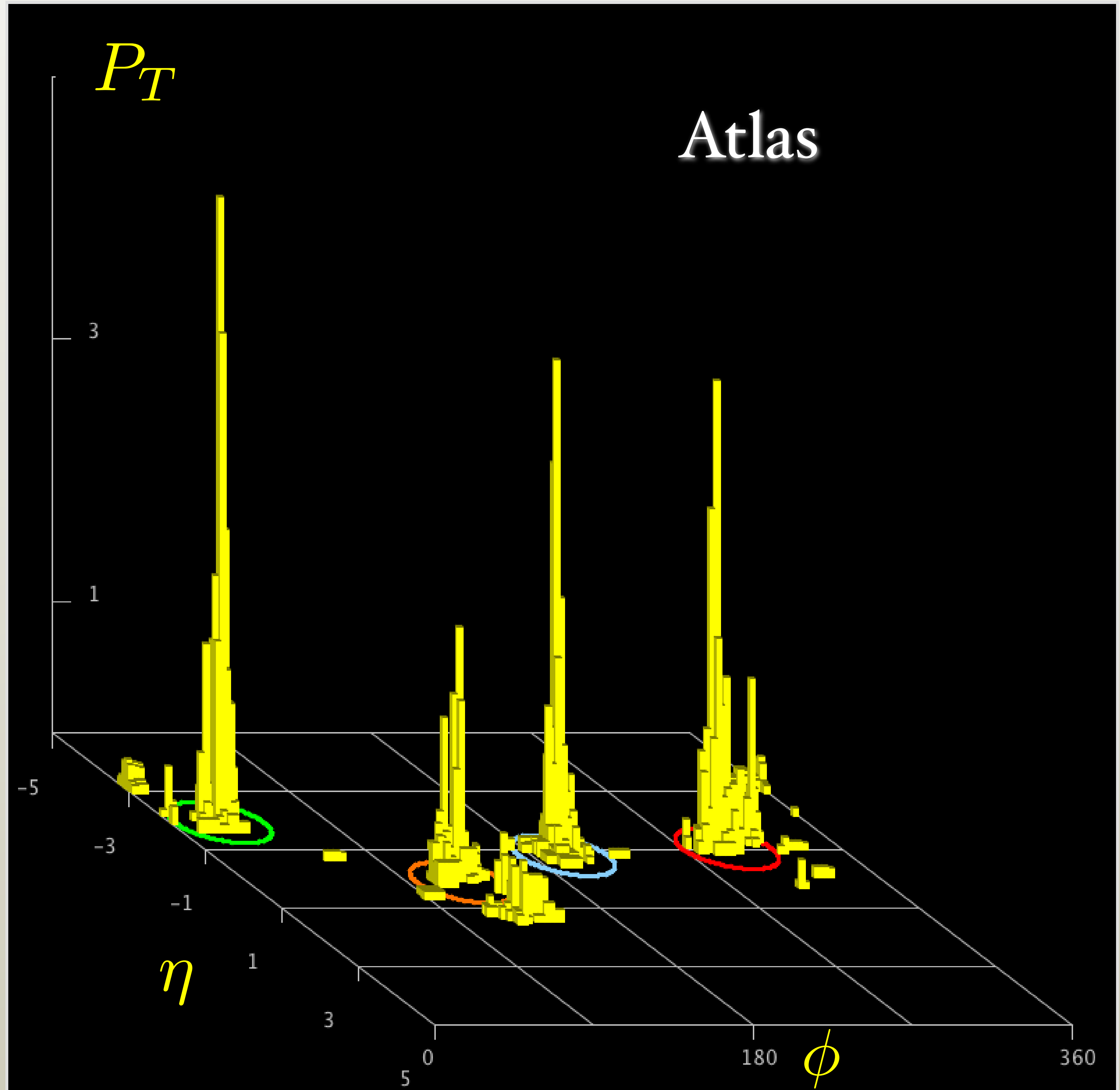
Outline

- Jets are real.
  - Experimental evidence
  - Why does this happen?
- Jets are a convention.
  - Infrared safety
  - Cone definitions
  - Iterative combination definitions

Jets are real

Experimental evidence

- An Atlas event with 4 high P_T jets.
- Highest P_T jet has P_T = 144 GeV.



DESY provided early evidence

- The PETRA accelerator had enough energy to make jets clearly visible.
- The PETRA experiments had 4π detectors, so that one could be convinced that two and three jet events existed with single event displays.

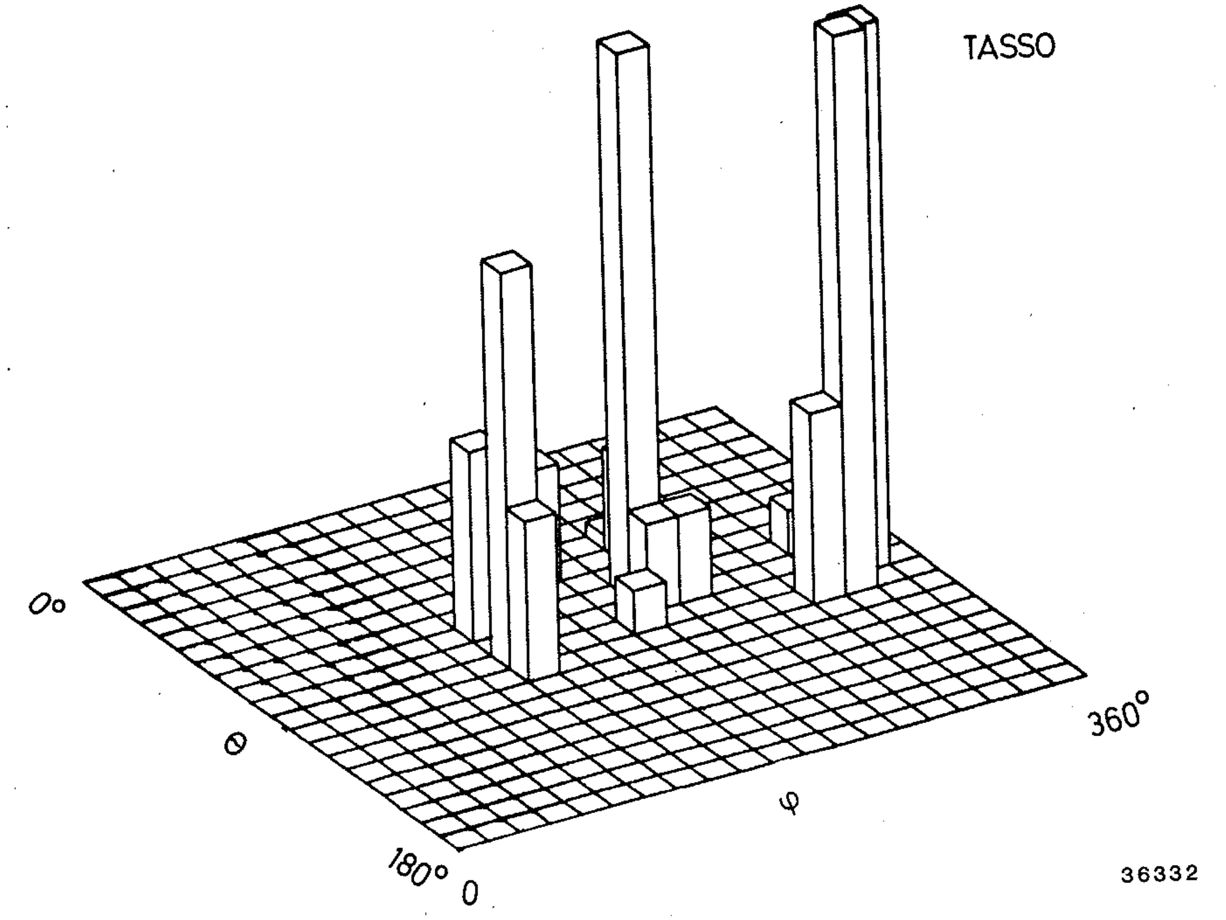


Fig.9 A typical three jet event

from G. Wolf, Multiparticle Conference, 1983

Why are there jets?

- Bjorken, Berman and Kogut (1971) had it figured out before jets were seen and before QCD.
- "... the isolated high P_T partons will communicate with the 'wee' partons by cascade emission of partons."

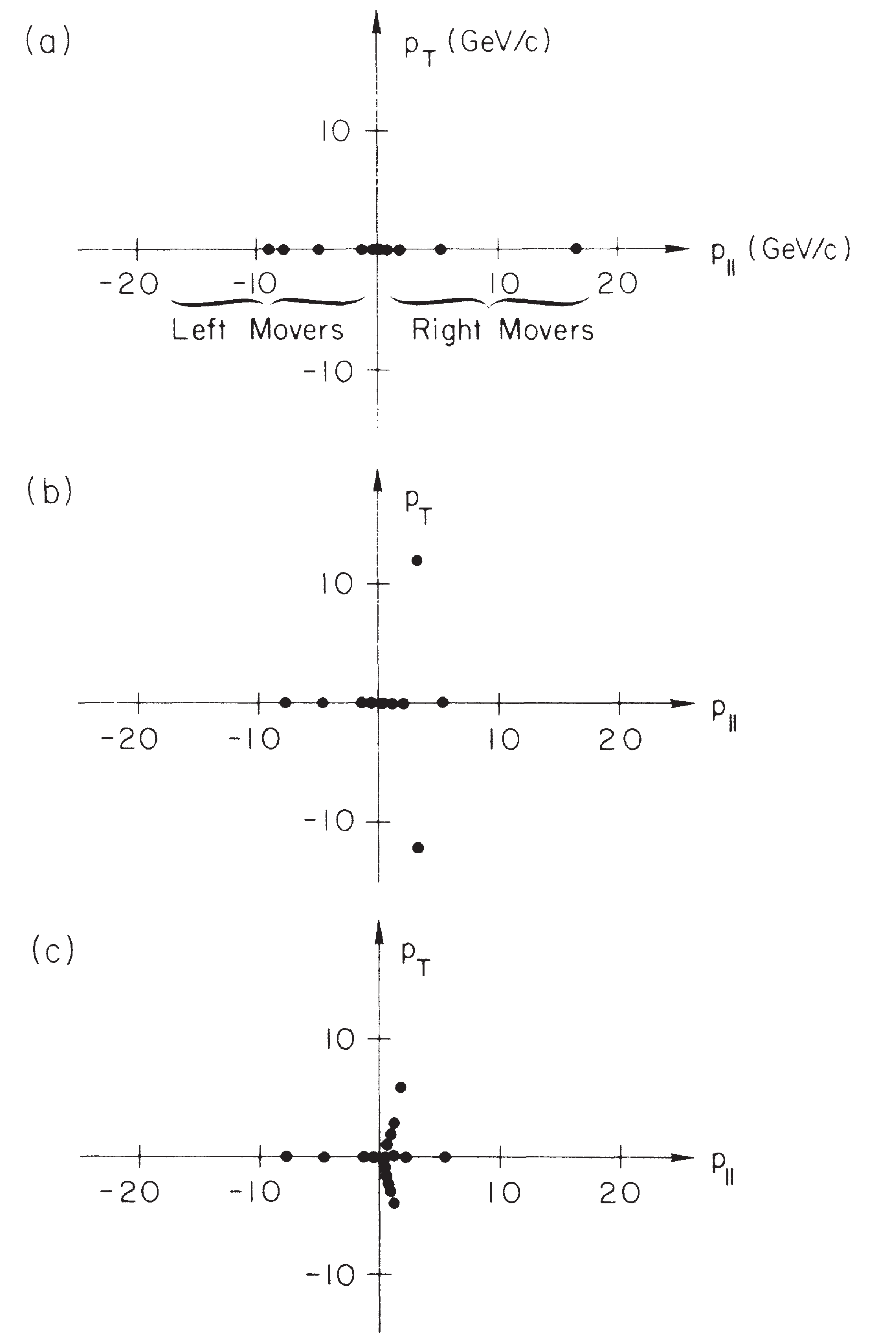
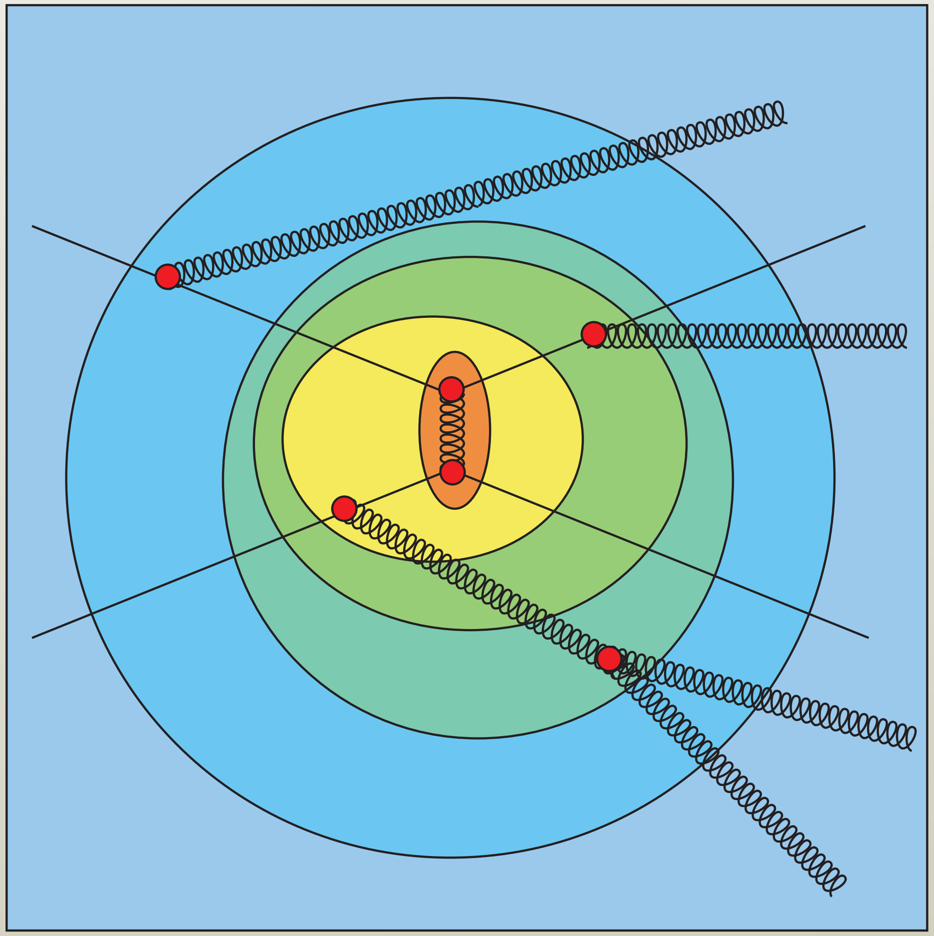


