Introduction to the Parton Model and Pertrubative QCD

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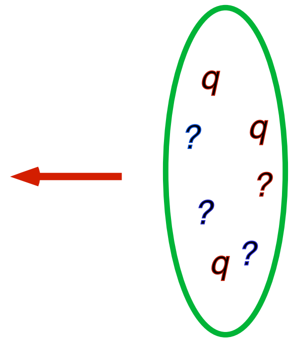
II. From the Parton Model to QCD

A. Color and QCD

B. Field Theory Essentials

C. Infrared Safety and Jets

IIA. From Color to QCD



- **Enter the Gluon**
- **If $\phi_{q/H}(x)$ = probability to find q with momentum xp,**
- **then,**

$$M_q = \sum_q \int_0^1 dx\ x\ \phi_{q/H}(x) \ = \ \text{total fraction of momentum carried by quarks.}$$

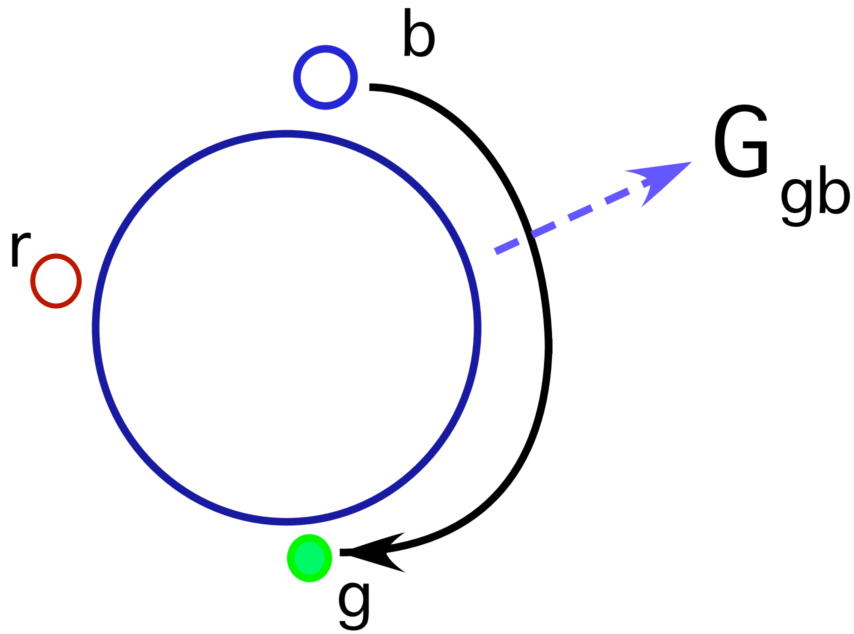
- **Experiment gave**

$$M_q \sim 1/2$$

- **What else? Quanta of force field that holds H together?**
- **'Gluons' – but what are they?**

- Where color comes from.
- Quark model problem:
  - $s_q = 1/2 \Rightarrow$ fermion $\Rightarrow$ antisymmetric wave function, but
  - (uud) state symmetric in spin/isospin combination for nucleons and
  - Expect the lowest-lying $\psi(\vec{x}_m, \vec{x}'_u, \vec{x}_d)$ to be symmetric
  - So where is the antisymmetry?
- Solution: Han Nambu, Greenberg, 1968: Color
- b, g, r, a new quantum number.
- Here's the antisymmetry: $\epsilon_{ijk}\psi(\vec{x}_u, \vec{x}'_u, \vec{x}_d)$, (i,j,k)= (b,g,r)

- **Quantum Chromodynamics: Dynamics of Color**
- **A globe with no north pole**



- **Position on 'hyperglobe' $\leftrightarrow$ phase of wave function**

  (Yang & Mills, 1954)
- **We can change the globe's axes at different points in space-time, and 'local rotation' $\leftrightarrow$ emission of a gluon.**
- **QCD: gluons coupled to the color of quarks**

  (Gross & Wilczek; Weinberg; Fritzsch, Gell-Mann, Leutwyler, 1973)