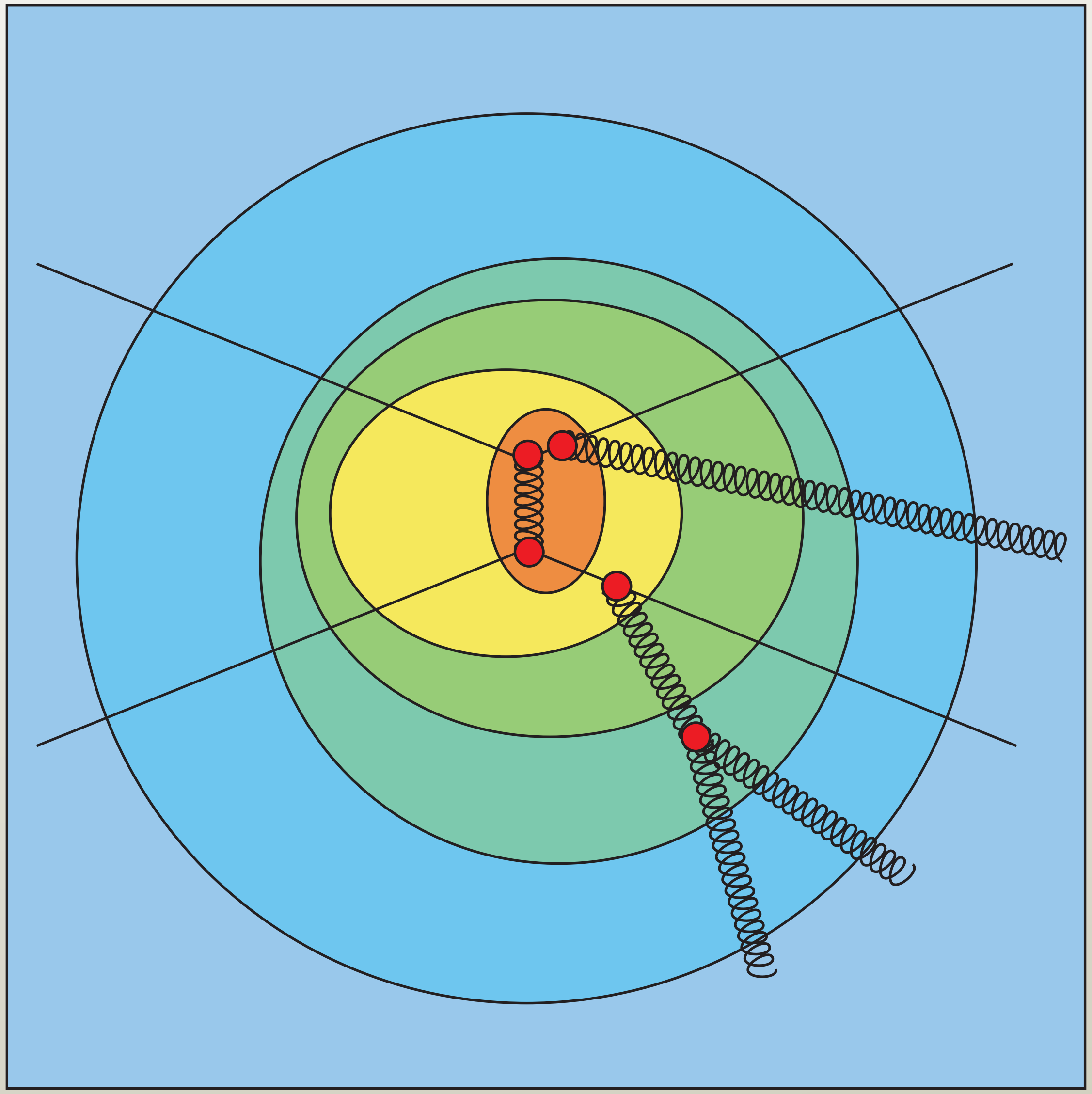
FIG. 4. A momentum-space visualization of hadron-hadron deep-inelastic scattering occurring in three steps.

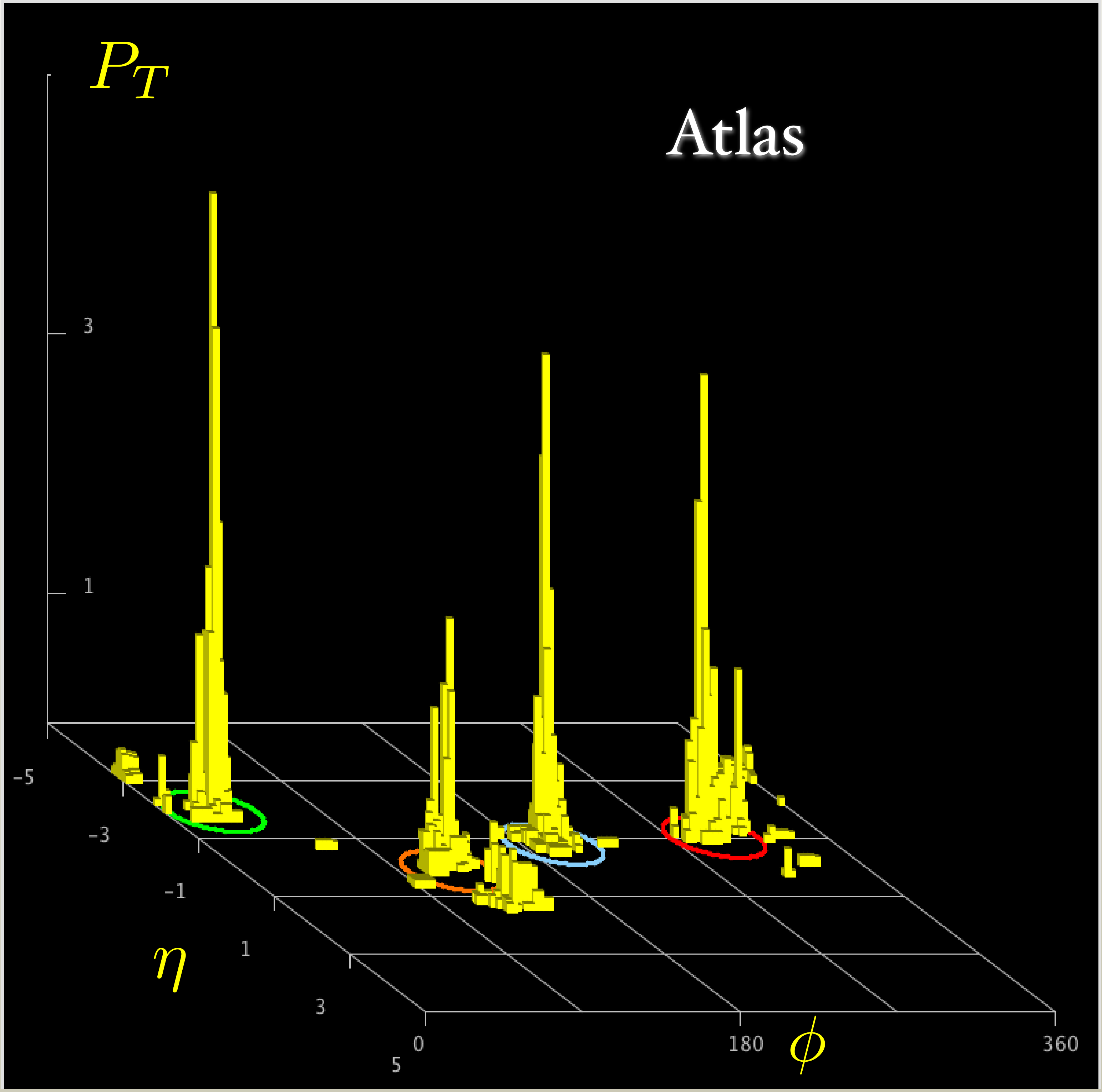
Jets from QCD showers

- Think of shower starting at hard interaction.
- Most probable: soft and collinear splittings.
- Such splittings from a high P_T parton builds up a jet.
- Very hard interactions happen with probability α_s.



- For example ...





Jets are a convention

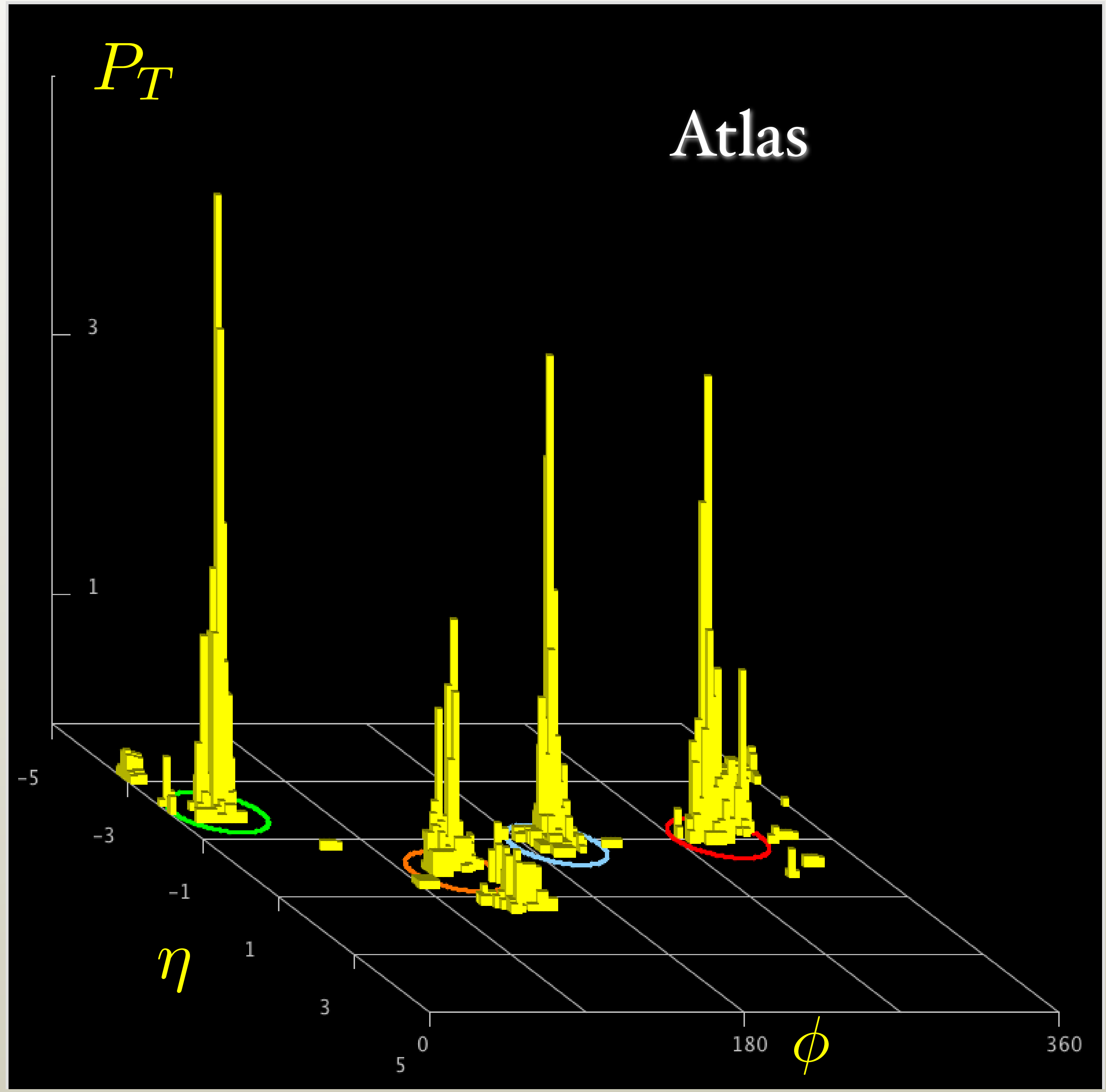
Let's measure a jet cross section

- Suppose we want a cross section for $pp \to 3\ \text{jets} + X$,

$$\frac{d\sigma}{d\eta_1\, dP_{T,1}\;\, d\eta_2\, dP_{T,2}\;\, d\eta_3\, dP_{T,3}}$$

- Here X could include other jets.
- If we want, we could define the cross section so that 1,2,3 are the highest P_T jets in the event.

- What is the P_T of the third jet here?
- Is it the P_T of what is in the blue circle?
- Or does the third jet include P_T from three subjets?
- We need a definition.



Infrared safety

- Our jet definition should be infrared safe.
- To see what this means, write the cross section as

$$\sigma = \frac{1}{2!}\int d\eta_1\, dp_2\, d\eta_2\, d\phi_2\ \frac{d\sigma[2\to 2]}{d\eta_1\, dp_2\, d\eta_2\, d\phi_2}\,\mathcal{S}_2(p_1,p_2)$$
$$+\frac{1}{3!}\int d\eta_1\, dp_2\, d\eta_2\, d\phi_2\, dp_3\, d\eta_3\, d\phi_3$$
$$\times\frac{d\sigma[2\to 2]}{d\eta_1\, dp_2\, d\eta_2\, d\phi_2\, dp_3\, d\eta_3\, d\phi_3}\,\mathcal{S}_3(p_1,p_2,p_3)$$
$$+\cdots$$

- $\mathcal{S}_n(p_1,\dots,p_n)$ defines the observable.

- I assume
- $\mathcal{S}_n(p_1, \ldots, p_n)$ is a symmetric function of its arguments.
- $\mathcal{S}_n(p_1, \ldots, p_n)$ is a smooth function of its arguments.

- Then we need

$$\mathcal{S}_{n+1}(p_1, \ldots, (1-\lambda)p_n, \lambda p_n) = \mathcal{S}_n(p_1, \ldots, p_n)$$

- If any two partons become parallel it is the same as if they were one.

$$\mathcal{S}_{n+1}(p_1, \ldots, p_n, 0) = \mathcal{S}_n(p_1, \ldots, p_n)$$

$$\mathcal{S}_{n+1}(p_1, \ldots, p_n, \lambda p_{\mathrm{A}}) = \mathcal{S}_n(p_1, \ldots, p_n)$$

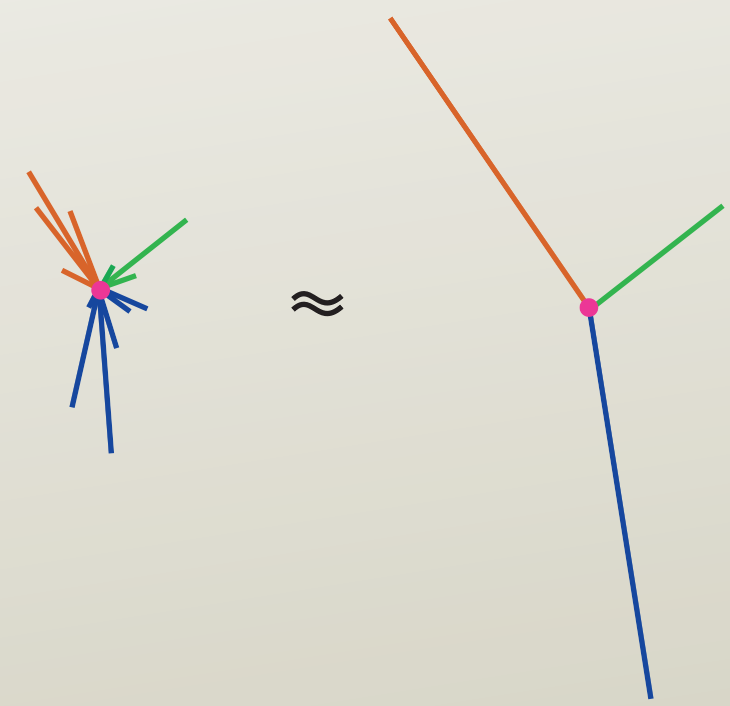
$$\mathcal{S}_{n+1}(p_1, \ldots, p_n, \lambda p_{\mathrm{B}}) = \mathcal{S}_n(p_1, \ldots, p_n)$$

- If any particle becomes soft or parallel to the beam it is the same as if it were not there.