IIB. Field Theory Essentials

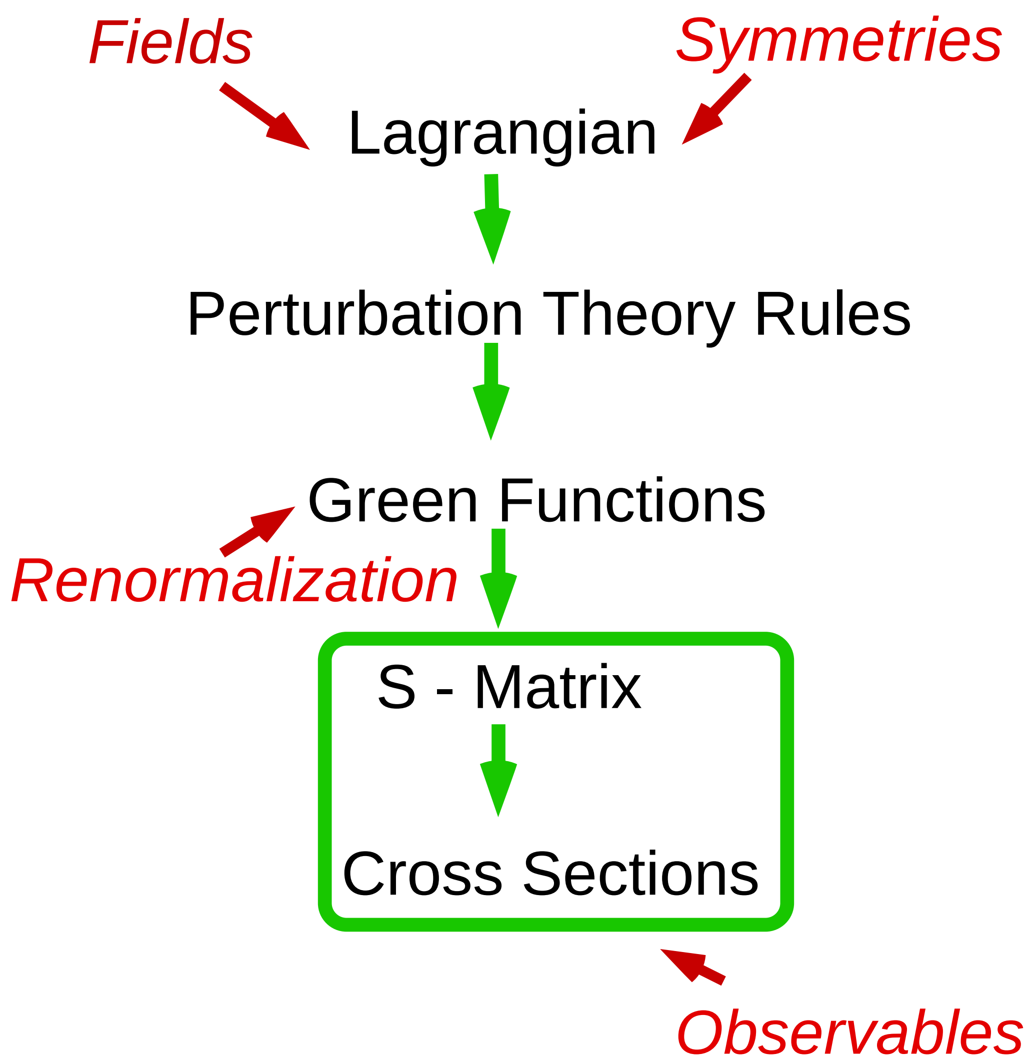
- **Fields and Lagrange Density for QCD**
- $q_f(x)$, $f = u, d, c, s, t, b$: **Dirac fermions (like electron) but extra** $(i, j, k) = (b, g, r)$ **quantum number.**
- $A^{\mu}_{a}(x)$ **Vector field (like photon) but with extra** $a \sim (g\bar{b}\ldots)$ **quantum no. (octet).**
- $\mathcal{L}$ **specifies quark-gluon, gluon-gluon propagators and interactions.**

$$\mathcal{L} = \sum_f \bar{q}_f \left(\left[i\partial_\mu - g A_{\mu a} T_a \right] \gamma^\mu - m_f \right) q_f - \frac{1}{4} \left(\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} \right)^2$$

$$- \frac{g}{2} \left(\partial_\mu A_{\nu a} - \partial_\nu A_{\mu a} \right) C_{abc} A^{\mu}_{b} A^{\nu}_{c}$$

$$- \frac{g^2}{4} C_{abc} A^{\mu}_{b} A^{\nu}_{c} C_{ade} A_{\mu d} A_{\nu e}$$

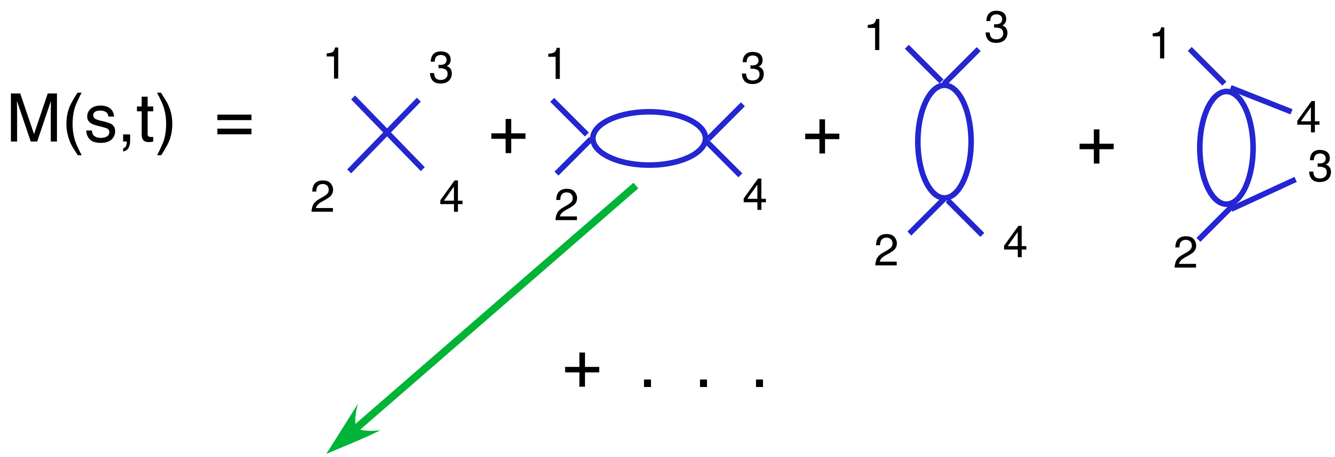
From a Lagrange density to observables, the pattern:



- **UV Divergences (toward renormalization & the renormalization group)**
- **Use as an example**

$$\mathcal{L}_{\phi^4} = \frac{1}{2}\left((\partial_\mu \phi)^2 - m^2\phi^2\right) - \frac{\lambda}{4!}\phi^4$$