- This ensures that you get a finite answer in a perturbative calculation.

What does IR safety mean?

- The physical meaning is that for an IR-safe quantity, the physical event with hadron jets should give approximately the same measurement as a parton event.



- It also means that in a Monte Carlo simulation the hadronization model and the underlying event model should not much matter.

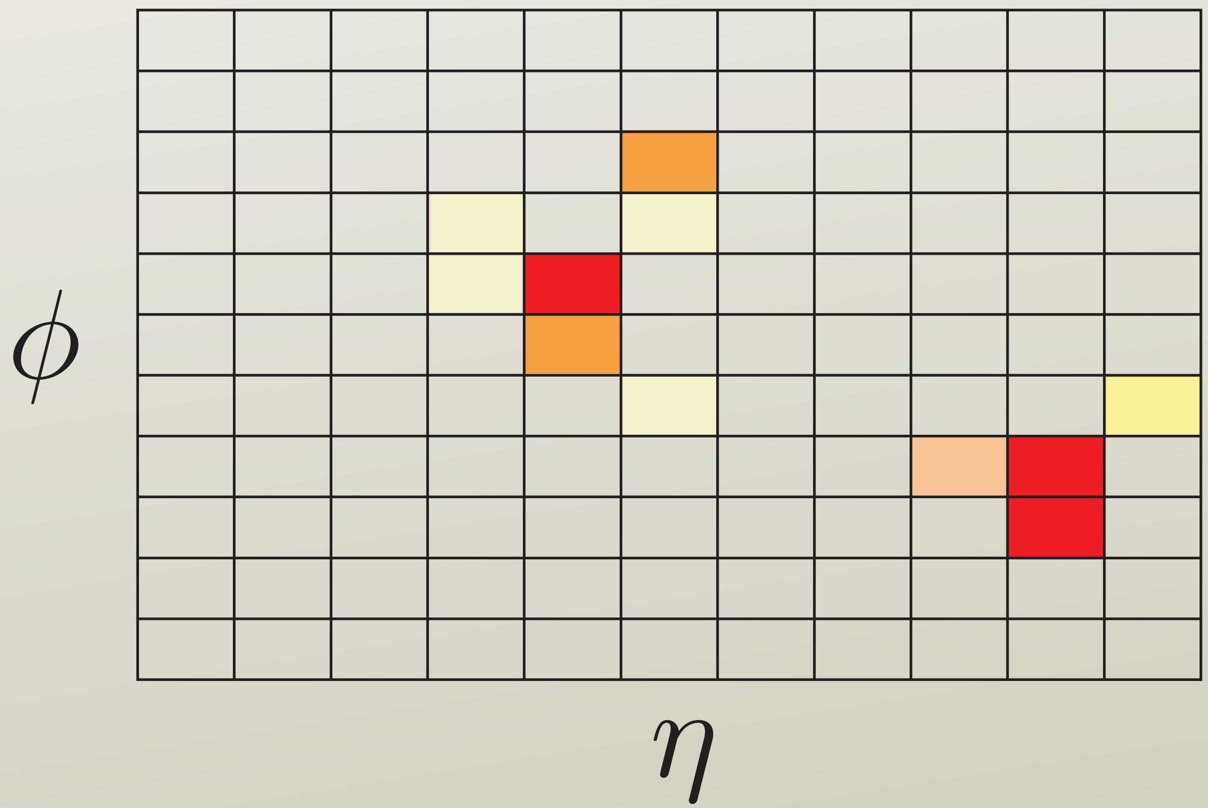
Example of infrared danger

From G. Wolf at Multiparticle Dynamics 1983.

- Use e^+e^- annihilation event shapes and hadron energy spectrum to measure strong coupling.
  - $\alpha_s(M_Z) = 0.13 \pm 0.01$, independent jet model.
  - $\alpha_s(M_Z) = 0.17 \pm 0.01$, string model.
- Why?
  - Measured quantities not infrared safe.
  - Theory mixed short and long distance physics.

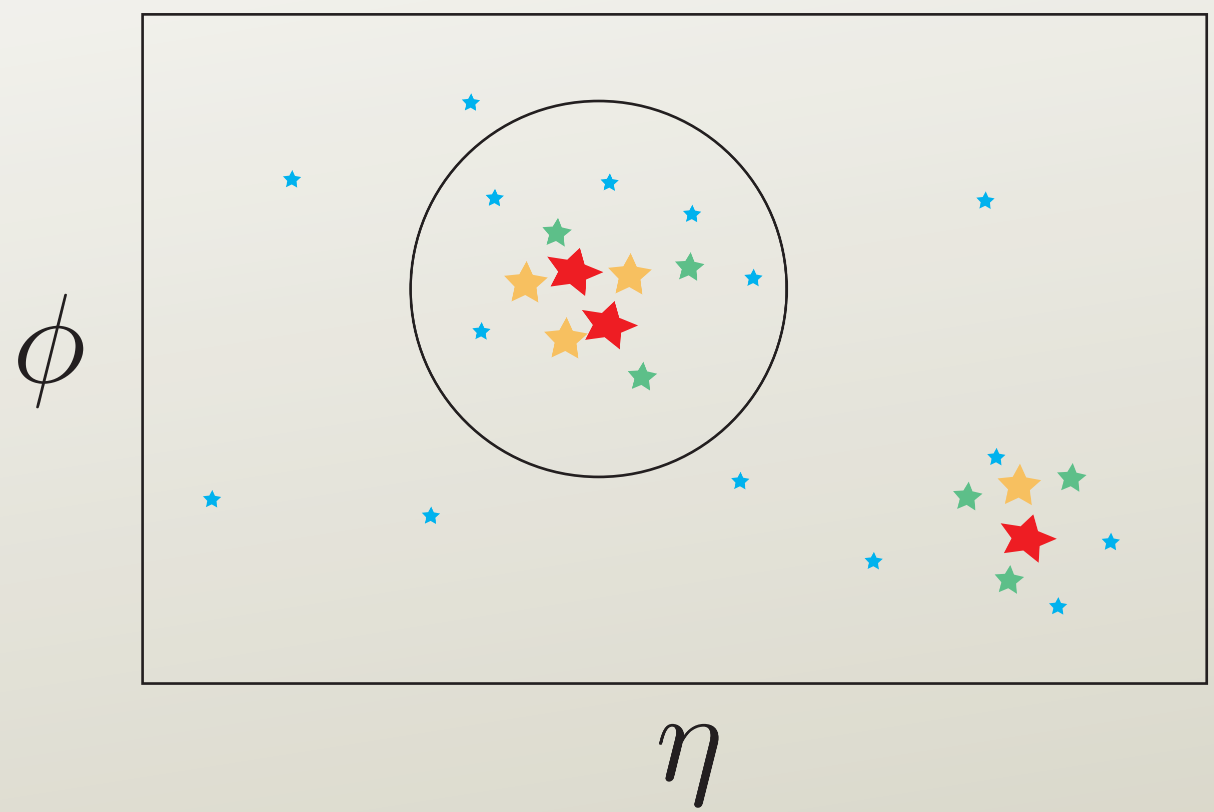
Cone definitions

- I recall the “Snowmass” definition.
- Divide calorimeter into cells *i*.



- Definition is based on cell variables $(P_{T,i}, \eta_i, \phi_i)$.

- There is a jet axis (η_J, ϕ_J).



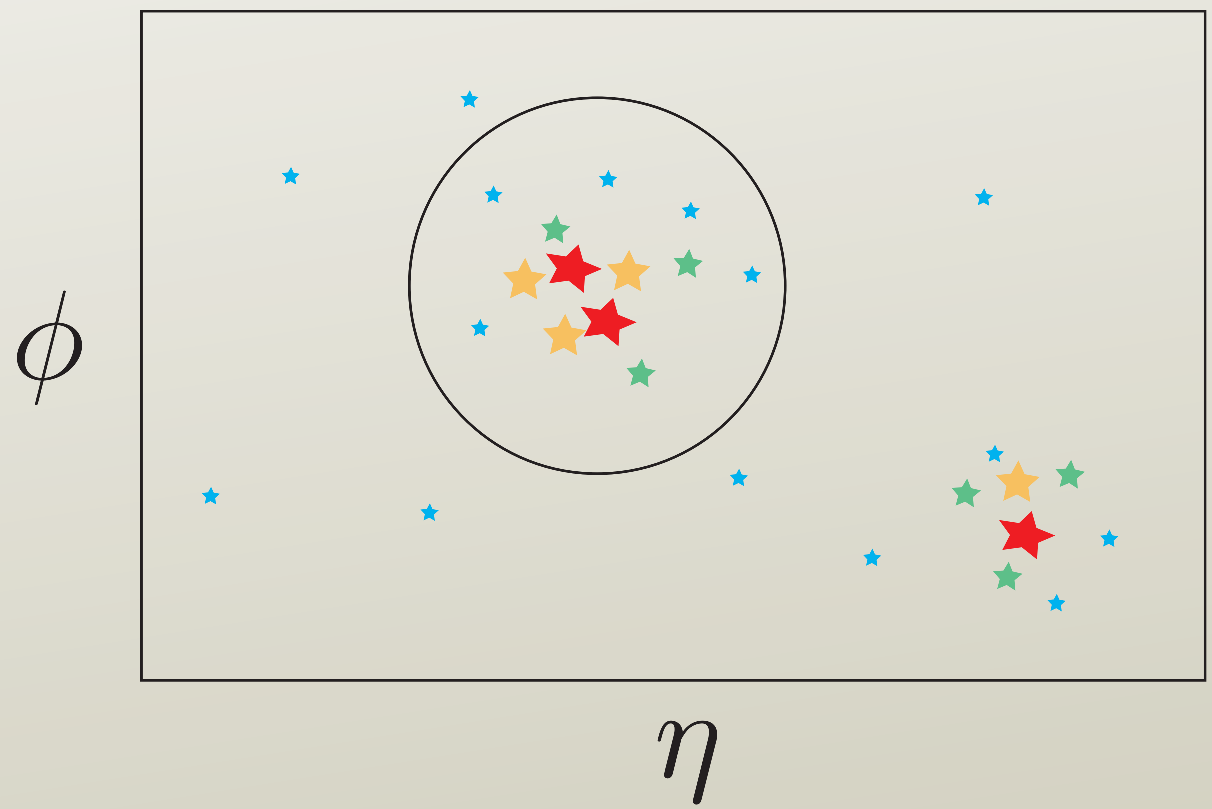
- A jet consists of all cells with $(\eta_i - \eta_J)^2 + (\phi_i - \phi_J)^2 < R^2$.

- The jet variables are defined by

$$P_{T,J} = \sum_{i \in \text{cone}} P_{T,i}$$

$$\eta_J = \frac{1}{P_{T,J}} \sum_{i \in \text{cone}} P_{T,i}\ \eta_i$$

$$\phi_J = \frac{1}{P_{T,J}} \sum_{i \in \text{cone}} P_{T,i}\ \phi_i$$



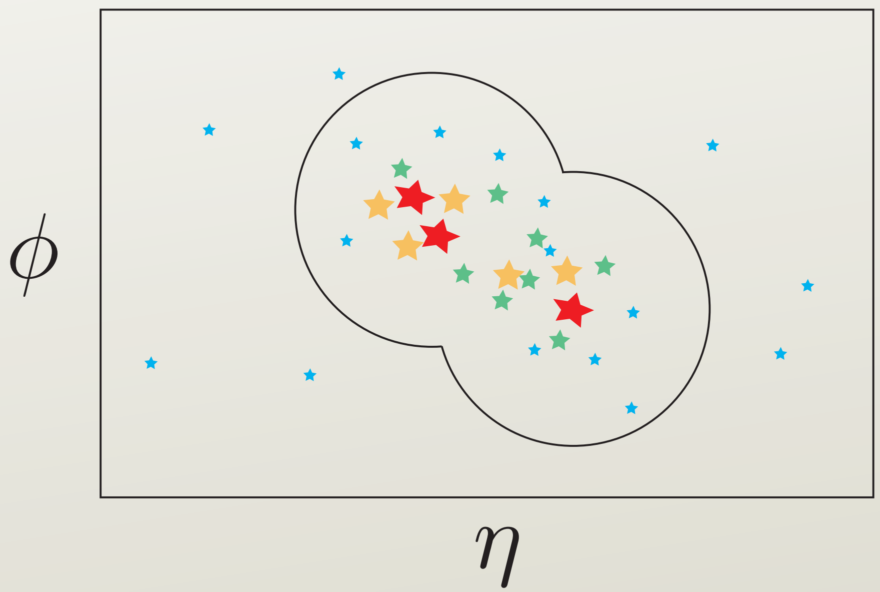
- Adjust (η_J, ϕ_J) to make this match.

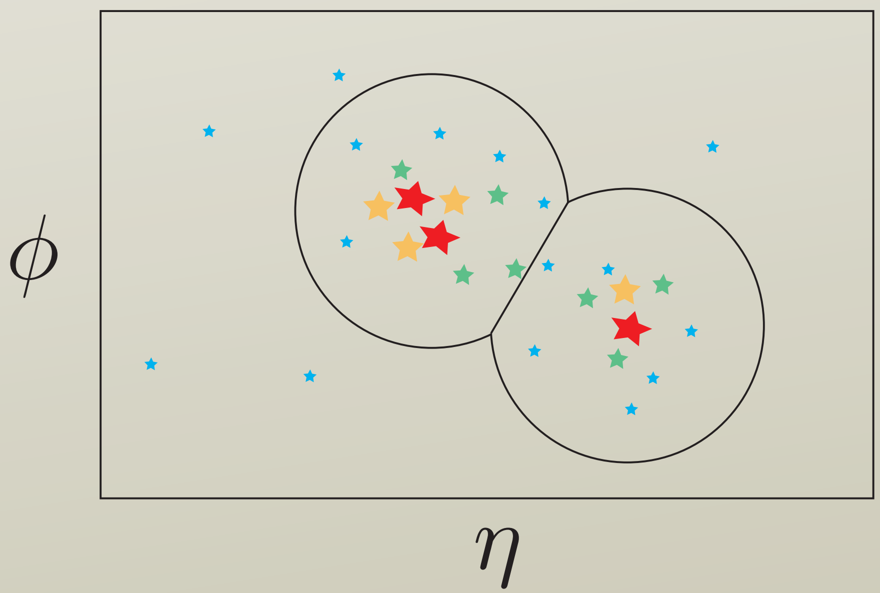
- Beyond this Snowmass convention, there is usually some fine print.