- **The "four-point Green function":**

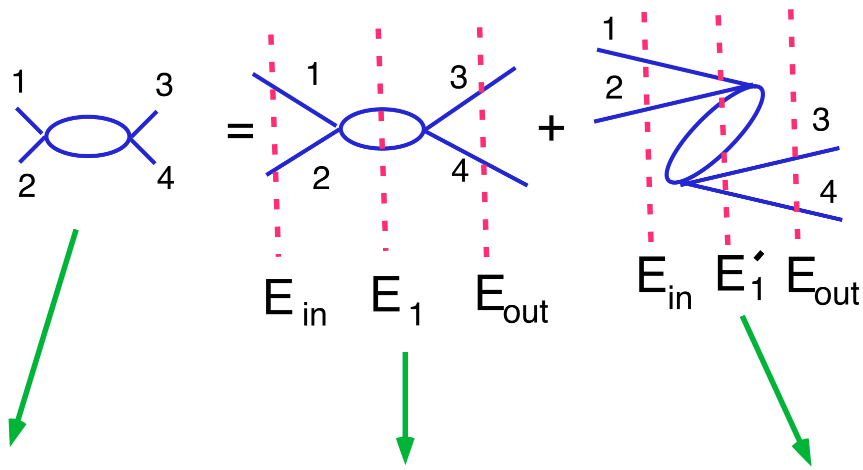


$$\int^\infty \frac{d^4k}{(k^2 - m^2)((p_1 + p_2 - k)^2 - m^2)} \sim \int^\infty \frac{d^4k}{(k^2)^2} \Rightarrow \infty$$

Interpretation: The UV divergence is due entirely states of high 'energy deficit',

$$E_{\text{in}} - E_{\text{state S}} = p_1^0 + p_2^0 - \sum_{i\,\in S} \sqrt{\vec{k}_i^{\,2} - m^2}$$

Made explicit in Time-ordered Perturbation Theory:

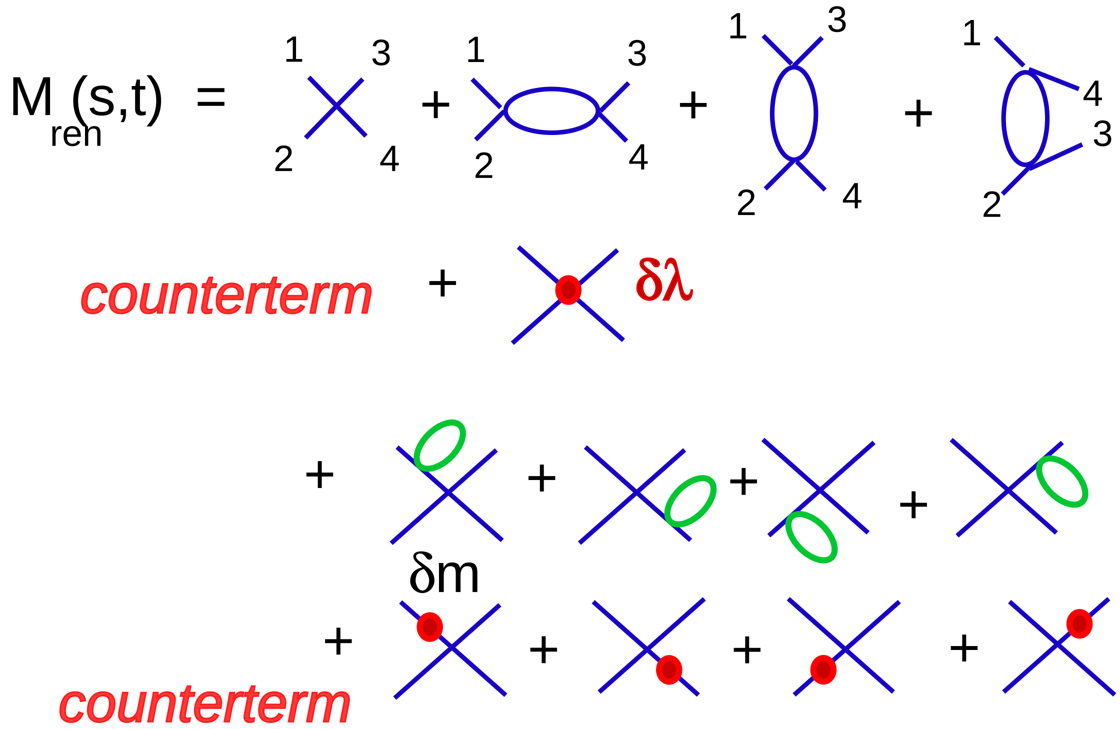


$$\int^{\infty} \frac{d^4k}{(k^2 - m^2)((p_1 + p_2 - k)^2 - m^2)} = \sum_{states} \left[\frac{1}{E_{\text{in}} - E_1} + \frac{1}{E_{\text{in}} - E_1'} \right]$$

Analogy to uncertainty principle $\Delta E \to \infty \Leftrightarrow \Delta t \to 0.$

- **This suggests: UV divergences are 'local' and can be absorbed into the local Lagrange density. Renormalization.**
- **For our full 4-point Green function, two new "counterterms":**

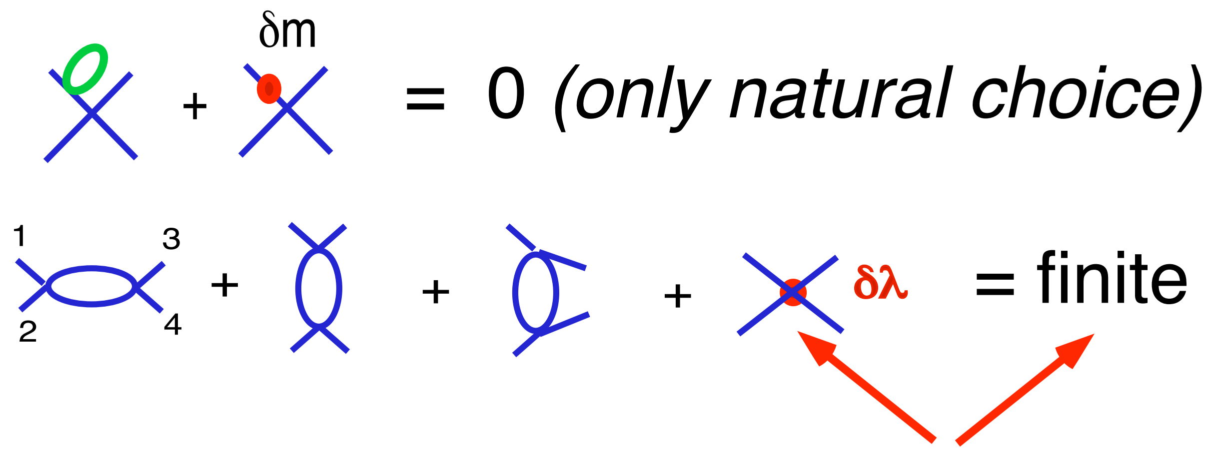
The renormalized 4-point function:



- **The combination is supposed to be finite.**

- **How to choose them? This is the renormalization “scheme”**

Renormalization:



But what should we choose for these?