- Cones can overlap.

- If there is a lot of P_T in the overlap region, join them.

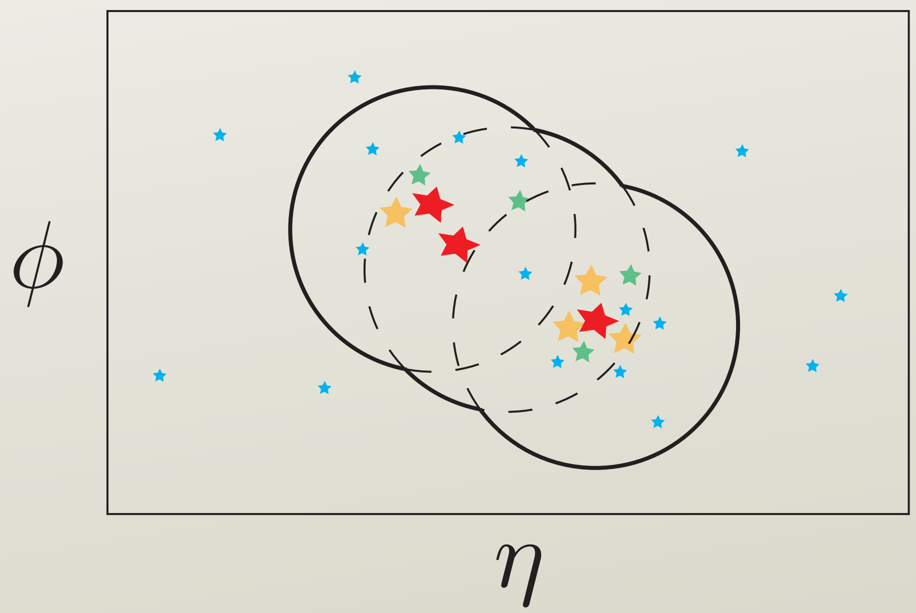
- Otherwise, split them.



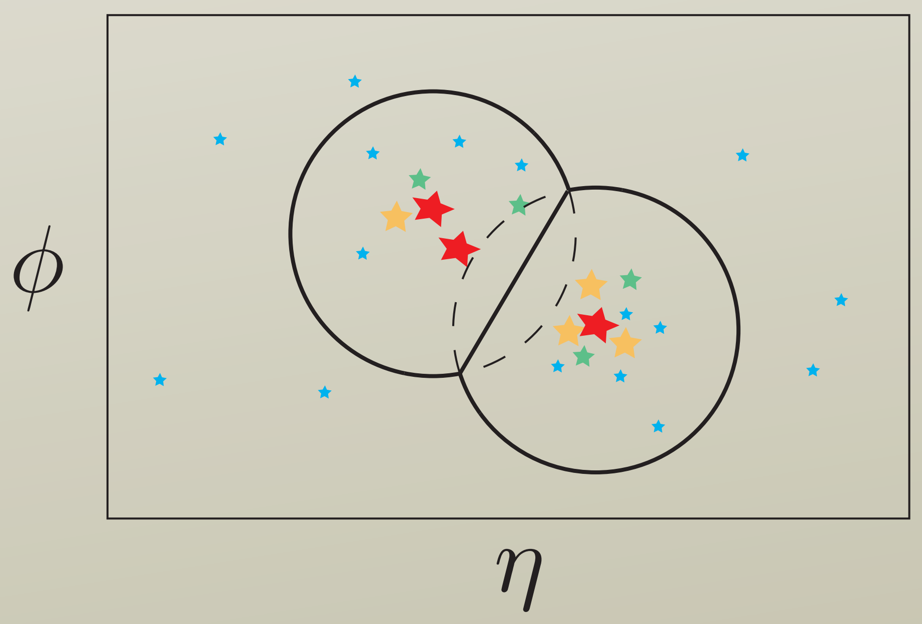


• This is IR safe if search for solutions does not use "seeds."

• With seed: three jets, merged.



• Without seed: two jets, split.

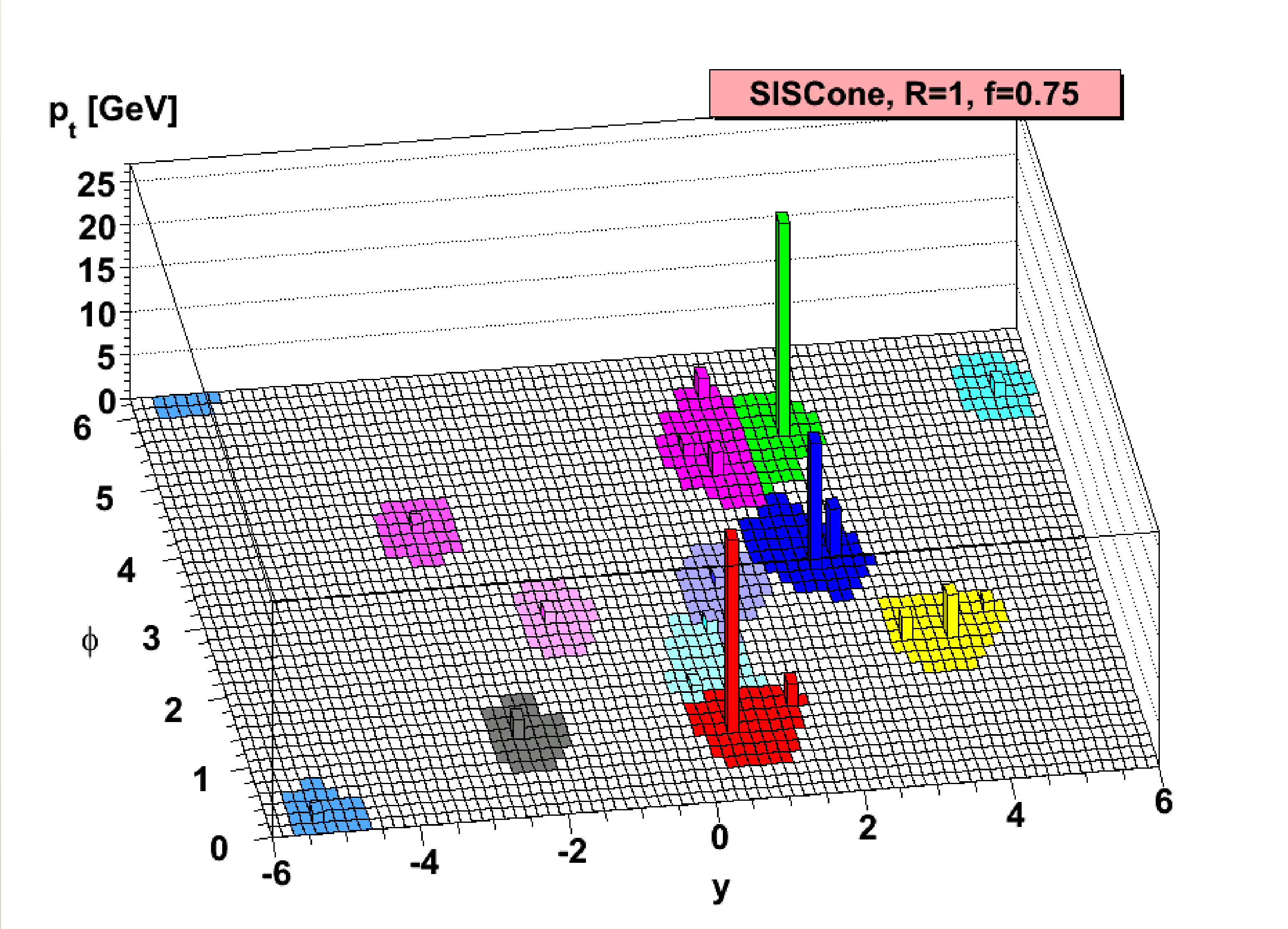


From a tiny seed...

a giant jet can grow.

Example with cone

- Here is an example event from Cacciari, Salam, and Soyez (2008).
- With a cone algorithm, we see what detector area goes into each jet.

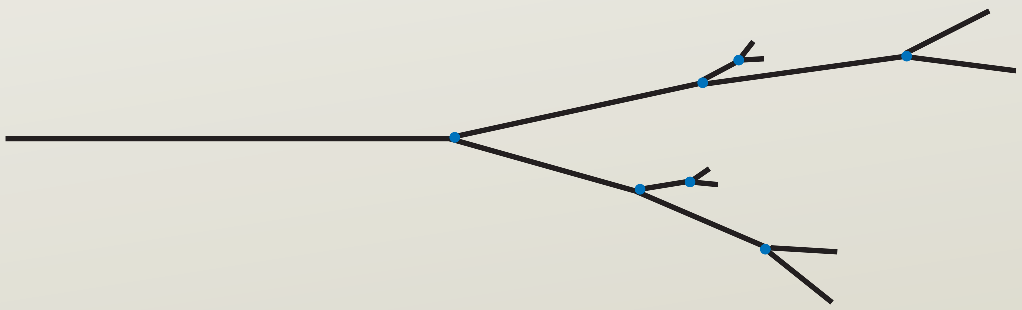


Successive combination algorithms

- These trace back to the Jade algorithm from the Jade Collaboration at DESY.

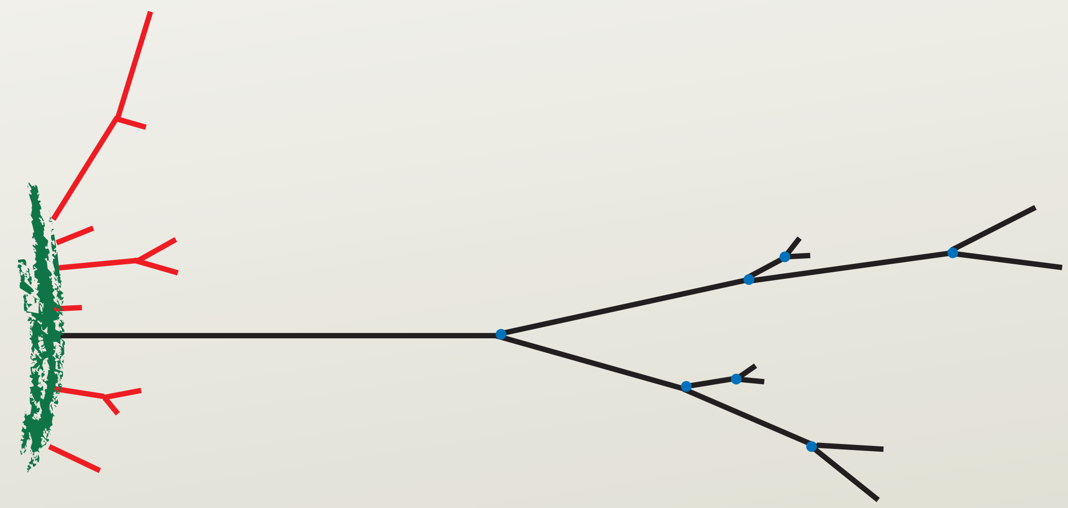
What people would like

- In some approximation, a final state is created by parton showers.



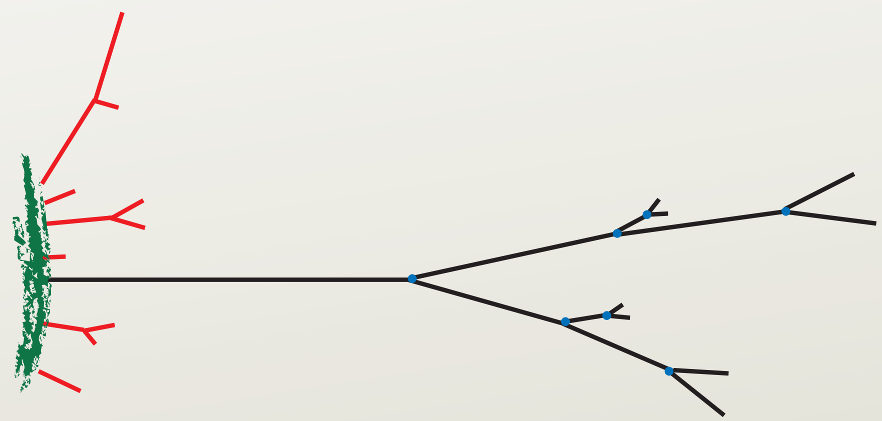
- You might want to deconstruct the shower to produce early state partons as jets.
- A resolution parameter defines how far back you want to go.

Beware of reality



- There can be many soft gluons that match your jet in angle but are from initial state radiation or radiated from other jets.
- In fact, more than one shower history can produce the same final state.

The k_T jet algorithm



- Choose a resolution parameter *R*.
- Start with a list of protojets, specified by their p_j^μ .
- Start with an empty list of finished jets.
- Result is a list of finished jets with their momenta.
- Many are low p_T debris; just ignore these.

1. For each pair of protojets define

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

For each protojet define

$$d_i = p_{T,i}^2$$

2. Find the smallest of all the d_{ij} and the d_i. Call it $d_{\rm min}$
3. If $d_{\rm min}$ is a d_{ij}, merge protojets i and j into a new protojet k with

$$p_k^\mu \quad = \quad p_i^\mu + p_j^\mu$$

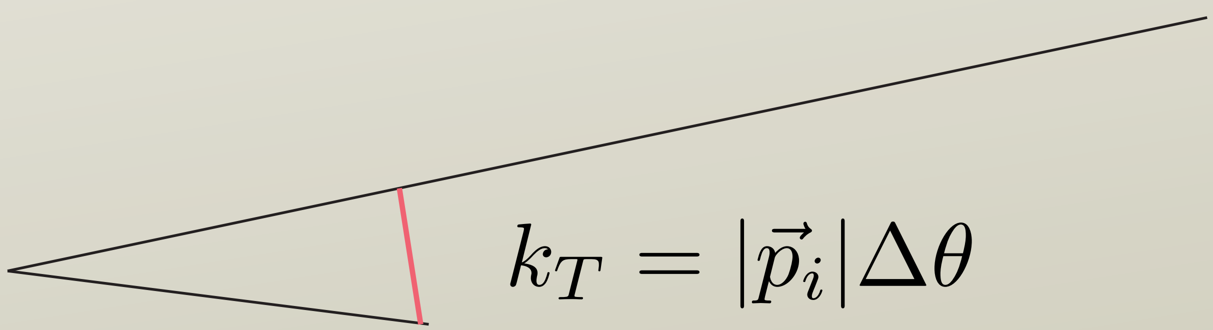
4. If $d_{\rm min}$ is a d_i, then protojet i is "not mergable." Remove it from the list of protojets and add it to the list of jets.
5. If protojets remain, go to 1.

Why the name?

$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2)\,[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

is essentially

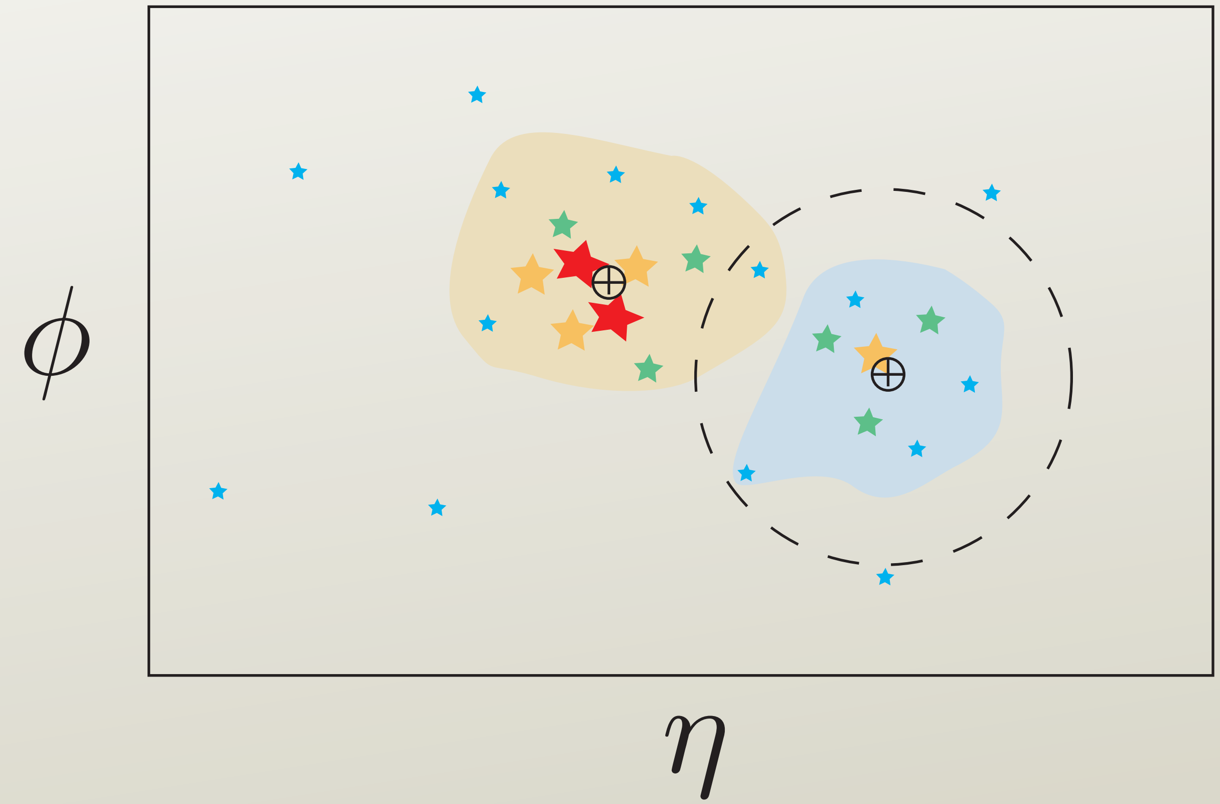
$$d_{ij} = k_T^2/R^2$$



$$k_T = |\vec{p}_i|\Delta\theta$$

The "no merge" condition

- Suppose $p_{T,i}^2 < p_{T,j}^2$.



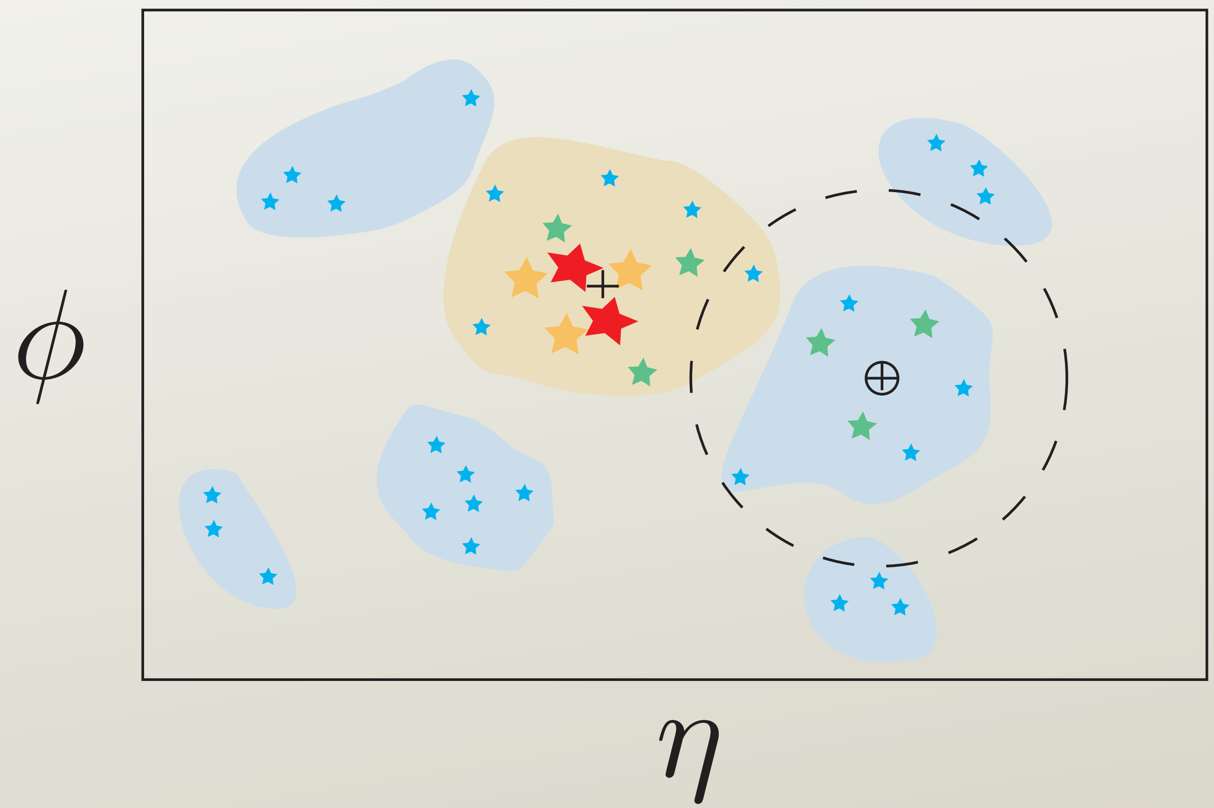
$$d_{ij} = \min(p_{T,i}^2, p_{T,j}^2) \left[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2\right]/R^2$$

$$d_i = p_{T,i}^2$$

- Protojet i is not mergable with parton j if $d_{ij} > d_i$. That is if

$$\left[(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2\right] > R^2$$

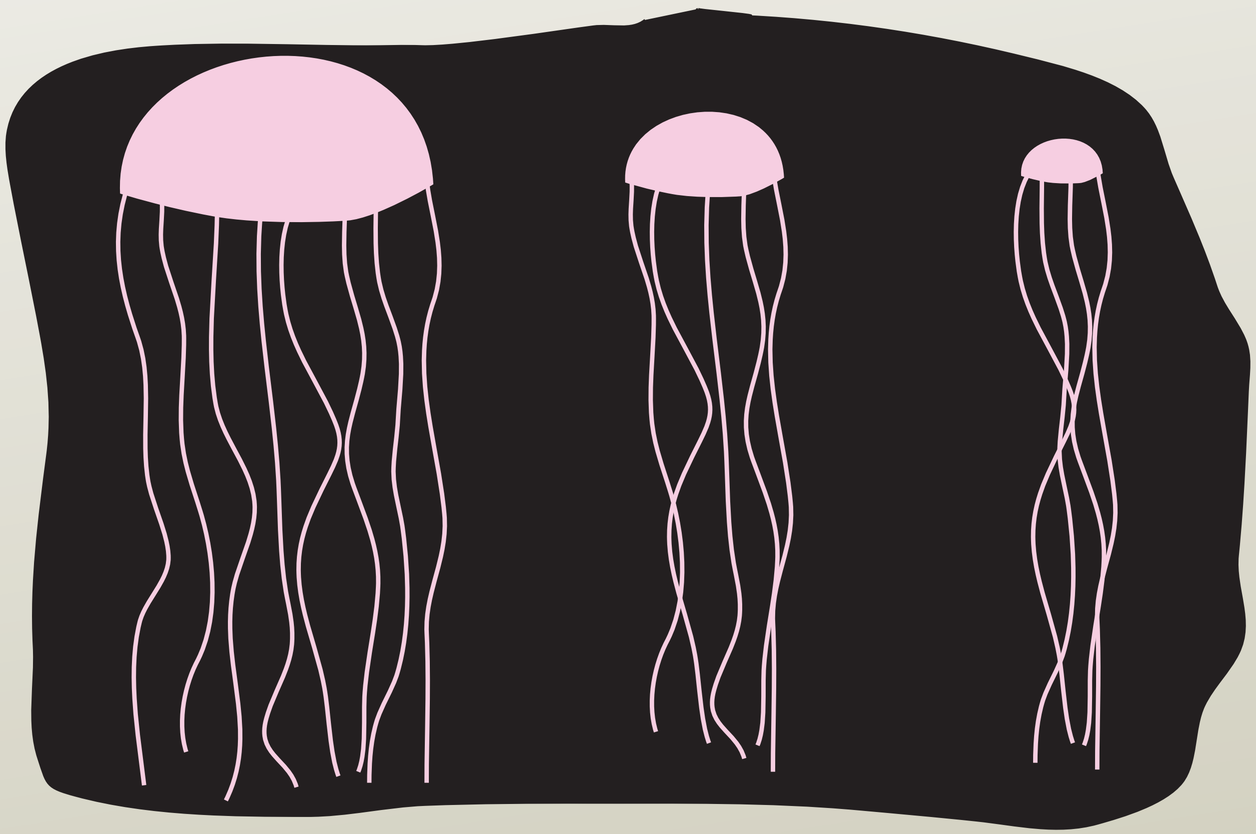
Why the no merge condition



- There will be many soft jets.
- They should not merge into a few giant jets.

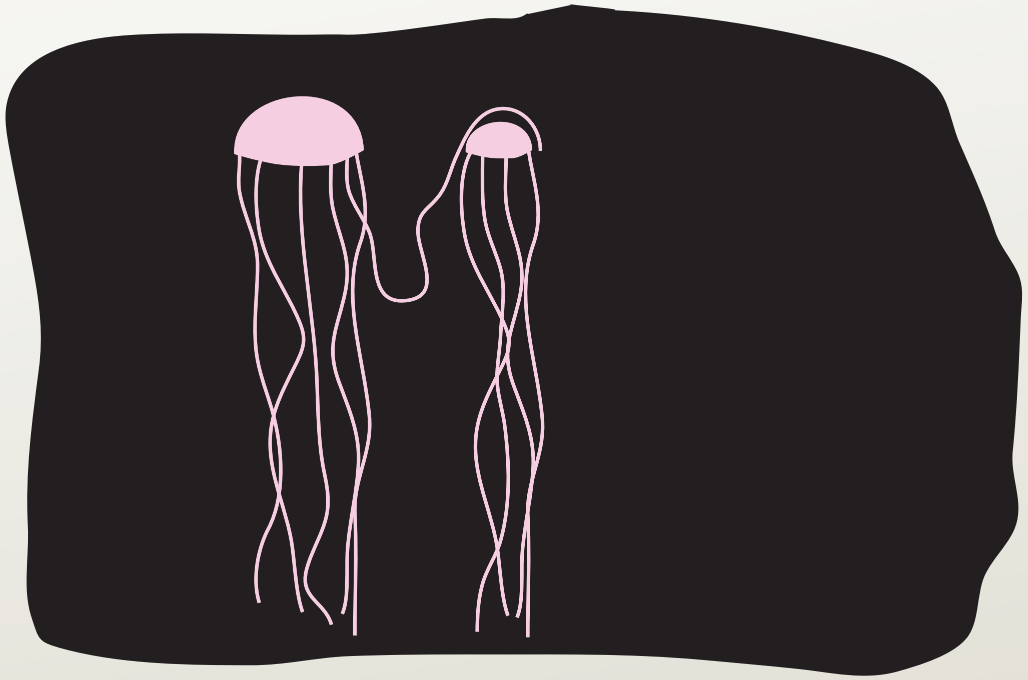
Cannibalistic jellyfish

Jellyfish of the species *protojetius cannibalis* come in a range of sizes.

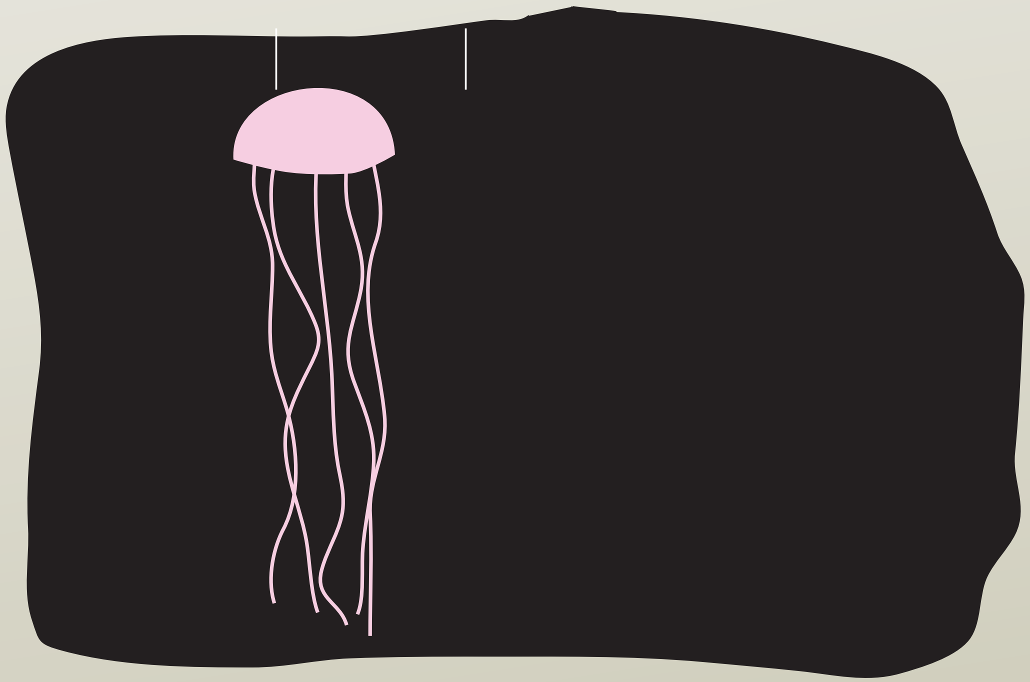


The tentacles are always the same size.