A B C D

- **For example: define A+B+C by cutting off $\int d^4k$ at $k^2 = \Lambda^2$ (regularization). Then**

$$A + B + C = a \ln \frac{\Lambda^2}{s} + b(s, t, u, m^2)$$

- Now choose:

$$D = -\, a \ln \frac{\Lambda^2}{\mu^2}$$

so that

$$A + B + C + D = a \ln \frac{\mu^2}{s} + b(s, t, u, m^2)$$

independent of Λ.

- Criterion for choosing μ is a "renormalization scheme":
  MOM scheme: $\mu = s_0$, some point in momentum space.
  MS scheme: same μ for all diagrams, momenta
- But the value of μ is still arbitrary. $\mu =$ renormalization scale.
- Modern view (Wilson) We hide our ignorance of the true high-E behavior.
- All current theories are "effective" theories with the same low-energy behavior as the true theory, whatever it may be.

- μ-dependence is the price we pay for working with an effective theory: The Renormalization Group
- As μ changes, mass m and coupling g have to change: $m = m(\mu)\ g = g(\mu)$ "*renormalized*" but . . .
- Physical quantities can't depend on μ:

$$\mu \frac{d}{d\mu} \sigma \left(\frac{s_{ij}}{\mu^2}, \frac{m^2}{\mu^2}, g(\mu), \mu \right) = 0$$

- The 'group' is just the set of all changes in μ.
- 'RG' equation (Mass dimension $[\sigma] = d_\sigma$):

$$\left(\mu \frac{\partial}{\partial \mu} + \mu \frac{\partial g}{\partial \mu} \frac{\partial}{\partial g} + \mu \frac{\partial m}{\partial \mu} \frac{\partial}{\partial m} + d_\sigma \right) \sigma \left(\frac{s_{ij}}{\mu^2}, \frac{m^2}{\mu^2}, g(\mu), \mu \right) = 0$$

$$\text{The beta function}: \quad \beta(g) \equiv \mu \frac{\partial g(\mu)}{\partial \mu}$$

- **The Running coupling**
- **Consider <u>any</u> σ ($m = 0$, $d_\sigma = 0$) with kinematic invariants $s_{ij} = (p_i + p_j)^2$:**

$$\mu \frac{d\sigma}{d\mu} = 0 \quad \rightarrow \quad \mu \frac{\partial \sigma}{\partial \mu} = -\beta(g) \frac{\partial \sigma}{\partial g} \tag{1}$$

- **in PT:**

$$\sigma = g^2(\mu)\sigma^{(1)} + g^4(\mu)\left[\sigma^{(2)}\left(\frac{s_{ij}}{s_{kl}}\right) + \tau^{(2)} \ln \frac{s_{12}}{\mu^2}\right] + \ldots \tag{2}$$

- **(2) in (1) $\rightarrow$**

$$\begin{aligned} g^4 \tau^{(2)} &= 2g\sigma^{(1)}\beta(g) + \ldots \\ \beta(g) &= \frac{g^3}{2}\frac{\tau^{(2)}}{\sigma^{(1)}} + \mathcal{O}(g^5) \equiv -\frac{g^3}{16\pi^2}\beta_0 + \mathcal{O}(g^5) \end{aligned}$$

- **In QCD:**

$$\beta_0 = 11 - \frac{2n_f}{3}$$

- $-\beta_0 < 0 \rightarrow g$ **decreases as** μ **increases.**

- **Asymptotic Freedom**: Solution for the QCD coupling

$$\mu \frac{\partial g}{\partial \mu} = -g^3 \frac{\beta_0}{16\pi^2}$$

$$\frac{dg}{g^3} = -\frac{\beta_0}{16\pi^2} \frac{d\mu}{\mu}$$

$$\frac{1}{g^2(\mu_2)} - \frac{1}{g^2(\mu_1)} = -\frac{\beta_0}{16\pi^2} \ln \frac{\mu_2}{\mu_1}$$

$$g^2(\mu_2) = \frac{g^2(\mu_1)}{1 + \frac{\beta_0}{16\pi^2} g^2(\mu_1) \ln \frac{\mu_2}{\mu_1}}$$

- Vanishes for $\mu_2 \to \infty$. Equivalently,

$$\alpha_s(\mu_2^2) \equiv \frac{g^2(\mu_2^2)}{4\pi} = \frac{\alpha_s(\mu_1)}{1 + \frac{\beta_0}{4\pi} \alpha_s(\mu_1) \ln \frac{\mu_2}{\mu_1}}$$

- **Dimensional transmutation:** $\Lambda_{\rm QCD}$
  - **Two mass scales appear in**

$$\alpha_s(\mu_2^2) = \frac{\alpha_s(\mu_1)}{1 + \frac{\beta_0}{4\pi}\alpha_s(\mu_1)\ln\frac{\mu_2}{\mu_1}}$$

but the value of $\alpha_s(\mu_2)$ can't depend on choice of μ_1.