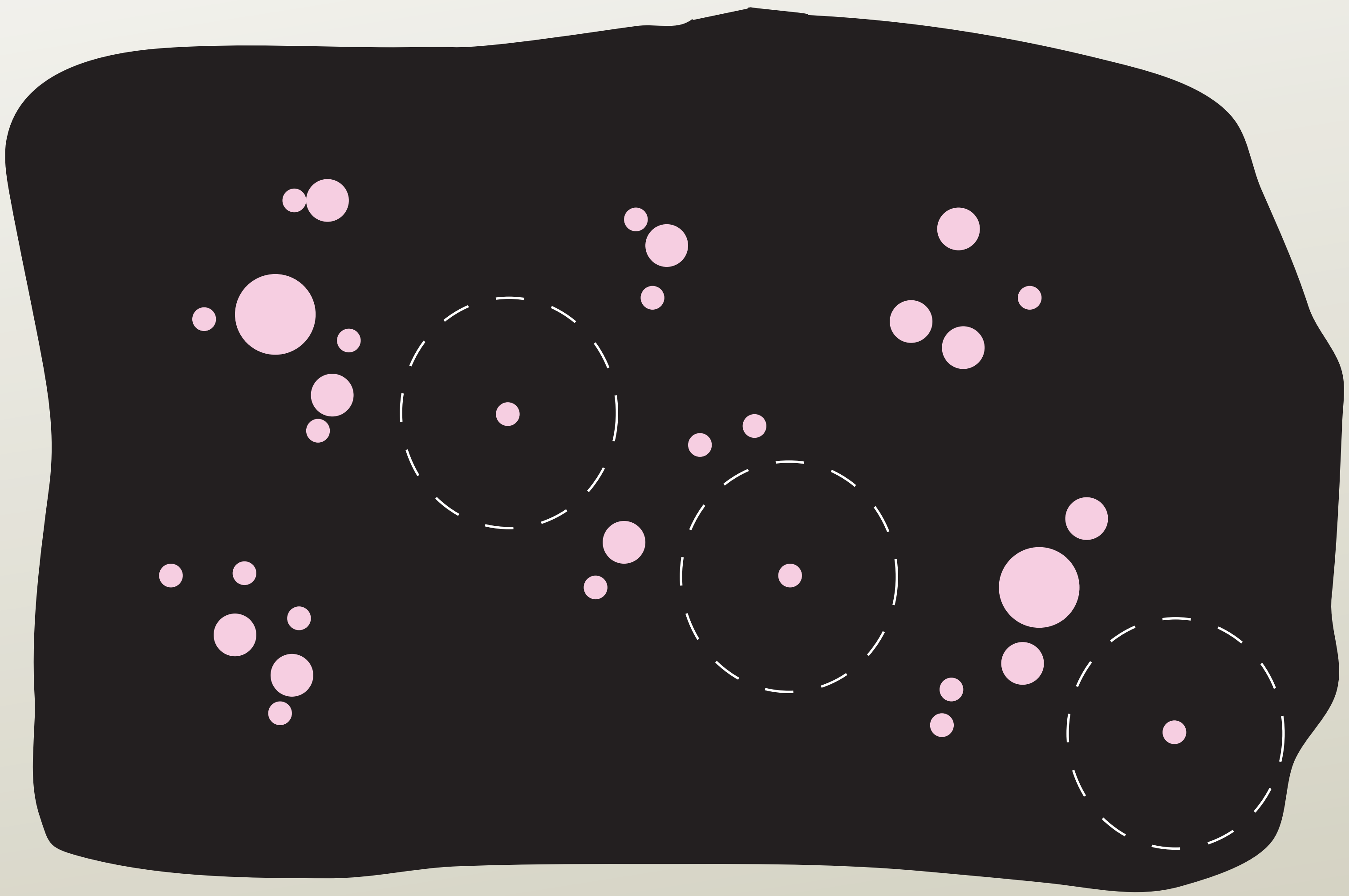
- Sometimes a big one has a little one for lunch.



- Then the big one is bigger.
- But the center-of-mass stays where it was.



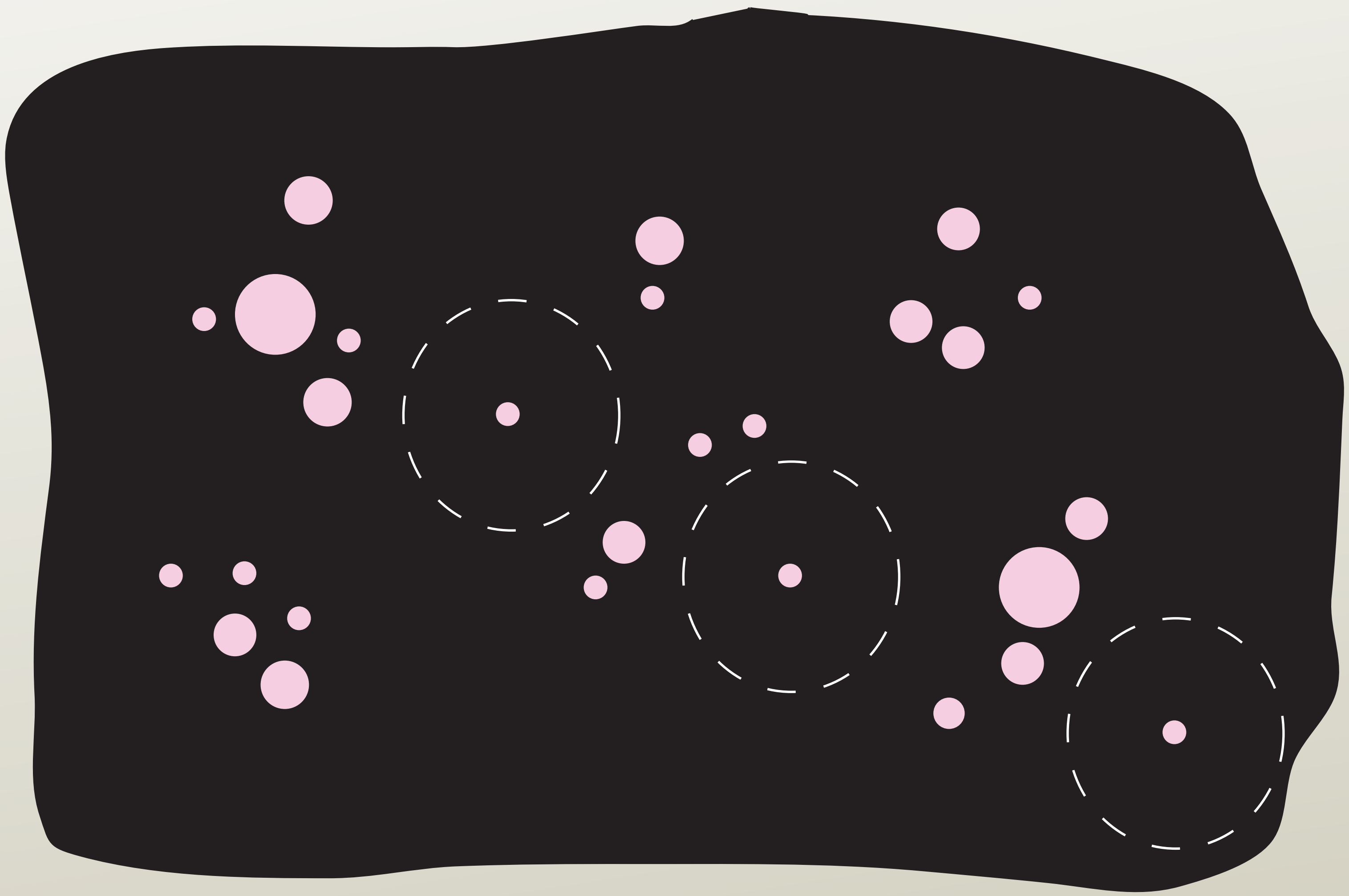
At sunrise, the jellyfish would like a snack.



They look for a jellyfish that is small and near.

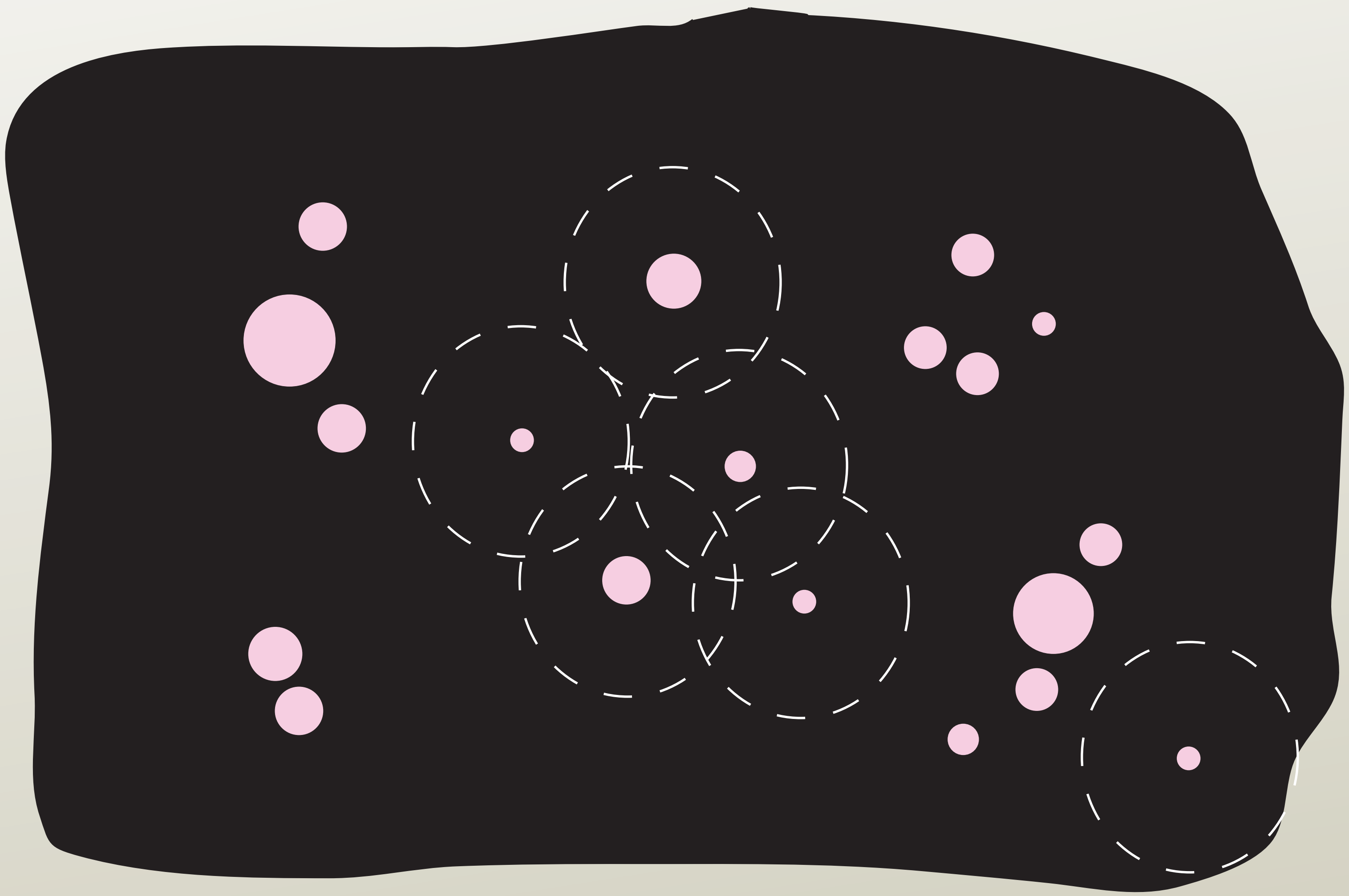
A few are out of range of predation.

After the snack, there are fewer jellyfish.



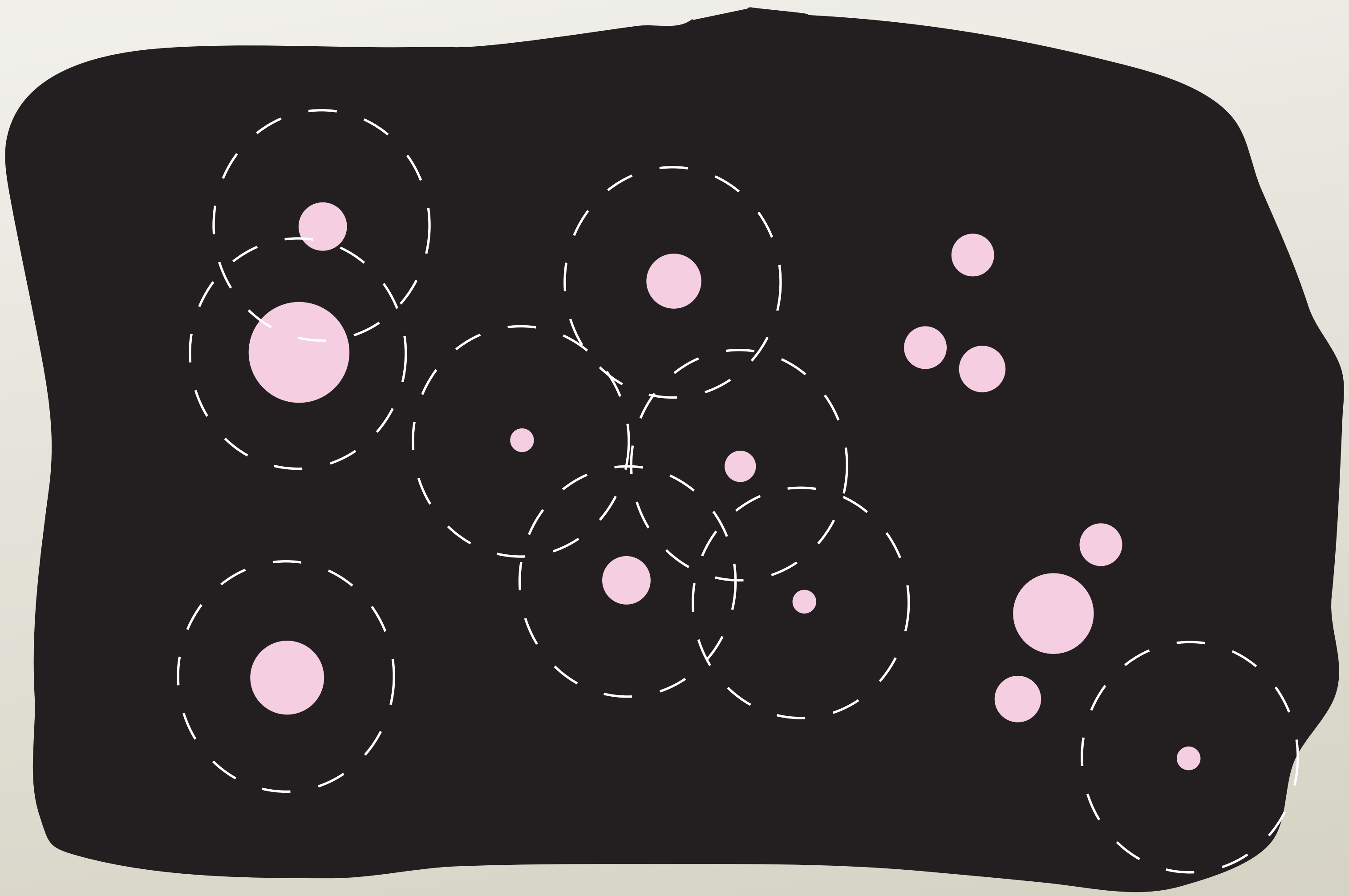
Then they get hungry for a real breakfast, and reach further and for bigger jellyfish.

By now, several jellyfish are out of reach from further predation.



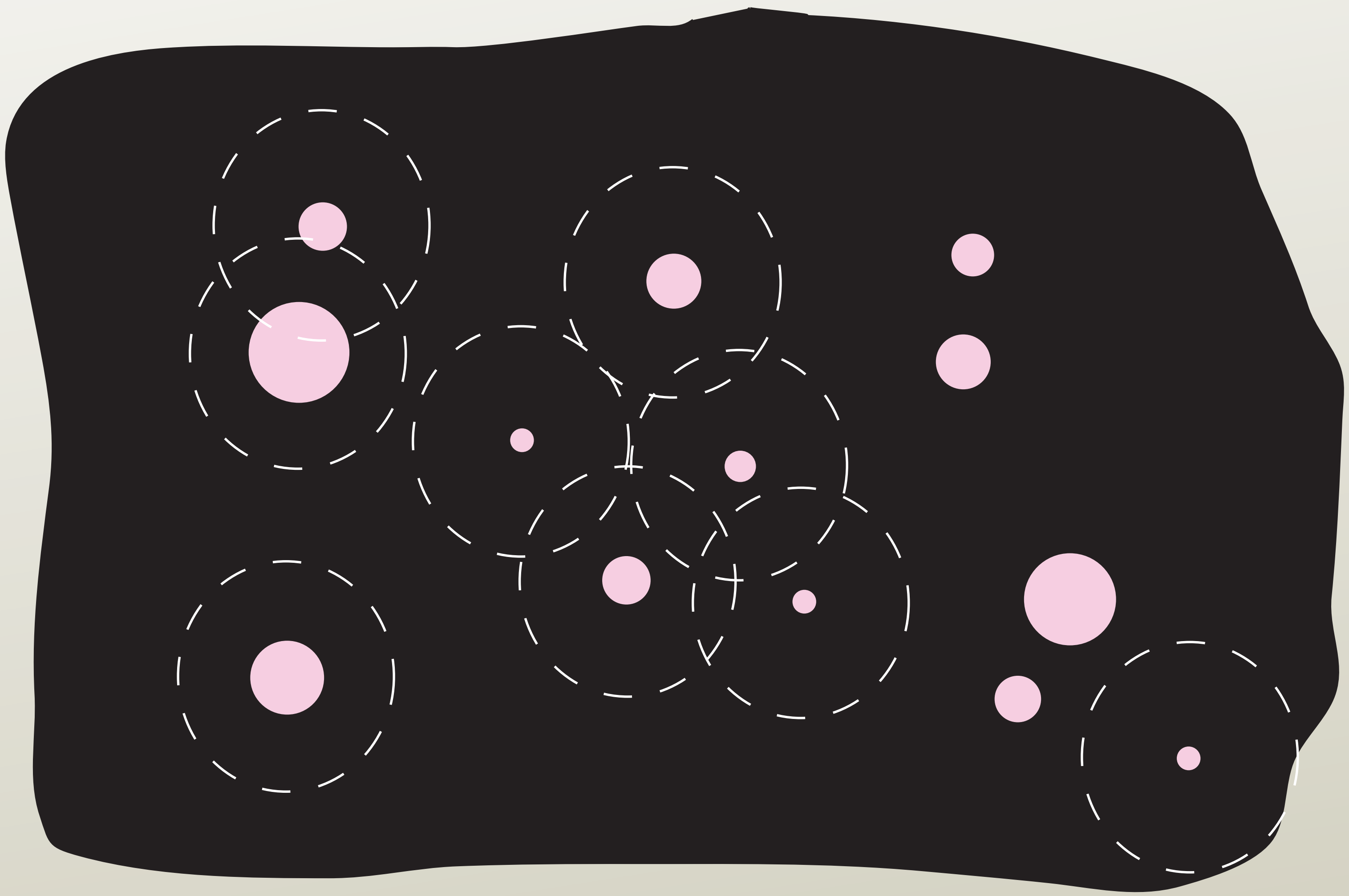
For lunch, they start reaching further and for bigger jellyfish.

Still, they are hungry...



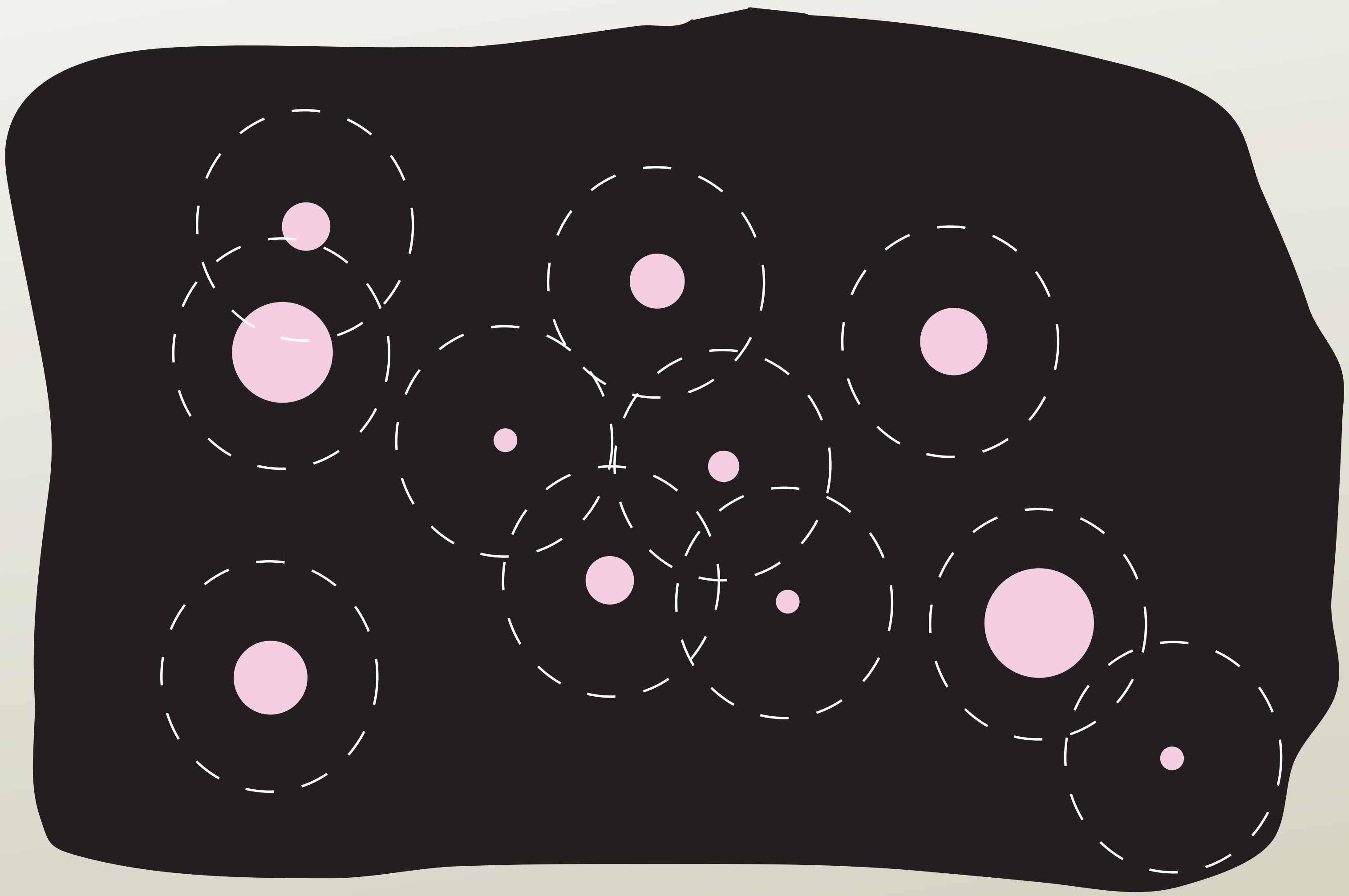
but only a few jellyfish are within reach.

Still, they are hungry...

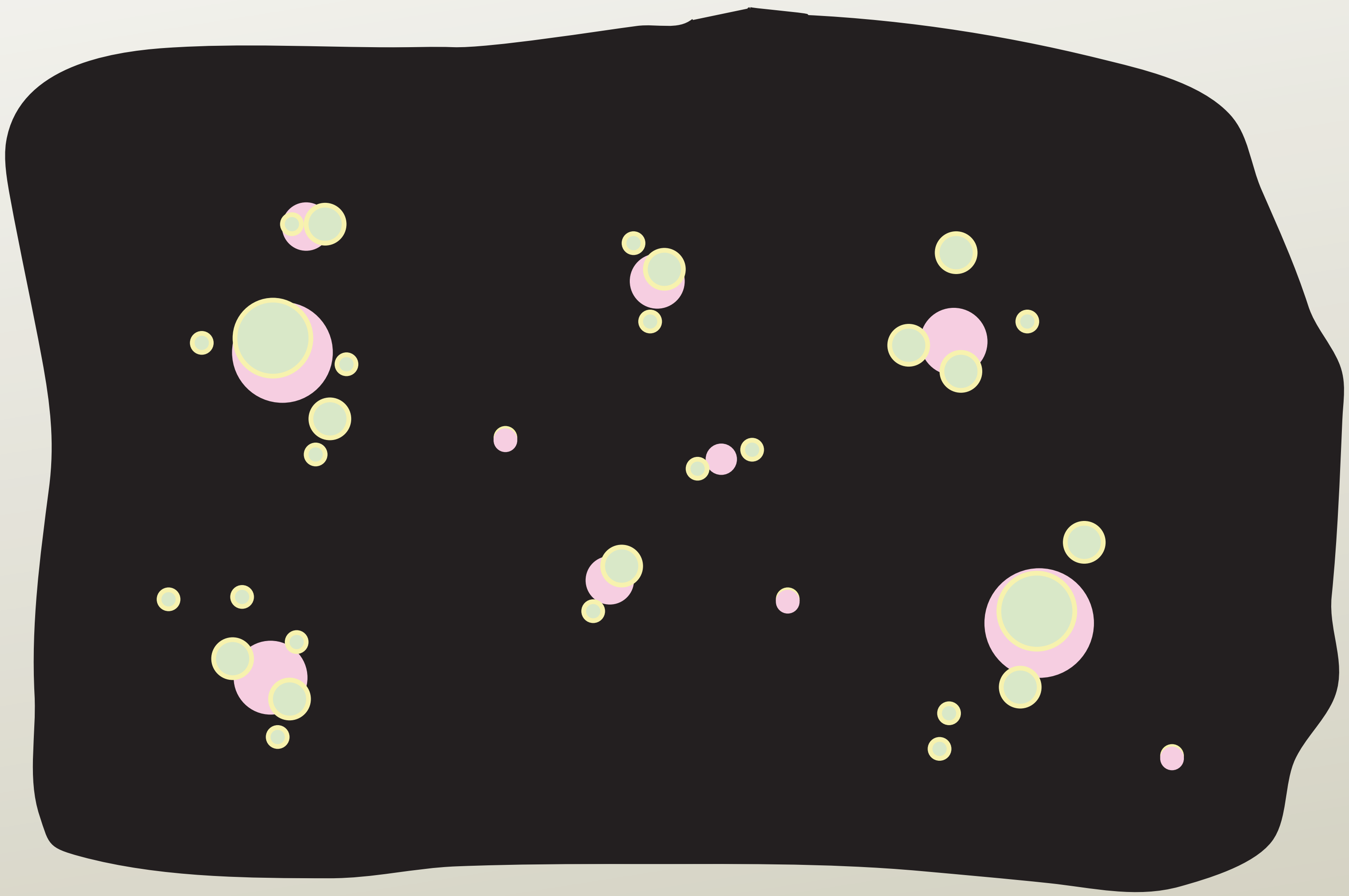


and one more feeding stage is possible.

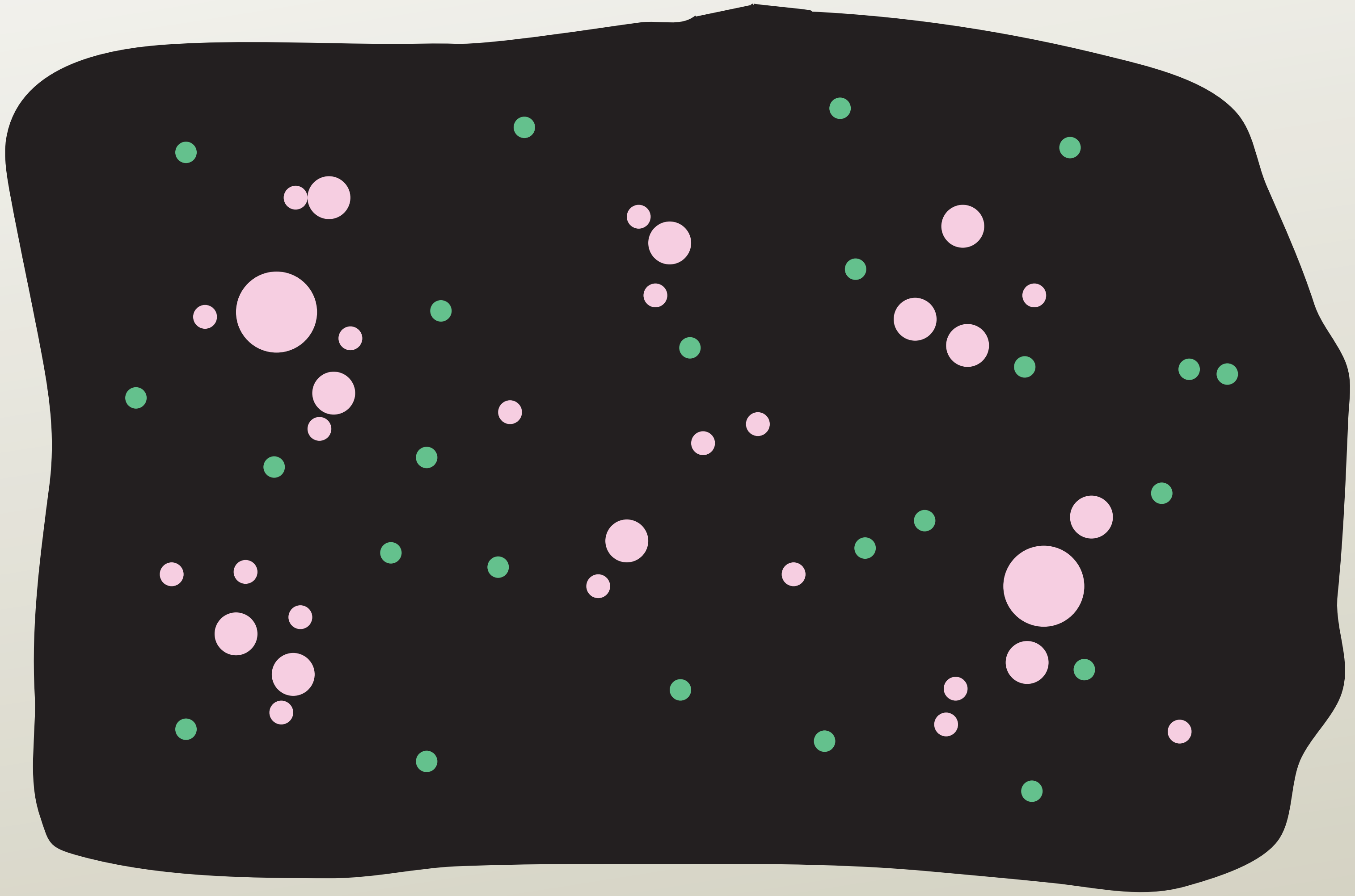
Feeding is over.