  - **Reduce it to one by defining $\Lambda \equiv \mu_1\, e^{-\beta_0/\alpha_s(\mu_1)}$, independent of μ_1. Then**

$$\alpha_s(\mu_2^2) = \frac{4\pi}{\beta_0 \ln\frac{\mu_2}{\Lambda^2}}$$

- **Asymptotic freedom strongly suggests a relationship to the parton model, in which partons act as if free at short distances. But how to quantify this observation?**

IIC. Infrared Safety and Jets

- To use perturbation theory, would like to choose μ 'as large as possible to make $\alpha_s(\mu)$ as small as possible.
- But how small is possible?
- A "typical" cross section, , define $Q^2 = s_{12}$ and $x_{ij} = s_{ij}/Q^2$,

$$\sigma\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu), \mu\right) = \sum_{n=1}^{\infty} a_n\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}\right) \alpha_s^n(\mu)$$

with m_i^2 all fixed masses – external, quark, gluon (=0!)

- Generically, the a_n depend logarithmically on their arguments, so a choice of large μ results in large logs of m_i^2/μ^2.

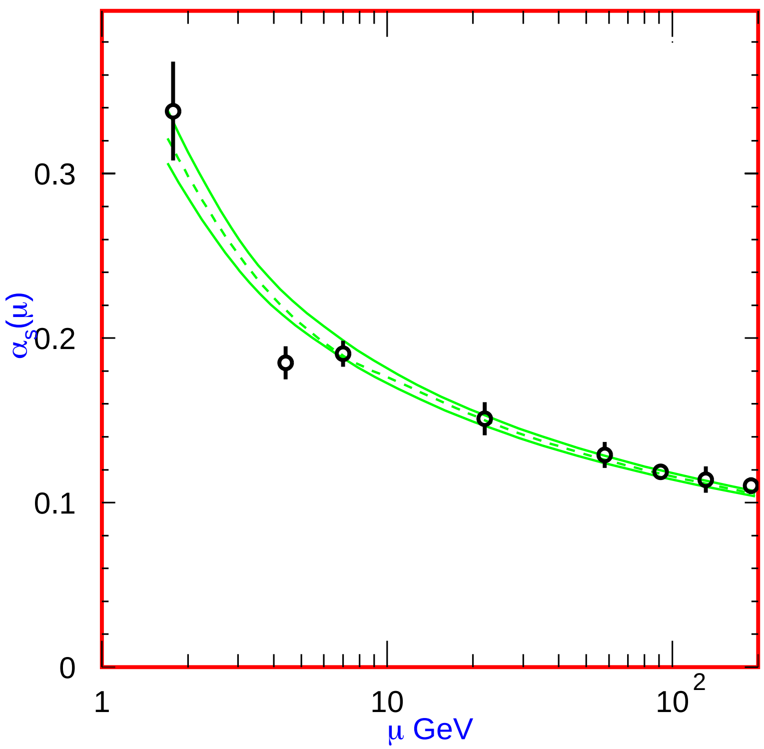
- But if we <u>could</u> find quantities that depend on $m_i's$ only through powers, $(m_i/\mu)^p, p > 0$, the large-μ limit would exist.

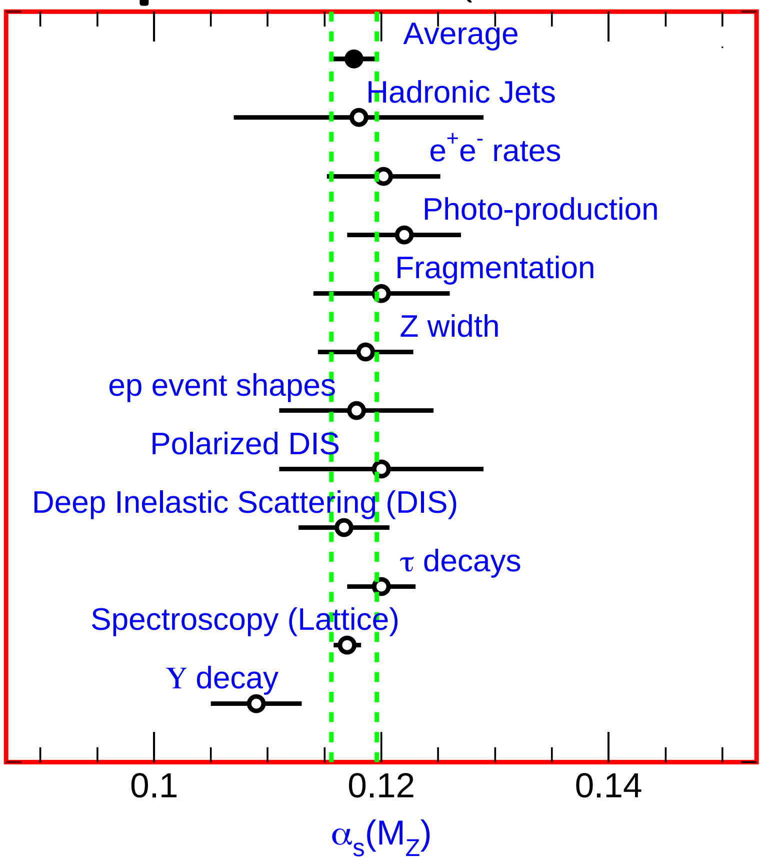
$$\sigma\left(\frac{Q^2}{\mu^2}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(Q), \mu\right) = \sigma\left(\frac{Q}{\mu}, x_{ij}, \frac{m_i^2}{\mu^2}, \alpha_s(\mu), \mu\right)$$

$$= \sum_{n=1}^{\infty} a_n\left(\frac{Q}{\mu}, x_{ij}\right) \alpha_s^n(\mu) + \mathcal{O}\left(\left[\frac{m_i^2}{\mu^2}\right]^p\right)$$

- Such quantities are called infrared (IR) safe.
- Measure $\sigma \rightarrow$ solve for α_s. Allows observation of the running coupling.
- Most pQCD is the computation of IR safe quantities.

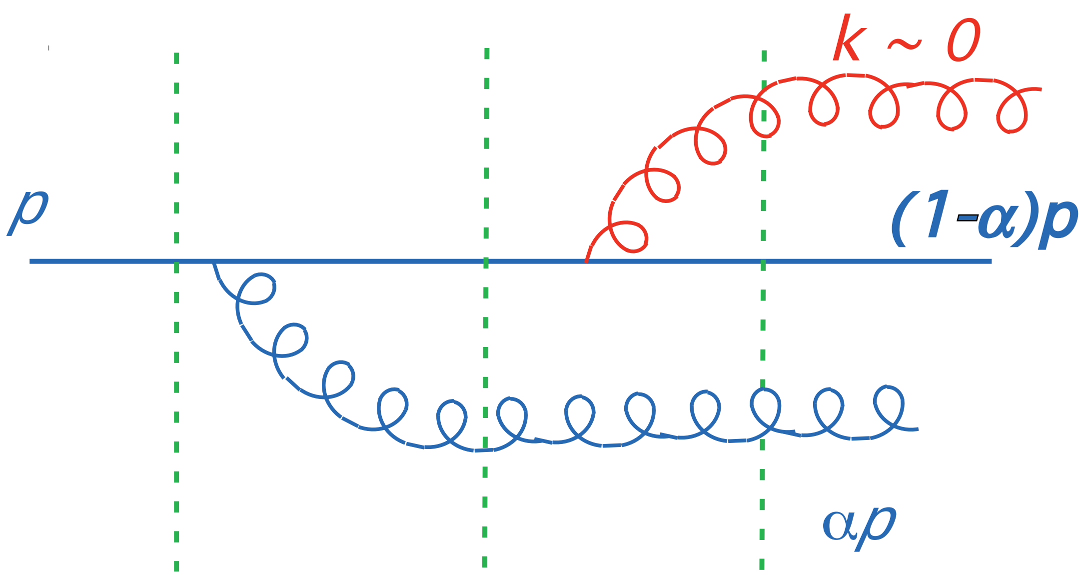
- **Consistency of $\alpha_s(\mu)$ found as above at various momentum scales**
  Each comes from identifying an IR safe quantity, computing it and comparing the result to experiment. **(Particle Data Group)**





- **To find IR safe quantities, need to understand where the low-mass logs come from.**

- To analyze diagrams, we generally think of $m \to 0$ limit in m/Q. Gives "IR" logs.
- Generic source of IR (soft and collinear) logarithms:



- IR logs come from degenerate states:
  Uncertainty principle $\Delta E \to 0 \Leftrightarrow \Delta t \to \infty$.
- For soft emission and collinear splitting it's "never too late".
  But these processes don't change the flow of energy ...
  Problems arise if we ask for particle content.

- **For IR safety, sum over degenerate final states in perturbation theory, and don't ask how many particles of each kind we have. This requires us to introduce another regularization, this time for IR behavior.**

- **The IR regulated theory is like QCD at short distances, but is better-behaved at long distances.**

- **IR-regulated QCD not the same as QCD except for IR safe quantities.**

- See how it works for the total $\mathrm{e}^+\mathrm{e}^-$ annihilation cross section to order α_s. Lowest order is $2 \to 2, \sigma_2^{(0)} \equiv \sigma_{\mathrm{LO}}$, σ_3 starts at order α_s.
  - Gluon mass regularization: $1/k^2 \to 1/(k^2 - m_G)^2$

$$\sigma_3^{(m_G)} = \sigma_{\mathrm{LO}} \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} - \frac{\pi^2}{6} + \frac{5}{2}\right)$$

$$\sigma_2^{(m_G)} = \sigma_{\mathrm{LO}}\left[1 - \frac{4}{3}\frac{\alpha_s}{\pi}\left(2\ln^2\frac{Q}{m_g} - 3\ln\frac{Q}{m_g} - \frac{\pi^2}{6} + \frac{7}{4}\right)\right]$$

which gives

$$\sigma_{\mathrm{tot}} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_{\mathrm{LO}}\left[1 + \frac{\alpha_s}{\pi}\right]$$

  - Pretty simple! (Cancellation of virtual (σ_2) and real (σ_3) gluon diagrams.)

– Dimensional regularization: change the area of a sphere of radius R from $4\pi R^2$ to $(4\pi)^{(1-\varepsilon)}\frac{\Gamma(1-\varepsilon)}{\Gamma(2(1-\varepsilon))}R^{2-2\varepsilon}$ with $\varepsilon = 2 - D/2$ in D dimensions.

$$\sigma_3^{(\varepsilon)} = \sigma_{\rm LO}\frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \times\left(\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}-\frac{\pi^2}{2}+\frac{19}{4}\right)$$

$$\sigma_2^{(\varepsilon)} = \sigma_{\rm LO}\,[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(\frac{(1-\varepsilon)^2}{(3-2\varepsilon)\Gamma(2-2\varepsilon)}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon} \times\left(\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}-\frac{\pi^2}{2}+4\right)]$$

which gives again

$$\sigma_{\rm tot} = \sigma_2^{(m_G)} + \sigma_3^{(m_G)} = \sigma_0\left[1+\frac{\alpha_s}{\pi}\right]$$

- This illustrates IR Safety: σ_2 and σ_3 depend on regulator, but their sum does not.