The shape of the feeding area for each final jellyfish is irregular.



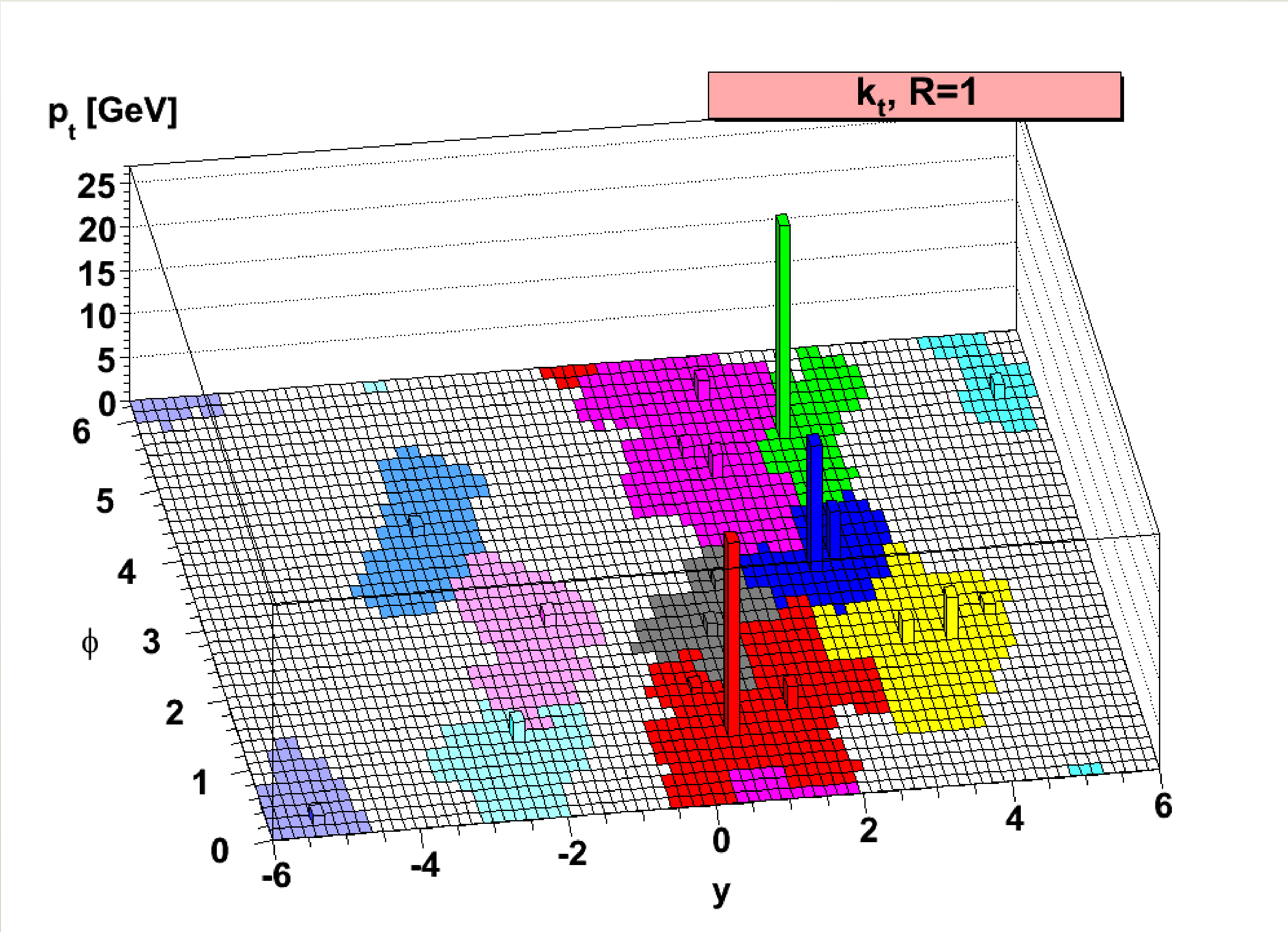
What would happen if there were lots of little green jellyfish also?



Is the final pattern much affected?

Example with k_T

- Here is the same example event from Cacciari, Salam, and Soyez (2008).
- With the k_T algorithm, we see what detector area goes into each jet.



The Cambridge-Aachen algorithm

- This is a variation on the general successive combination plan.
- Use

$$d_{ij} = [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

$$d_i = 1$$

- Keep everything else the same.

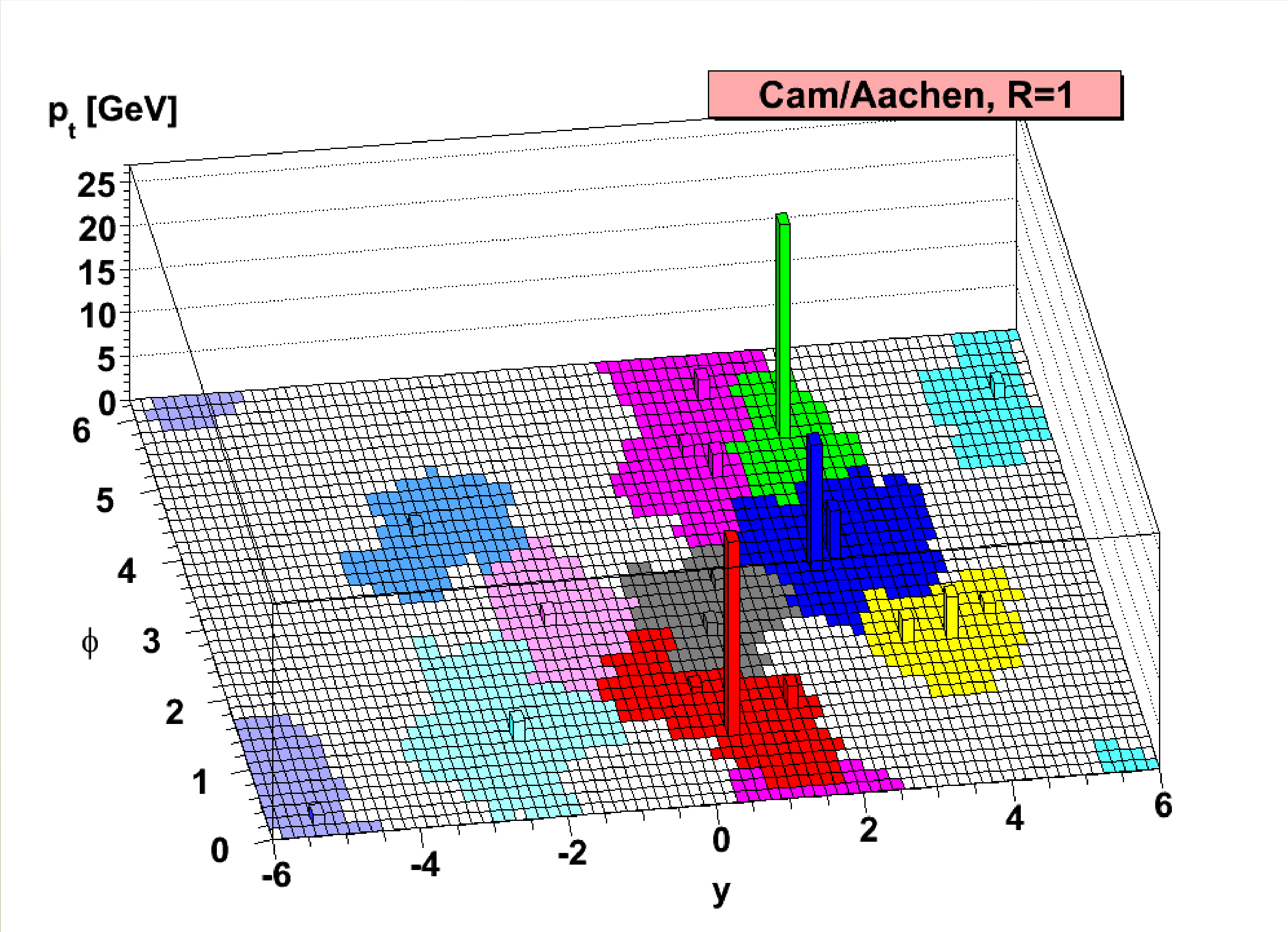
$$d_{ij} = [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

$$d_i = 1$$

- Thus we merge the protojets that are closest in angle first.
- We stop at *angle* = *R*.
- This is a little like the inverse of a HERWIG shower.

Example with *C-A*

- Here is the same example event from Cacciari, Salam, and Soyez (2008).
- With the Cambridge-Aachen algorithm, we see what detector area goes into each jet.



The anti-k_T algorithm

- This is another variation on the general successive combination plan.
- Use

$$d_{ij} = \min\left(\frac{1}{p_{T,i}^2}, \frac{1}{p_{T,j}^2}\right) [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

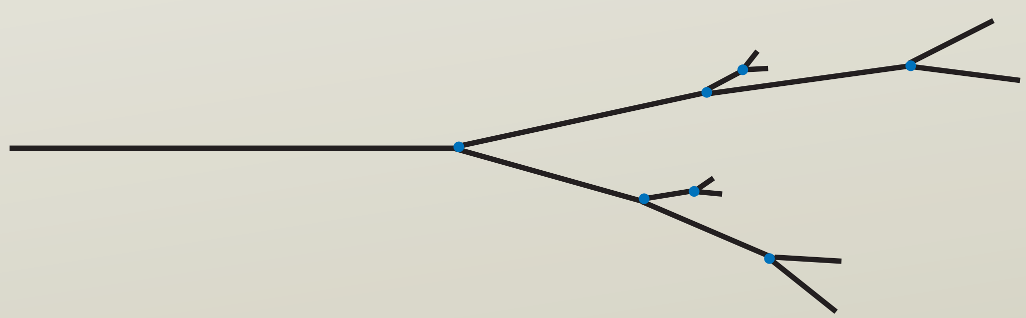
$$d_i = \frac{1}{p_{T,i}^2}$$

- Keep everything else the same.

$$d_{ij} = \min\left(\frac{1}{p_{T,i}^2}, \frac{1}{p_{T,j}^2}\right) [(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2]/R^2$$

$$d_i = \frac{1}{p_{T,i}^2}$$

- This puts protojets together in an order that is nothing like the order that any shower Monte Carlo would generate splittings.

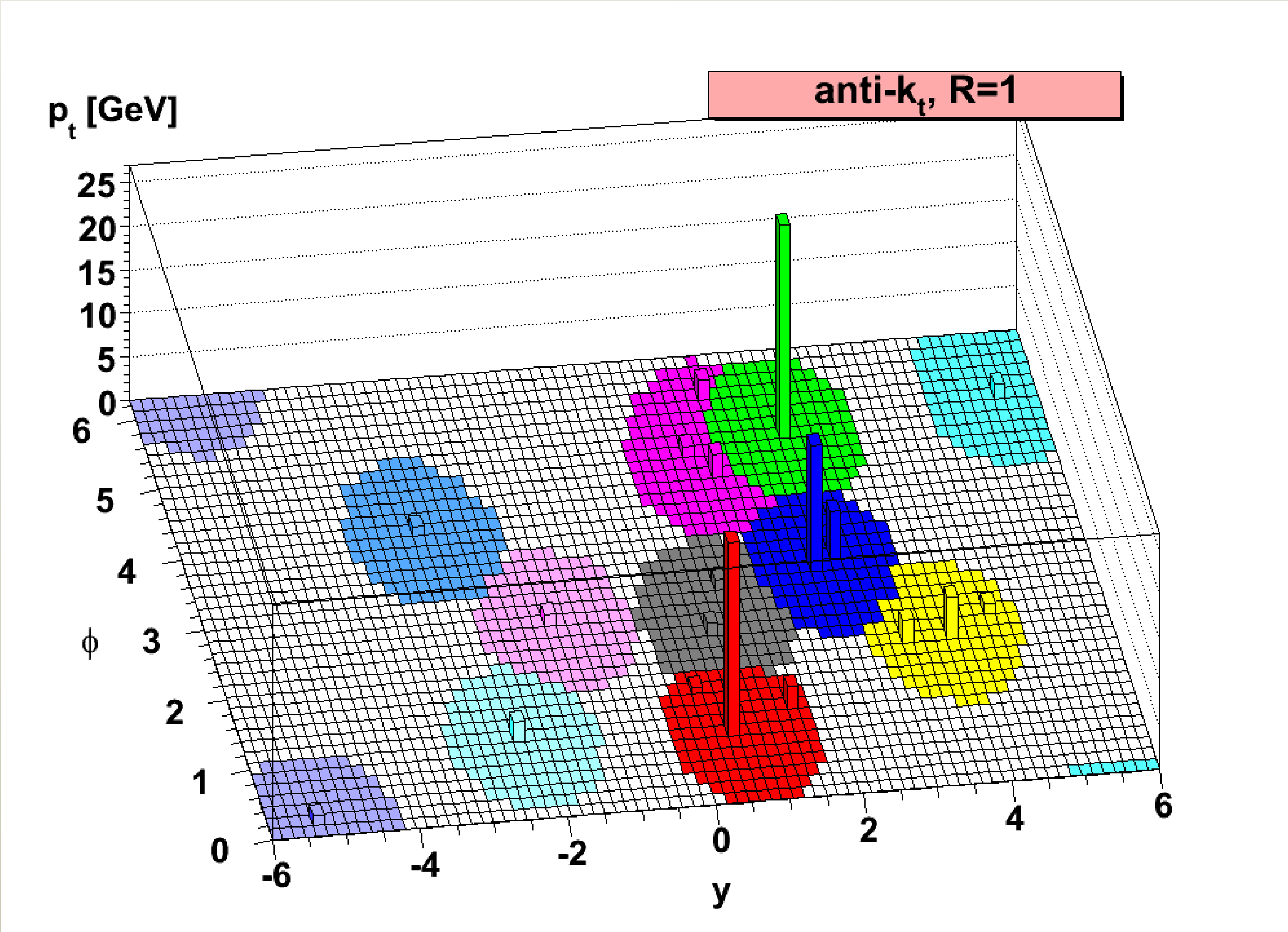


- The highest P_T protojet has priority to absorb nearby softer protojets (out to radius R).

Example with anti-k_T

- Here is the same example event from Cacciari, Salam, and Soyez (2008).

- With the anti-k_T algorithm, we see what detector area goes into each jet.



Conclusions

- To measure jet cross sections, you need a careful definition of a jet.
- The definition needs to be infrared safe.
- Definitions typically use an angular size parameter *R*.
- The conceptually simplest kind of definition successively combine small protojets into bigger ones.