- **Generalized IR safety: sum over all states with the same flow of energy into the final state. Introduce** *IR safe weight* "$e(\{p_i\})$"

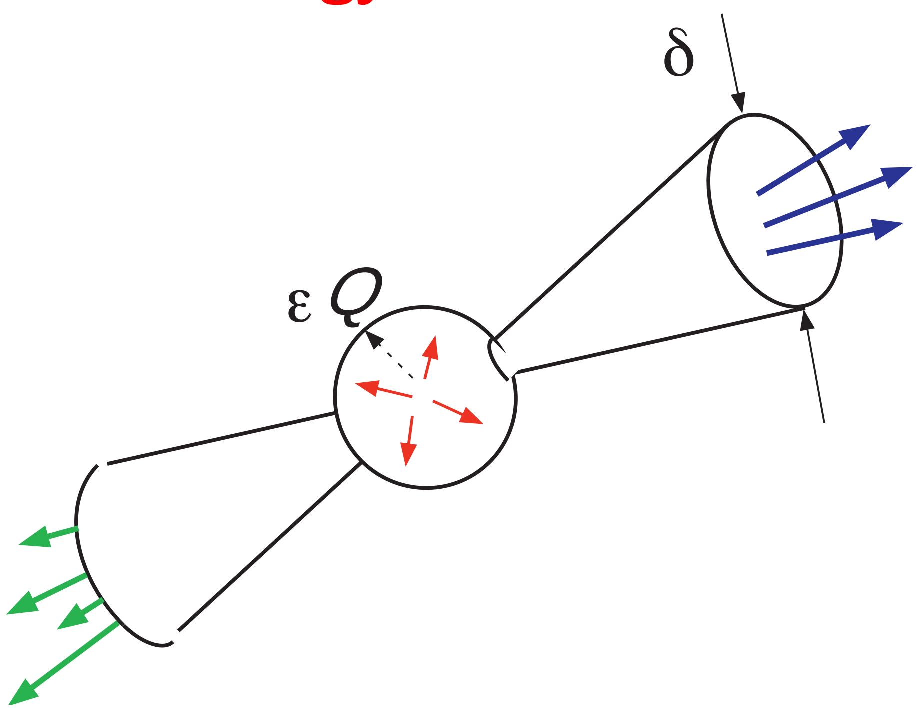
$$\frac{d\sigma}{de} = \sum_n \int_{PS(n)} |M(\{p_i\})|^2 \delta\left(e(\{p_i\}) - w\right)$$

with

$$e(\dots p_i \dots p_{j-1}, \alpha p_i, p_{j+1} \dots) = e(\dots (1+\alpha) p_i \dots p_{j-1}, p_{j+1} \dots)$$

- **Neglect long times in the initial state for the moment and see how this works in** e^+e^- **annihilation: event shapes and jet cross sections.**

- **"Seeing" Quarks and Gluons With Jet Cross Sections**
- **Simplest example: cone jets in $\mathrm{e}^{+}\mathrm{e}^{-}$ annihilation. All but fraction ϵ of energy flows into cones of size δ.**



- **Intuition: eliminating long-time behavior $\Leftrightarrow$ recognize the impossibility of resolving collinear splitting/recombination of massless particles**

- No factors Q/m or $\ln(Q/m)$ **Infrared Safety**.

- In this case,

$$\sigma_{2J}(Q,\delta,\epsilon) = \frac{3}{8}\sigma_0(1+\cos^2\theta) \times \left(1 - \frac{4\alpha_s}{\pi}\left[4\ln\delta\ln\epsilon + 3\ln\delta + \frac{\pi^2}{3} + \frac{5}{2}\right]\right)$$

- Perfect for QCD: asymptotic freedom $\rightarrow d\alpha_s(Q)/dQ < 0$.
- No unique jet definition. $\leftrightarrow$ Each event a sum of possible histories.
- Relation to quarks and gluons always approximate but corrections to the approximation computable.

- The general form of an $\mathrm{e}^+\mathrm{e}^-$ annihilation jet cross section:

$$\sigma_{\text{jet}} = \sigma_0 \sum_{n=0}^{\infty} c_n(y_i, N, C_F)\alpha_s^n(Q)$$

- Dimensionless variables y_i include direction and information about the 'size' and 'shape' of the jet:
- δ, cone size as above
- To specify the jet direction, may use a Shape variable, e.g. thrust

$$T = \frac{1}{s}\max_{\hat{n}} \sum_i |\hat{n} \cdot \vec{p}_i| = \frac{1}{s}\max_{\hat{n}} \sum_i E_i \,|\cos\theta_i|$$

with θ_i the angle of particle i to the "thust" axis, which we can define as a jet axis.

- $T = 1$ for "back-to-back" jets.

$$T = \frac{1}{s}\max_{\hat{n}} \sum_i E_i \,|\cos\theta_i|$$

- The thrust is IR safe precisely because it is insensitive to collinear emission (split energy at fixed θ_i) and soft emission ($E_i = 0$).

- Once jet direction is fixed, we can generalize thrust to any smooth weight function:

$$\tau[f] = \sum_{\text{particles } i \text{ in jets}} E_i \, f(\theta_i)$$

- Using thrust to define a jet axis is useful mostly to describe two, back-to-back, jets (no wide-angle gluon emission – the majority, but by no means all events in $\mathrm{e}^+\mathrm{e}^-$).

- **The distribution as seen at high energies, compared to experiment** (Davison & Webber, 0809)**:**

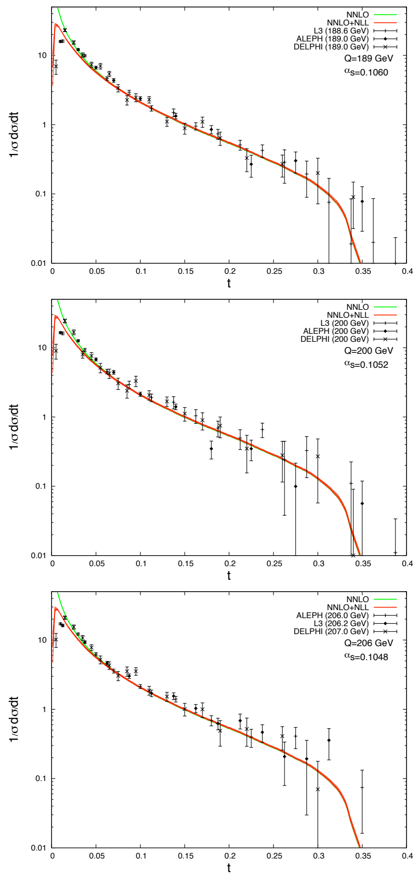


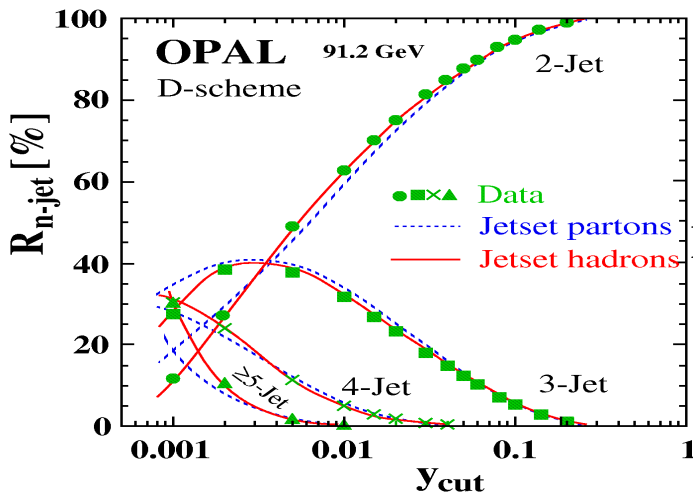
Fig. 3. Fixed-order (NNLO), resummed (NNLO+NLL) and experimental thrust distributions: $Q = 189 - 207$ GeV.

- **Strongly peaked near, but not at, $T = 1$, due to radiation.**

- For possibly multi-jet events, "cluster algorithms".
- $y_{\rm cut}$ Cluster Algorithm: Combine particles i and j into jets until all $y_{ij} > y_{\rm cut}$, where (e.g., "Durham alogrithm" for e^+e^-):

$$y_{ij} = 2\min\left(E_i^2, E_j^2\right)\left(1 - \cos\theta_{ij}\right)$$

- The number of jets depends on the variable $y_{\rm cut}$, and the dependence on the number of jets was an early application of jet physics. (Reproduced from Ali & Kramer, 1012)



- **To anticipate: for hadronic collisions, jets are only well-defined away from the beam axis, so (instead of energy, E_i) use kinematic variables defined by the beam directions:**
transverse momentum, azimuthal angle and rapidity:

$$k_t$$
$$\phi$$
$$y = \frac{1}{2} \ln \left(\frac{E + p_3}{E - p_3} \right)$$

- **The beams define the '3-axis'.**

- **Cluster variables for hadronic collisions:**

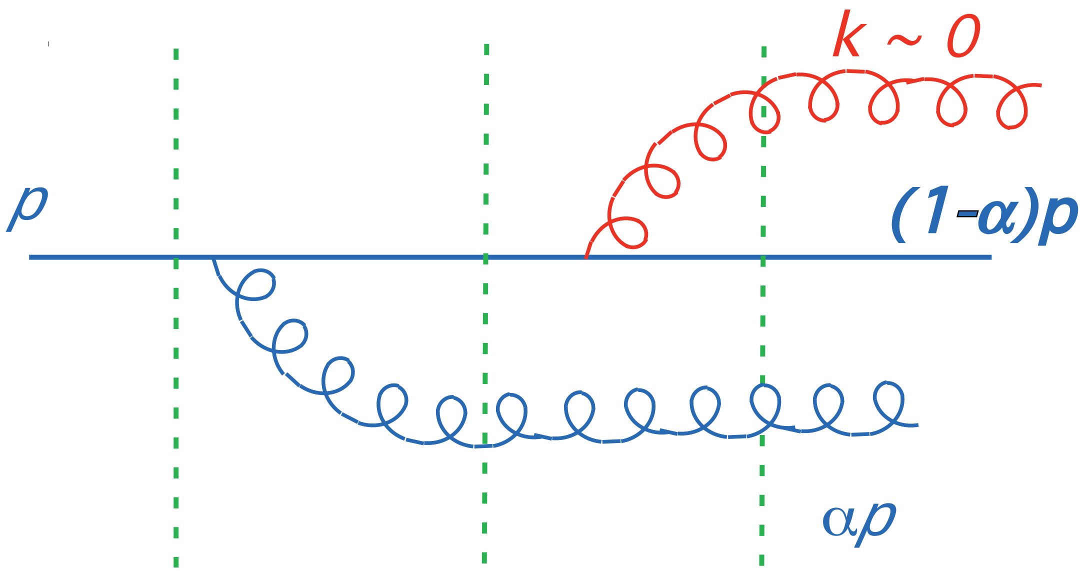
$$d_{ij} = \min\left(k_{ti}^{2p}\,,\,k_{tj}^{2p}\right)\frac{\Delta_{ij}^2}{R^2}$$

$\Delta_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$. R **is an adjustable parameter.**

- **The "classic" choices:**
  - $p = 1$ **"k_t algorithm:**
  - $p = 0$ **"Cambridge/Aachen"**
  - $p = -1$ **"anti-k_t"**

Summarize: what makes a cross section infrared safe?

- **Independence of long-time interactions:**



More specifically: should depend on only the flow of energy into the final state. This implies independence of collinear re-arrangements and soft parton emisssion.

But if we prepare one or two particles in the initial state (as in DIS or proton-proton scattering), we will always be sensitive to long time behavior inside these particles. The parton model suggests what to do: factorize. This is the subject of Part III.